Review
of
*Théorie Analytic Probabilités**

No author indicated.

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THÉORIE ANALYTIQUE

DES PROBABILITÉS,

DEDICATED

A S. M. L'EMPEREUR ET ROI

By M. le comte LAPLACE,

Chancelier du Sénat-Conservateur, Grand-Officier de la Légion-d'Honneur, Member de l'Institut impérial, et du Bureau des Longitudes, etc.

We will be no better able to render account of this Work, than by transcribing the presentation that the author made of it at the beginning.

"I myself propose, he says, to give here the analysis and the principles necessary in order to resolve the problems concerning probabilities. This analysis is composed of two theories that I have given, thirty years ago, in the *Mémoires de l'Académie des Sciences*. The one of them is the *Théorie des Fonctions génératrices*: the other is the *Théorie de l'Approximation des formules fonctions de très-grands nombres*. They are the object of the first book, in which I present them in a manner yet more general than in the Memoirs cited. Their reconciliation shows with evidence, that the second is only an extension of the first, and that they are able to be considered as two branches of one same calculus, that I designate by the name *Calculus of generating Functions*. This calculus is the foundation of my *Théorie des Probabilités*, which makes the object of the second book. The questions relative to the events due to chance, are brought back most often with facility, to some linear equations in simple or

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partial differences: the first branch of the calculus of generating functions gives the most general method in order to integrate this kind of equations. But when the events that one considers are in great number, the expressions to which one is led, are composed of one so great multitude of terms and of factors, that their numerical calculation becomes impracticable; it is therefore then indispensable to have a method which transforms them into convergent series. This is that which the second branch of the calculus of the generating functions makes with so much more advantage, as the method becomes more necessary.

"My object being to present the methods and the general results of the theory of probabilities, I treat specially the most delicate, most difficult, and at the same time most useful questions of this theory. I am attached especially to determine the probabilities of the causes and of the results indicated by the events considered in large number, and to seek the laws according to which this probability approaches some limits in measure as the events are multiplied. This research merits the attention of the geometers, by the analysis that it requires: it is there principally that the theory of the approximation of formulas functions of great numbers, finds its most important applications. This research interests observers, by indicating to them the means that they must choose among the results of their observations, and the probability of the errors that they have yet to fear. Finally, it merits the attention of the philosophers, by showing how regularity completes by being established in the same things which would appear to us entirely delivered from chance, and by revealing the hidden, but constant, causes of them, on which this regularity depends. It is out of the regularity of the mean results of the events considered in great number, which raise diverse establishments, such as annuities, tontines, assurances, etc. The questions which are related to them, such as to the inoculation of vaccine and to the decisions of assemblies, offer no difficulty according to my theory. I limit myself here to resolve the most general; but the importance of these objects in civil life, the moral considerations of which they complicate themselves, and the numerous observations that they suppose, require a work apart.

"If one considers the analytic methods to which the series of probabilities has already given birth, and those that it is able to yet give birth; the justice of the principles which serve it at base; the rigorous and delicate logic which their use requires in the solution of the problems; the establishment of public utility which is supported on it: if one observes next that in the same things which are able to be submitted to the calculus, this theory gives the most certain outlines which are able to guide us in our judgments, and that it teaches to preserve oneself from the illusions which often mislead us; one will see that there is no science at all more worthy of our meditations, and of which the results are more useful. It owes birth to two French geometers of the seventeenth century, so fecund in great men and in great discoveries, and perhaps of all the centuries the one which gives the most honor to the human spirit. Pascal and Fermat themselves proposed and resolved some problems on the probabilities: Huygens reunited these solutions, and extended them in a small treatise on the same matter which next had been considered in a more general manner by Bernoulli, Montmort, Moivre, and by many celebrated geometers of these last times."

