

PROBLÈME
SUR LA LOTERIE DE LORAINÉ
PROPOSITION XLIII

PIERRE RENARD DE MONTMORT EXTRACTED FROM THE
ESSAY D'ANALYSE SUR LES JEUX DE HAZARD
2ND EDITION OF 1713, §206–207, PP. 257–260

*I have inserted that which was in the Journal des Sçavans in the month
of March of the year 1711.*

“There was drawn last year at Paris a Lottery contained under the name the Lottery of Lorraine, of which the Public has not been subject to be satisfied. When it was published I myself perceived first that one ran the risk of being the dupe of it, & that one should have obliged the Director of the Lottery to give good and sufficient caution, since according to the conditions to which he was obligated, it was possible that he had to render 424950 livres beyond the 500000 livres which he had received. I saw roughly that his part was not good, & thence I suspected that one had designs to seize the silver of the Public, that which happened.

“I did not go further then, & I postponed for a time where I had more leisure to examine at base the disadvantage of the one who held the Lottery. Having found little since, I have believed that it was not useless to propose the research on it to the Geometers. Those who have most esteem for Algebra & Analysis, know not at all enough how it has usage with respect to the things of civil life. It is good, it seems to me, to give to them here a new proof, & at the same time, to make known to the Magistrates who could have decided on a matter of the nature of this one, which is of their competence, that the Geometers the only ones of whom they can receive some certain decisions.

RULES OF THE LOTTERY.

“The tickets were ten sols, & there were one million. For the 500000 livres which the one who held the Lottery received from the Public, he rendered to them 425000 in twenty thousand lots. Two conditions made the novelty & the singularity of this Lottery.

“1 ° The one who held the Lottery, in order to compensate the Public of the 75000 livres which he received, he obligated himself to render 25 livres to each of those who had taken 50 tickets in sequence, had no lot in their 50 tickets.

“2 ° Here is in what manner the Lottery was drawn. All the tickets or numbered tickets were in a box, & the black tickets in another. One drew at the same time a black ticket and a numbered ticket, & after one had written what numbered ticket was on such lot, one cast aside the black ticket, & one replaced the numbered ticket into the box of the numbered tickets, so that in this manner of drawing the lots, the same numbered ticket could win many lots, or even all of them.

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PROBLEME

“By supposing that all those who put into the Lottery will take either 50 tickets, or 100 tickets, or 150 tickets, &c. (this supposition would be entirely admissible;) *We ask what is the advantage or the disadvantage of the one who held the Lottery?* It is easy to observe, 1° That the Director of the Lottery will win 75000 livres if all those who have put into the Lottery have a lot in each half hundred of tickets. 2° That he will lose 424950 livres if one alone of all those who will have put into the Lottery carry away all the lots. 3° That he will neither lose nor win, if three thousand persons only have not at all lots in their 50 tickets. Whence it follows that this Lottery is a kind of game of chance, where the one who holds the Lottery can lose or win. The solution of this Problem is cached under this Anagram *4a, 5e, 5i, 13o, 3u, 2l, 2n, 2p, 4s, 32, c, d, m, r*, of which I will give the explication when one will wish it.”

SOLUTION.

207. It is the same thing in the Lottery of Lorraine to suppose that there are one million tickets to draw, & 20000 persons who take 50 tickets each with 10 franc coin, or to suppose twenty thousand tickets, & twenty thousand persons who each take a ticket with twenty-five livre coin that will make always 500000 livres for the fund of the Lottery & will produce no difference in the interests of those who put into it. Thus for the simplicity of the Problem, we will hold ourselves to this last supposition. The question therefore to know how much has the prejudice to the one who holds the Lottery the condition to which he obligates himself to render 25 livres to each of those who having taken a ticket will have no lot at all. In order to resolve this Problem, I myself propose one here which is not different.

I cast at random 20000 dice which have each 20000 faces, one marked with an ace, the other with a two, the other with a three, & so forth up to 20000. We ask how much are the odds that I will not bring forth an ace.

We will be convinced that this Problem is the same as the one of the Lottery, if we observe that in this last the question is to know how much are the odds that drawing twenty thousand times a ticket from among the 20000, or that which is the same thing, that throwing at the same time 20000 dice, which have each 20000 faces marked with an ace, with a two, with a three, &c. no ticket is found marked with my number, or any die of which the upper face is, for example, an ace. Now this similitude put, we know by art. 37, that my lot will be $\frac{20000-1^{20000}}{20000}$, & since there are 20000 persons who are in the case to ask again their silver, that is to say 25 livres, when their numbered ticket comes not at all in the 20000 tickets, it follows that the disadvantage of the one who holds the Lottery, based on this condition in which he is obligated to render the silver to those who will have no lot at all, will be $\frac{19999^{20000}}{20000} \times 25 \times 20000$, that which by the Table of logarithms gives 184064 livres for the sought disadvantage.

