

## PREFACE

ABRAHAM DE MOIVRE  
THE DOCTRINE OF CHANCES

*'Tis now about Seven Years, since I gave a Specimen in the Philosophical Transactions, of what I now more largely treat of in this Book. The occasion of my then undertaking this Subject was chiefly owing to the Desire and Encouragement of the Honourable Francis Robartes Esq; who, upon occasion of a French Tract, called, L'Analyse des Jeux de Hazard, which had lately been published, was pleased to propose to me some Problems of much greater difficulty than any he had found in that Book; which having solved to his Satisfaction, he engaged me to methodize those Problems, and to lay down the Rules which had led me to their Solution. After I had proceeded thus far, it was enjoined me by the Royal Society, to communicate to them what I had discovered on this Subject: and thereupon it was ordered to be published in the Transactions, not so much as a matter relating to Play, but as containing some general Speculations not unworthy to be considered by the Lovers of Truth.*

*I had not at that time read any thing concerning this Subject, but Mr. Huygen's Book de Rationciniis in Ludo Aleae, and a little English Piece (which was properly a translation of it) done by a very ingenious Gentleman,<sup>1</sup> who, tho' capable of carrying the matter a great deal farther, was contented to follow his Original; adding only to it the computation of the Advantage of the Setter in the Play called Hazard, and some few things more. As for the French Book, I had run it over but cursorily, by reason I had observed that the Author chiefly insisted on the Method of Huygens, which I was absolutely resolved to reject, as not seeming to me to be the genuine and natural way of coming at the Solution of the Problems of this kind. (However, had I allowed my self a little more time to consider it, I had certainly done the Justice to its Author, to have owned that he had not only illustrated Huygen's Method by a great variety of well chosen Examples, but that he had added to it several curious things of his own Invention.*

*Tho'I have not followed Mr. Huygens in his Method of Solution, 'tis with very great pleasure that I acknowledge the Obligations I have to him; his Book having Settled in my Mind the first Notions of this Doctrine, and taught me to argue about it with certainty.*

*I had said in my Specimen, that Mr. Huygens was the first who had published the Rules of this Calculation, intending thereby to do justice to a Man who had well deserved of the Public; but what I then said was misinterpreted, as if I had designed to wrong some Persons who had considered this matter before him: and a Passage was cited against me out of Huygen's Preface, in which he saith, "Sciendum vero quod iam pridem, inter Praestantissimos tota Gallia Geometras, Calculus hic fuerit agitatus; ne quis indebtam mihi primae Inventionis gloriam hac in re tribuat."<sup>2</sup> But what follows immediately after, had it*

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*Date:* Published 1717.

<sup>1</sup>Arbutnot's translation.

<sup>2</sup>It is necessary to know besides that there is already a certain time that a few of the more Celebrated Mathematicians of all France occupied themselves in this kind of Calculus, so that a person not attribute to me the honor of the first Invention which will not belong to me.

*been minded, might have cleared me from any Suspicion of injustice. The words are these, "Caeterum illi difficillimis quibusque Quaestionibus se invicem exercere soliti, methodum suam quisque occultam retinere, adeo ut a primis elementis hanc materiam evolvere mihi necesse fuerit."*<sup>3</sup> *By which it appears, that tho' Mr. Huygens was not the first who had applied himself to those sorts of Questions, he was nevertheless the first who had published Rules for their Solution; which is all that I affirmed. (Since the printing of my Specimen, Mr. de Monmort, Author of the Analyse des jeux de Hazard, Published a Second Edition of that Book, in which he has particularly given many proofs of his singular Genius, and extraordinary Capacity; which Testimony I give both to Truth, and to the Friendship with which he is pleased to Honor me.)*

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*Having explained the common Rules of Combination, and given a Theorem which may be of use for the Solution of some Problems relating to that Subject, I lay down a new Theorem, which is properly a contraction of the former, whereby several Questions of Chance are resolved with wonderful ease, tho' the Solution might seem at first sight to be of insuperable difficulty.*

*It is by the Help of the Theorem so contracted, that I have been able to give a compleat Solution of the Problems of Pharaon and Bassette, which was never done before me: I own that some great Mathematicians had already taken the pains of calculating the advantage of the Banker, in any circumstance either of Cards remaining in his Hands, or of any number of times that the Card of the Ponte is contained in the Stock: But still the curiosity of the Inquisitive remained unsatisfied; the Chief Question, and by much the most difficult, concerning Pharaon or Bassette, being, What it is that the Banker gets per Cent. of all the Money adventured at those Games: which now I can certainly answer is very near Three per Cent. at Pharaon, and three fourths per Cent. at Bassette, as may be seen in my 33d Problem, where the precise Advantage is calculated.*

