

# Recherches sur la probabilité des jugements, principalement en matière criminelle\*

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*Comptes Rendus*, I., (1835), pp. 473–494.  
*L'Institut*, III., 1835, pp. 416–419.

“Condorcet is the first who has tried to determine the probability of judgments and, in general, of decisions rendered by the plurality of votes. The book which he has written on this subject<sup>1</sup> had been undertaken in the lifetime and at the request of minister Turgot, who conceived all the advantage that the moral sciences and public administration are able to get back from the calculus of probabilities, of which the indications are always precise, even though, for lack of sufficient data of observation, it cannot lead to complete solutions of the questions. This work contains a quite extended preliminary discourse, where the author exposes, without the help of analytic formulas, the numerous results that he has obtained, and where are developed with care the proper considerations to demonstrate the utility of this kind of researches.

“In his *Traité des Probabilités*, Laplace himself is also occupied in calculating the chances of error to fear in the judgment rendered by a known majority, against an accused, by a tribunal or a jury composed of a number of persons equally known. The solution that he has given of this problem, one of the more delicate in the theory of probabilities, is based on the principle which serves to determine the probabilities of the diverse causes to which one can attribute the observed facts; a principle which Bayes has presented first under a form a little different, and of which Laplace has made next the most fortunate use, in his memoirs and in his treatise, in order to determine the probability of future events, according to the observation of some past events: however, in that which concerns the problem of the probability of judgments, it is just to say that it is to Condorcet who is due the ingenious idea to make his solution depend on the principle of Bayes, by considering successively the culpability and the innocence of the accused, as the unknown cause of the pronounced judgment, which is then the observed fact, of which the question is to deduce the probability of this cause. The exactitude of this principle is demonstrated in total rigor; his application to the question which occupies us can no longer leave any doubt, but for this application, Laplace makes a hypothesis which is not incontestable: he supposes that the probability that a juror will not be misled, is susceptible to all the equally possible degrees, from certitude, represented by unity, to indifference, designated in the calculus by the fraction  $\frac{1}{2}$ ,

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<sup>1</sup>*Essai sur l'Application de l'analyse à la probabilité des décisions rendues à la pluralité des voix.*

and which corresponds to an equal chance of error and of truth. The illustrious geometer founds this hypothesis on this that the opinion of a juror has without doubt more tendency towards the truth than towards error; that which one must admit effectively in general; but there exists an infinity of different laws of probability of the errors which satisfy this condition, without which it is necessary to suppose that the probability that a juror will not be misled, could never descend beneath  $\frac{1}{2}$ , and that above this limit, all its values are equally possible. Independently of the particular hypothesis that Laplace has made on the probability of the opinion of a juror and that I have not admitted, no more than any other, I digress again from the method which he has followed to solve the problem, with other points which it will be difficult to indicate in this preamble, but which will be examined scrupulously in the continuation of the work. The different solutions what one finds, either in the *Traité des Probabilités*<sup>2</sup>, or in the first *Supplément* to this great work<sup>3</sup>, have always left much in doubt in my mind; it is to the illustrious author that I would have submitted them, if I had occupied myself with this problem during his life: the authority of his name has made a duty of it to me, as his friendship, of which I myself will glorify always, he would have rendered easily to fill me. One will imagine without pain that it is only after long reflections, that I myself am decided to contemplate the question under another point of view; and one will permit me to expose, before going farther, the principal reasons which have caused me to abandon the last solution at which Laplace himself had stopped, and to which he has inserted the numerical results in the *Essai philosophique sur les Probabilités*.

“The formula of Laplace, in order to express the probability of an error of a judgment, depends only on the majority to which it has been pronounced, and on the total number of judges; it contains nothing which is relative to their more or less extensive knowledge of the material which has been submitted to them. It follows therefore that the probability of the error of a decision rendered by a jury, in the majority of seven votes against five, for example, would be the same, whatever was the class of persons where twelve jurors would have been chosen; a consequence which would seem to me already sufficient so that one was based to admit at no point the formula from which it is deduced.