The explication of the Anagram consists in these words: *20000 moins 1, divisé par 20000 élevé à la puissance 20000.*

*Extract of the Letter of Nicolaus Bernoulli to M. de Montmort
From Basel this 26 February 1711.*

The problem which you have had plan to propose to the Geometers has no difficulty: here is how I have concluded the thing. The question is to find how often the condition of rendering their 25 livres to those who having taken 50 tickets would have won no lot in their 50 tickets, gives advantage or disadvantage to the one who holds the Lottery, that

which is the same thing as if we wished to seek the lot of the one who would undertake to bring forth with 20000 dice with 1000000 faces, of which 50 alone are marked with some points, in a single trial at least one of the marked faces; now the number of cases that this will not happen is 999950^{20000} & the number of all the cases is 1000000^{20000} ; whence it follows that this condition to render the silver to each of those who win no lot in their 50 tickets, is worth $\left(\frac{999950}{1000000}\right)^{20000} \times 25$ livres that which makes in all $\left(\frac{999950}{1000000}\right)^{20000} \times 500000$ livres=(that which is found by logarithms) 184064 livres. & about ten sols. Therefore the disadvantage of the Banker, who retains only 75000 livres will be $109064\frac{1}{2}$ livres so that he must not be amazed at all if the one who has held one such Lottery has been bankrupted. One can by this same method & by two words resolve proposition 44 of your Book.¹

*Extract of the letter of Mr. de Montmort to Mr. Nicolas Bernoulli
at Montmort 10 April 1711.*

The anagram which I give for the solution of the Problem which I propose on the Lottery of Loraine, contains these words *20000 moins un divisé par 20000 élevé à l'exposant 20000*, that which gives a solution conformed to yours. This problem has always appeared to me more curious than difficult; nonetheless its difficulty is such to my opinion, that it can stop some persons who could be not at all as you & Mr. your Uncle some Geometers of the first order, & capables of the greatest things: Many Geometers of my friends have worked uselessly. Besides the solution of this Problem is only a particular case of the formula which I have sent to Mr. your Uncle in my last Letter

$$\frac{m-1}{m-1}^{p-q} \times p \cdot \frac{p-1}{2} \cdot \frac{p-2}{3} \cdot \frac{p-3}{4} \text{ \&c. divided by } m^p,$$

but beyond that one has not yet much thought to these sorts of Problems of combinations, it was necessary to be advised to reduce the Problem of the Lottery to a question of dice.

*Extract of the Letter of Nicolaus Bernoulli to M. de Montmort
From Basel this 10 November 1711.*

As you have invited me to render public my solution of your Problem on the Lottery of Loraine, I have sent it to Mr. Varignon there is four months to insert it into the Journal des Sçavans, where it appeared the thirteenth of July, that which you know perhaps already. For that which is your solution, I have remarked that beyond that your Anagram *4a, 5e, 5i, 13o, 3u, 2l, 2n, 2p, 4s, 3, 2, c, d, m, r*, does not contain exactly these words: *20000 moins un divisé par 20000 élevé à l'exposant 20000*; because the Anagram will have no correct sense, & will not give at all the sought value 184064. Moreover that which you say that the solution of this Problem is only a particular case of the formula

$$m - 1^{p-q} \times p \times \frac{p-1}{2} \times \frac{p-2}{3} \times \frac{p-3}{4} \times \text{\&c. divided by } m^p,$$

is only true when the number which expresses how many of the tickets one must draw, finally that having won no lot in all these tickets, one can redraw his silver, is correctly a fractional part of the number of all the tickets; because in order to reduce the cases where this is not found in the Problem for the dice, in order to bring forth a certain number of points, it would be necessary to suppose that each die has many faces marked with a point which one proposes to bring forth, this is to what your formula extends not at all. But to

¹Proposition XXXIX of the 2nd Edition.

what serves it to go seeking so far the manner of solving this Problem, doesn't one see first & most easily that it is only a particular case of proposition 44² of your Book?

*Extract of the Letter of M. de Montmort to Nicolaus Bernoulli
At Paris 1 March 1712.*

In the explication which I have sent to you on my Anagram, it is necessary to read *20000 moins 1, divisé par 20000 élevé à la puissance 20000*. I had put through distraction *exposant* in the place of *puissance*: this is that which has prevented you from understanding, because besides it is clear that it is necessary to multiply this number by 25×20000 , & this goes without saying.

²Proposition XXXIX of the 2nd Edition.