*In the 35th and 36th Problems, I explain a new sort of Algebra, whereby some Questions relating to Combinations are solved by so easy a Process, that their Solution is made in some measure an immediate consequence of the Method of Notation. I will not pretend to say that this new Algebra is absolutely necessary to the Solving of those Questions which I make to depend on it, since it appears that Mr. Montmort, Author of the Analyse des Jeux de Hazard, and Mr. Nicholas Bernoulli have solved, by another Method, many of the cases therein proposed: But I hope I shall not be thought guilty of too much Confidence, if I assure the Reader, that the Method I have followed has a degree of Simplicity, not to say of Generality, which will hardly be attained by any other Steps than by those I have taken.*

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*The 44th and 45th Problems, having in them a Mixture of the two Methods of Combinations and Infinite Series, may be proposed for a pattern of Solution, in some of the most difficult cases that may occur in the Subject of Chance, and on this occasion I must do that*

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<sup>3</sup>But these scholars, although they put to proof the one the other of them in proposing many difficult questions to resolve, have however hidden their methods. I have therefore owing to examine and to delve myself into all this matter to begin with the elements, and it is impossible to me for the reason that I just mentioned to assert that we have set out from the same first principle.

*Justice to Mr. Nicholas Bernoulli, to own he had sent me the Solution of those Problems before mine was Published; which I had no sooner received, but I communicated it to the Royal Society, and represented it as a Performance highly to be commended: Whereupon the Society order'd that his Solution should be Printed; which was accordingly done some time after in the Philosophical Transactions, Numb. 341, where mine was also inserted.*

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*When I first began to attempt the general Solution of the Problem concerning the Duration of Play, there was nothing extant that could give me any light into that Subject; for altho' Mr. de Montmort, in the first Edition of his Book, gives the Solution of this Problem, as limited to three Stakes to be won or lost, and farther limited by the Supposition of an Equality of Skill between the Adventurers; yet he having given no Demonstration of his Solution, and the Demonstration when discovered being of very little use towards obtaining the general Solution of the Problem, I was forced to try what my own Enquiry would lead me to which having been attended with Success, the result of what I found was afterwards published in my Specimen before mentioned.*

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*Mr. de Monmort, and Mr. Nicholas Bernoulli, have each of them separately given the Solution of my xxxixth Problem, is a Method differing from mine, as may be seen in Mr. de Monmort's second Edition of his Book. Their Solutions, which in the main agree together, and vary little more than in the form of Expression, are extreamly beautiful; for which reason I thought the Reader would be well pleased to see their Method explained by me, in such a manner as might be apprehended by those who are not so well versed in the nature of Symbols: In which matter I have taken some Pains, thereby to testify to the World the just Value I have for their Performance.*

REMARK. (Edition of 1756, pp. 210-211)

In the first attempt that I had ever made towards solving the general Problem of the Duration of Play, which was in the Year 1708, I began with the Solution of this LXV<sup>th</sup> Problem, well knowing that it might be a Foundation for what I farther wanted, since which time, by a due repetition of it, I solved the main Problem: but as I found afterwards a nearer way to it, I barely published in my first Essay on those matters, what seemed to me most simple and elegant, still preserving this Problem by me in order to be published when I should think it proper. Now in the year 1713 Mr. *de Monmort* printed a Solution of it in a Book by him published upon Chance, in which was also inserted a Solution of the same by Mr. *Nicolas Bernoulli*, and as those two Solution seemed to me, at first sight, to have some affinity with what I had found before, I considered them with great attention; but the Solution of Mr. *Nicolas Bernoulli* being very much crouded with Symbols, and the verbal Explication of them too scanty, I own I did not understand it thoroughly, which obliged me to consider Mr. *de Monmort's* Solution with very great attention; I found indeed that the was very plain, but to my great surprise I found him very erroneous; still in my Doctrine of Chances I printed that Solution, but rectified and ascribed it to Mr. *de Monmort*, without the least intimation of any alterations being made by me; but as I had no thanks for so doing, I resume my right, and now print it as my own: but to come to the Solution.