The work that we announce contains all that which has been done of importance on this branch of human knowledge, that the author appears to us to have perfected, either by the generality of his analysis, or by the novelty and the difficulty of the problems that he has resolved. Among these numerous problems, those which concern the means that it is appropriate to choose among the results of observations, in the same way the probability of phenomena, of their causes, and of future events, deduced from observed events, seems to us should fix particularly the attention of the geometers. After having exposed how the observations had often rebuked the analysts, by making them sense the necessity to rectify their observations, and as he is arrived himself by the considerations of the probabilities, to the great periodic and secular inequalities of the celestial movements; the author adds:

"One sees thence how it is necessary to be attentive to the indications of nature, when they are the result of a great number of observations, although besides, they appear inexplicable by the known means. I engage thus the astronomers, to follow with a particular attention, the lunar inequality in long period, which depends on the longitude of the perigee of the moon, added to the double of the longitude of its nodes; and that already the observations indicate with much likelihood. If the sequence of observations continues to verify it, it will force the geometers to return again onto the lunar theory, by making enter the consideration of the difference which is able to exist between the northern and southern hemispheres of the earth, a difference on which this inequality appears to me principally to depend. Thus one is able to say that nature itself has concurred in the perfection of the theories founded on the principal of universal gravity, and it is in my sense, one of the strongest proofs of the truth of this admirable principal.

"One is able still, by the analysis of probabilities, to verify the existence or the influence of certain causes of which one has believed to note the action on other organic beings. Of all the instruments that we are able to employ in order to understand the imperceptible agents of nature, the most sensible are the nerves, especially when their sensibility is magnified by particular circumstances. It is by their means, that one has discovered the feeble electricity that the contact of two heterogeneous metals develop; that which has opened a vast field to the researches of physicists and chemists. The singular phenomena which result from the extreme sensibility of the nerves in some individuals, have given birth to diverse opinions on the existence of a new agent one has named *animal magnitism*, on the action of ordinary magnetism and the influence of the sun and of the moon, in some nervous affections; finally, on the impressions that the proximity of metals or of running water are able give birth. It is natural to think that the action of these causes is very weak, and is able to easily be troubled by a great number of accidental circumstances; thus of that which, in any case, it is not at all manifested, one must not conclude that is never exists. We are so distant from knowing all the agents of nature, that it would be little philosophical to deny the existence of phenomena, uniquely because they are inexplicable in the actual state of our knowledge. Only we must examine them with an attention so much more scrupulous, as it appears more difficult to admit

them; and it is here that the analysis of the probabilities become indispensable in order to determine to what point it is necessary to multiply the observations or the experiences, finally to have in favor of the agents that they seem to indicate, a probability superior to all the ratios that one has besides to reject the existence of them.

"The same analysis is able to be extended to the diverse results of medicine and political economy, and even to the influence of moral causes; because the action of these causes, when it is repeated a great number of times, offers in its results so much regularity as that of physical causes."

One of the most remarkable phenomena of the system of the World, is the one of the nearly circular movements in the same sense and very nearly in the same plane, of the planets and of their satellites, while the comets move in some very eccentric orbits, and indifferently in all the senses and all the inclinations to the ecliptic. Mr. Count Laplace submits to the analysis of probabilities, the existence of this singular phenomenon, by supposing it the effect of chance; and he finds for its probability, a fraction excessively small, whence he concludes this phenomenon indicates a particular cause, with a probability superior to those of the greatest number of historical facts, on which one is permitted no doubt. He has shown in his *Exposition du Systèm du Monde*, that his cause has been able to be only the solar atmosphere originally extended beyond the orbits of the planets, and that the cooling and the attraction of the sun has successively condensed. Seen at the distance of the stars, this star would appear to us now to shine as they; but in an original state where the author supposes it, it would resemble at this distance, the nebulas that the telescopes show us composed of a core more or less shining, surrounded by a nebulosity which, being condensed by a sequence of times to the surface of the core, will end by transforming it into a star. By conceiving by analogy, all the stars formed in this manner; one is able to imagine their former state of nebulosity, preceded itself by some successive states in which the nebulous matter would be more or less diffuse, the core being less luminous: one arrives thus, in showing thus far that it is possible, in a nebulosity so diffuse, that one is able with pain to suspect the existence of it. Such is, in fact, the first state of the nebulas that Mr. Herschell has observed with a particular care, by means of his powerful telescopes, and in which he has followed the progress of the condensation, not on one alone, this progress being able to become sensible for us only after some centuries, but on their assemblage; very nearly as one is able to follow in a vast forest, the increase of the trees, on the individuals of diverse ages, that it contains. He has observed first the nebulous matter spread in diverse clusters, in the different parts of the sky of which it occupies a great extent. He has seen in some of these clusters, this matter weakly condensed about one or many faint cores. In other nebulas, these cores shine more relative to the nebulosity which surround them. The atmospheres of each core, being separated by an ulterior condensation, there results from it some multiple nebulas formed by a shining core, surrounded by an atmosphere. Sometimes the nebulous matter, by being condensed in a uniform matter, has produced that nebulas which one names *planetary*. Finally a greater degree of condensation transforms all these nebulas into stars. It is