“This same formula supposes that before the decision of the jury, there was no presumption that the accused was guilty; so that the more or less great probability of his culpability, would be concluded uniquely from the decision which will be rendered against him. But this is again inadmissible: the accused, when he arrives at the Assize Court,<sup>4</sup> has already been the object of an arrest of prevention and of an arrest of accusation, which establishes against him a probability greater than  $\frac{1}{2}$ , that he is guilty; and certainly, a person would not hesitate to wager, in an equal game, rather for this guilt than for his innocence. Now, the rules which serve to increase the probability of an observed event with that of its cause, and which are the base of the theory of which we occupy ourselves, requires that we have regard to all presumption previous to the observation, when we do not suppose, or when we have not demonstrated that nothing exists of it. One such presumption being, on the contrary, evident in the criminal pro-

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<sup>2</sup>Page 460.

<sup>3</sup>Page 32.

<sup>4</sup>*Translator's note:* The criminal court.

cedures, I have ought to take account in the solution of the problem; and we will see, indeed, that by setting it aside, it would be impossible to accord the consequences of the calculus with the constant results of observation. This presumption is similar to that which takes place in civil matters, when one of the plaintiffs appeals on a first judgment before a superior court: he appears with a presumption contrary to his cause; and one would be misled gravely, if one has no regard to this circumstance, in calculating the probability of the error to be feared in the definitive arrest.

“Finally, Laplace himself is restricted to consider the probability of the error of a judgment rendered by a known majority; however the danger that the accused incurs to be condemned in a wrong by this majority, when he is led before the jury, depends not only on this probability; it depends also on the chance that such a conviction will be pronounced. Thus, by admitting for a moment that the probability of the error of a judgment rendered by the majority of seven votes against five, is expressed by a fraction very nearly equal to  $\frac{2}{7}$ , as it would result from the formula of Laplace, it is necessary also to observe that, after the experience, the number of convictions by the jurors which have taken place each year in France, at this majority, is only 0.07 of the total number of accused; the danger for an accused to be maljudged by the majority in question, will have therefore for measure the product of the two fractions  $\frac{2}{7}$  and  $\frac{7}{100}$ , or  $\frac{1}{50}$ ; because, in all the things possible, the fear of a loss or the hope of a gain has for expression the product of the value of the thing that one fears or that one hopes, multiplied by the probability that it will take place. This consideration would reduce therefore already to one out of fifty the proportion of the non-guilty accused who would be convicted annually by the smallest majority of juries; this will be without doubt again too much, if all these accused were really innocent; but it is here convenient to demonstrate the true sense that one must attach, in this theory, to the words *guilty* and *innocent*, and that Laplace and Condorcet have effectively attributed themselves.

“One would never know how to arrive at the mathematical proof of the guilt of an accused; his confession even can be regarded only as a probability close to certitude; the most enlightened and humane juror therefore pronounces a conviction only on a strong probability, frequently less, nevertheless, only that which would result from the confession of the guilty. There is between him and the judge in civil matters, an essential difference; when a judge, after the thorough examination of an affair,<sup>5</sup> has been able to recognize, seeing the difficulty of the question, only a feeble probability in favor of one of the two parties, which suffices why he convicts the adverse party; whereas a juror must pronounce a vote of conviction only when, in his eyes, the probability that the accused is guilty attains a certain limit, and surpasses by much the probability of his innocence. Since all chance of error cannot be avoided, whatever one does, in the criminal judgments, to what must it be reduced, in order to assure to the innocent the greatest guarantee possible? This is a question to which it is difficult to respond in a general manner. According to Condorcet, the chance to be convicted unjustly would be equivalent to that of a danger which we judge small enough in order to not even seek ourselves to escape during the habits of life; because, says he, society has well the right, for its safety, to expose one of its members to a danger of which the chance to him is, so to speak, indifferent; but this consideration is much too subtle in a question

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<sup>5</sup>*Translator's note:* That is, a lawsuit or a criminal case.

