ENCYCLOPÉDIE OU *DICTIONNAIRE* RAISONNÉ DES SCIENCES, DES ARTS ET DES MÉTIERS

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LOTERIE

LOTTERY, (*Arithmetic*.) a kind of game of chance in which different shares of goods or different sums of money are deposited in order to form some prizes & some benefits to those to whom the favorable tickets fall. The object of *lotteries* & the manner of drawing them, are some things too common in order that we stop ourselves here. Our French *lotteries* commonly have for object to raise some funds destined to some pious works or to some need of the state; but *lotteries* are very frequent in England & in Holland, where one can make them only by permission of a magistrate.

M. Leclerc has composed a treatise on *lotteries*, where he shows that they contain the laudable & blamable. Grégorio Leti also gave a work on the *lotteries*, & Father Menetrier has published in 1700 a treatise on the same subject, where he shows the origin of the *lotteries*, & their usage among the Romans; he distinguishes various kinds of *lotteries*, & thence takes occasion to speak of chances & to resolve many cases of conscience which have relationship there. *Chambers*.

Let there be a lottery of n tickets if which m is the price of the ticket, mn will be the money of all the *lottery*; & as this money never returns in total into the purse of the interested parties taken together, it is apparent that the *lottery* is always a disadvantageous game. For example, let there be a *lottery* of 10 tickets at 20 livres per ticket, & let there be only a share of 150 livres, the expectation of each interested party is only $\frac{150}{10}$ livre$=$ 15 livre & his stake is 20 livre therefore he loses a quarter of his stake, & could sell his expectation for only 15 livre. *See* **Jeu**, **Avantage**, **Probabilité**, &c.

In order to calculate in general the advantage or the disadvantage of any *lottery*, there is only to suppose that an individual takes to himself alone all the *lottery*, & to see the ratio of that which he has disbursed to that which he will receive: let m be the money disbursed, or the sum of the value of the tickets, & n the sum of the shares which is always less, it is apparent that the disadvantage of the *lottery* is

$$\frac{m-n}{m}$$

See **Avantage**, **Jeu**, **Pari**, **Probabilité**, &c.

If a *lottery* contains n tickets & m shares, one asks what probability there is that one has a share, if one takes r tickets. Take an example: one supposes in all 20 tickets, 15 shares, & consequently 15 tickets which must be drawn, & that one has taken 4 tickets: one will represent these 4 tickets by the first four letters of the alphabet, a, b, c, d, & the 20 tickets by the first twenty letters of same alphabet. It is clear, 1. that the question is

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reduced to knowing how many times 20 letters are able to be taken fifteen by fifteen; 2. what probability there is that one of the 4 tickets is found among the 15. Now the *article* **Combination** teaches that twenty things are able to be combined fifteen by fifteen in the number of times represented by a fraction of which the denominator is $1 \cdot 2 \cdot 3 \cdot 4$ &c. to 15. & the numerator $6 \cdot 7 \cdot 8 \ldots$ &c. to $6 + 14$ or 20. In regard to the second question, it is reduced to knowing how many times the 20 tickets (excepting the four a, b, c, d) can be taken fifteen by fifteen, that is to say how many times 16 tickets are able to be taken fifteen by fifteen, this which is expressed (*See the article* **Combination**) by a fraction of which the denominator is $1 \cdot 2 \cdot 3 \cdot 4$ &c. to 15. & the numerator $2 \cdot 3 \cdot 4$ &c. to $2 + 14$ or 16. Therefore the sought probability is in ratio to the first of these two fractions, less the second to the first; because the difference of the two fractions expresses evidently the number of cases where one of the tickets a, b, c, d, will leave the wheel. Therefore this probability is in ratio of $6 \cdot 7 \cdot 8 \cdots 20 - 2 \cdot 3 \cdot 4 \cdots 16$ to $6 \cdot 7 \cdot 8 \cdots 20$, that is to say of $17 \cdot 18 \cdot 19 \cdot 20 - 2 \cdot 3 \cdot 4 \cdot 5$ to $17 \cdot 18 \cdot 19 \cdot 20$.

Therefore in general the sought probability is expressed by the ratio of $(n - m + 1.n - m + 2 \ldots n) - (n - r - m + 1.n - r - m + 2 \ldots n - r)$ to $(n - m + 1.n - m + 2 \ldots n)$ Whence one sees that if $n - r - m + 1 = 0$ or is negative, one will play at gambling without risk. If, for example, in the previous case instead of 4 tickets one would take 6 of them, then one would have $n - r - m + 1 = 20 - 6 - 15 + 1 = 0$; & there would be certainty to have a share, this which is evident, since if of 20 tickets one takes 6 of them & if 15 of them must be drawn from the wheel, it is infallible that there will leave from it one of the 6, the others making together only 14. *See* **Jeu**, &c. (*M. d'Alembert*)