necessary to follow in the same Memoir that Mr. Herschell just published, the progressions of condensation of the nebulas which, classified according to this very philosophical view, indicate with an extreme likelihood, the transformation of the nebulas into stars, and the former state of nebulosity of the existing stars. We will confirm the proofs drawn from these analogies, by the following remark:

For a long time the particular disposition of some visible stars to the simple view, has struck some philosophical observers, Mr. Michell has already noted how little it is probable that the six stars of the Pleiades, for example, had been tightened in the narrow space which contains them, by the sole chances of hasard; and he has concluded from it that this gorup of stars and the similar groups that the sky presents to us, are the effects of an original cause, or of a general law of nature. Now, these effects are a necessary continuation of the condensation of these nebulas to many cores, that Mr. Herschell has described; because it is visible that nebulous matter being attracted without ceasing by these diverse cores, they must form at length, a group of stars, parallel to the one of the Pleiades. The condensation of the nebulas to two cores, will form similarly from the stars turning very nearly about one another, parallels to that of which Mr. Herschell has already considered the respective movements. Such are further, the 61st of Cignus and its following, in which Mr. Bessel just recognized the proper movements, so considerable and so little different, that the proximity of these stars among them, and their movement their common center of gravity, must leave no doubt. Thus Mr. Count Laplace and Mr. Herschell are arrived by some opposite routes, to the consideration of the sun surrounded formerly by a vast atmosphere; the first, by ascending to this state of the sun, by the consideration of the singular phenomena of the solar system; the second, by descending through the progress of the condensation of the nebulous matter. This encounter, by making the proofs agree that they have both produced, from their ideas, gives to them together, a probability quite near to certitude.

By rendering to the good researches of Mr. Herschell, the justice which is due to them; we will modify in some regards, his opinion on the cause of the movements of rotation of the sun and of the stars. A cluster of molecules, all originally immobile, are not able by being condensed, to produce as he seems to believe, a star endowed with a movement of rotation. Mr. Count Laplace has demonstrated in his *Méchanique Céleste*, that if all these molecules, by being reunited, come to form a body endowed with a movement of rotation; the axis of rotation will be necessarily the straight line perpendicular to the invariable plane of the *maximum* of the areas, and passing through the center of gravity of the entire mass; and the movement of rotation will be such, that the sum of the areas described by each molecule projected onto this plane, will always remain the same as in the original; whence it follows that this movement will be null, if all the molecules have been originally in repose. One is able to see in the Work cited, that this constancy of the areas maintain the uniformity of the movement of rotation of the earth and of the duration of the day which, from Hipparchus to us, has not varied by a hundredth of a second, despite the winds, the currents of the Ocean, and all the interior convulsions of the globe. But in a nebula with many cores, nothing is opposed to this that the stars which result

from it, have movements of rotation, provided that they turn in some different sense; because it is not true, as many celebrated philosophers have advanced, that the universal attraction is not able to produce in a system of originally immobile bodies, any permanent movement, and that it must at length, reunite them all to their common center of gravity.

This work of Mr. Herschell gives to him anew rights to the recognition of the astronomers, as so many important discoveries have merited to him a long time. One of the principal is the discovery of the planet Uranus and of six satellites that the power of his telescopes has made him perceive about it. Two alone of among them have been able to be recognized by some other observers. It is well to desire that this celebrated astronomer publish the observations that without doubt; he has made in order to note the existence of thes stars and in order to determine their movements.