so grave. Laplace gives a definition, much more proper to illuminate the question, of the chance of error which one is forced to admit in the judgments in criminal matter. According to him, this probability must be such that there is more danger for the public safety, by the acquittal of a guilty, than to fear the conviction of an innocent; as he says it expressly, it is this question, rather than the guilt even of the accused, that each juror is called to decide, in his manner, according to his wisdom and his opinion; so that the error of his vote, either that he convict, or that he absolve, can proceed from two different causes: either from this which he estimates wrongly the proofs contrary or favorable to the accused, or from this that he fixes too high or too low the limit of the probability necessary to conviction. Not only is this limit not the same for all the persons called to judge, but it changes also with the nature of the accusations, and depends even on the circumstances where one is found: in the army, in presence of the enemy, and for a crime of espionage, it will be without doubt much less elevated than in the ordinary cases.

“The decisions of juries is related therefore to the opportunity of convictions or of acquittals: one will render the language more exact by substituting the word *convictable*, which is entirely true, for the *guilty*, which has need of explication, and which we continue to use in order to conform us to the usage. Thus, when we will find, that on a very great number of judgments, there is a certain proportion of erroneous convictions, it will not be necessary to understand that this proportion is that of convicted innocents; this will be the proportion of the convicted who have been it by a too feeble probability, not in order to establish that they are rather guilty ones than innocents, but in order that their conviction was necessary to the public safety. To determine among these convicted, the number of those who really were not guilty, this is not the object of our calculations; however there is place to believe that this number is happily of very little consequence, at least outside of political suits: one can judge it, in the ordinary cases, by the very small number of convictions pronounced by juries, against which the public opinion itself is raised; by the small number of complete pardons which have been accorded; and by the number, also very small, of cases where the Assize Courts have used the right that the law gives them, to annul the conviction pronounced by a jury, and to send back the accused before other jurors, when they judge that the oral debate had destroyed the accusation, and that the accused is not guilty.

“The results relative to the chances of error of criminal judgments, to which Laplace has arrived, have seemed exorbitant, and in discord with the general ideas; this which will be contrary to the promises of the author, that *the theory of probabilities is, at basis, only good sense reduced to calculus*. They have been badly interpreted; and one is too anxious to conclude from it that the mathematical analysis is not applicable to this kind of questions, nor generally to the things which one calls morals. This is a prejudice that I have observed with reluctance shared by good minds; and, in order to destroy it, I think useful to recall here some general considerations, which will be proper, moreover, to make well known the object of the special problem which I myself have proposed in this work, and to advance some points of similitude with other questions where a person contests only the use of the calculus be legitimate and necessary.

“Things of every nature are subject to a universal law which one can call *the law of large numbers*. It consists in this that, if one observes some very considerable numbers

of events of a like nature, depending on causes which vary irregularly, as many in one sense, as in another, that is without their variation be progressive in any determined sense, one will find, among these numbers, some very nearly constant ratios. For each nature of things, these ratios will have a special value from which they will deviate less and less, in measure as the series of observed events will increase further, and that they would attain rigorously if it were possible to prolong this series to infinity. Accordingly as the amplitudes of variation of the irregular causes will be more or less great, there must be of the numbers also more or less great events so that their ratios arrive sensibly to permanence; the same observation will be made known, in each question, if the series of experiences has been sufficiently prolonged; and according to the numbers of constant facts, and the size of the deviations which will remain yet among their ratios, the calculus will furnish certain rules in order to determine the probability that the special value towards which these ratios converge is contained between some limits as narrowed as one will wish. If one makes new experiences, and if one finds that these same ratios diverge notably from their special value, determined by the preceding observations, one will be able to conclude from it that the causes on which the observed facts depend, have experienced a progressive variation, or even some abrupt change, in the interval of the two series of experiences. However, without the help of the calculus of probabilities, one would risk much to be mistaken on the necessity of that conclusion; but this calculus leaves nothing vague in this regard, and furnishes us also the rules necessary in order to determine the chance of a changing in the causes, indicated by the comparison of observed facts in different epochs.

“This law of large numbers is observed in the events which we attribute to a blind chance, defect in knowing the causes of it, or because they are too complicated. Thus, in the games where the circumstances which determine the arrival of a card or of a die, vary to infinity and cannot be submitted to any calculus, the different trials present themselves however according to constant ratios, when the series of trials has been prolonged a long time. Moreover, when one will have been able to calculate according to the rules of the game, the respective probabilities of the trials which can happen, one will verify that they are equal to these constant ratios, conforming to the known theorem of Jacques Bernoulli. But in most of the questions of eventuality, the determination *a priori* of the chances of these diverse events is impossible, and these are, on the contrary, the observed results which are made known of them: one would not know, for example, how to calculate in advance the probability of the loss of a vessel on a voyage of long course; one supplied it therefore by the comparison of a number of disasters to that of the voyages: when this is very great, the ratio of the one to the other is very nearly constant, at least for every sea and for every nation in particular; its value can be taken for the probability of future disasters; and it is on this natural consequence of the law of large numbers, which are based the maritime assurances. If the insurer operated only on a very small number of affairs, it would be a simple wager, which would have no value on which he could compute; if he operates on very great numbers, it is a speculation of which the success is nearly certain.

“The same law rules equally the phenomena which are produced by some known forces, concurrently with some accidental causes of which the effect have no regularity. The risings and successive fallings of the sea in the ports and on the coasts, offer an example of it of a remarkable precision. Despite the inequalities that the winds pro-

duce, and which would make the laws of the phenomenon vanish in some isolated or observations few in number, if one takes the means of a great number of observed tides in a same place, one finds that they are very nearly conformed to the laws of *flow* and of *ebb*, resulting from the attractions of the moon and of the sun, and the same as if the winds have had no influence. The means deduced from observations made at the beginning and at the end of the last century, or separated by an interval of one hundred years, have presented only small differences, that one can attribute to some intervening changes in the localities.

“For example of the law I considered, I will cite again the length of the mean life in the human species. Out of a considerable number of infants born in some places and in some epochs near enough, there will be of them who will die at young age, others who will live a longer time, others who will attain the limits of longevity; now, despite the vicissitudes of the life of men, who put so great differences between the ages of the dying, if one divides the sum of these ages by their number supposed very great, the quotient, or what one calls *mean life*, will be a quantity independent of this number. Its duration will not be able to be the same for the two sexes; it will be able to differ in the different countries, and in different epochs, because it depends on climate, and without doubt also on the welfare of the peoples: it will increase if a malady just disappears, as the smallpox by the blessing of the vaccine; and, in all cases, the calculus of the probabilities will demonstrate if the variations recognized in this period, are great enough and result from a great enough number of observations, in order that it is necessary to attribute them to some changes happening in the general causes. The ratio between the number of masculine and feminine annual births, in a country of great extent, has equally a constant value, which seems not to depend on the climate, but which, by a singularity of which it would not be perhaps difficult to assign a likely cause, seems different for the legitimate infants and for the infants born outside of marriage.

“The constitution of the bodies formed of disjoint molecules which separate the empty spaces of ponderable matter, offer also an application, of a particular nature, of the law of large numbers. By a point taken in the interior of a body and following a determined direction, if one draws a straight line, the distance of this point to which it will encounter a first molecule, although very small in every sense, will vary nonetheless in a very great ratios with its direction: it will be able to be ten times, twenty times, one hundred times, . . . greater in one sense than in another. Around each point, the distribution of molecules will be able to be very irregular, and very different from one point to another; it will change even incessantly by the effect of internal oscillations of the molecules; because a body in repose is nothing other than an assemblage of molecules which execute some continual vibrations of which the amplitudes are insensible, but comparable to the intermolecular distances. Now, if one divides each portion of the volume, of insensible greatness, by the number of molecules that it contains, which number will be extremely great by reason of their excessive smallness, and if one extracts the cube root of the quotient, there will result from it a *mean interval* of the molecules, independent of the irregularity of their distribution, which will be constant in every extension of a homogeneous body, throughout at the same temperature, and setting aside the unequal compression of its parts, produced by its own weight. It is out of some similar considerations that the calculus of the molecular forces and of

the thermal radiation in the interior of the body is based, such as I have presented in other works.

“Now, this universal law of large numbers, of which we just gave some examples of all sorts, that we might have been able, in case of need, to multiply and to vary yet more; this law, we say, is the base of all the applications of the calculus of probabilities. Now, it is evident that it agrees equally to the moral things which depend on the will of man, on his intelligences and on his passions; because there is no question here of the nature of the causes, nor even of the variation of their isolated effects, and of the numbers of cases necessary in order that these irregularities balance themselves in the mean results. The magnitude of these numbers cannot be calculated in advance; it will be different in the diverse questions, and, as I have said above, as much more considerable as the irregularities of the observed facts will have greater amplitude. But, in this regard, one must not believe that the effects of spontaneous will, of the blinding of the passions, of the defect of intelligence, vary on a greater scale than human life, from the infant who dies at birth, to the one who will become centenarian; that they are more difficult to predict than the circumstances which make a vessel to perish on a long voyage; more capricious than the kind which brings a card or a roll of die. The exposition which I have just made of the data of experience, on which I have supported myself on the question of the probability of judgments, will confirm plainly these general considerations. One will see that, under the control of one same legislation, the ratio of the number of convictions to that of the accused in all of France, has varied little from one year to another: so that it is sufficient to consider around 7000 cases, that is the number of judgments pronounced each year by the juries, in order that this ratio attained sensibly to permanence; while on some other question, and, for example, on that of mean life, that I just cited, a parallel number will be quite far to be great enough to conclude a constant result. One will see also, in a striking manner, the influence of the general causes on the ratio in question, which has varied every time the legislation has changed.

“The government has published the *Comptes généraux de l'administration de la justice criminelle*, during the nine years elapsed from 1825 to 1833; it is in this genuine compilation, and presented with a remarkable care, that I have drawn all the documents of which I have made use. The number of the affairs judged annually by the Assize Courts, has been around 5000, and that of the accused around 7000. From 1825 to 1830 inclusively, the legislation has not changed, and the decisions of the juries have been rendered by the majority of at least seven votes against five, save the intervention of the court in the case of two votes only of difference. In 1831, this intervention has been eliminated, and one has required the majority of at least eight votes against four, this which had ought to render acquittals more frequent. The ratio of their number to that of the accused during the interval of the six first years is found equal to 0.39, by neglecting the thousandths; one sole year, it is lowered to 0.38, and one other year, it is raised to 0.40; whence there results that in this period, it has varied, from one year to another, only one hundredth on both sides of its mean value. One can therefore take 0.39 for the value of this ratio, and 0.61 for the ratio of the number of convictions to that of the accused, under the control of the legislation anterior to 1831. In this same epoch, the ratio of the number of convictions pronounced at the *minimum* majority of seven votes against five, to the total number of the accused, has been 0.07, and it has varied very little from year to year. By subtracting this fraction from 0.61, there

remains 0.54 for the proportion of convictions which have taken place at more than seven votes against five; the ratio of the number of acquittals to that of the accused, would have therefore been 0.46, if one had required a majority of at least eight votes against four; now, it is effectively this which has happened during the year 1831, so that the difference between this ratio concluded from the preceding years and the one which has been observed, is found only in the thousandths, which I have neglected in these citations.

“In 1832, by conserving the same *minimal* majority as in 1831, the law has prescribed the question of the *extenuating circumstances*, carrying, in the case of the affirmative, a diminution of punishment; the effect of this measure must have been to render more easily the convictions by the juries; but in what proportion? It is that which experience alone would be able to inform, and what one would not be able to calculate in advance, as the increase of the acquittals, which had taken place by a change in the least majority. Experience has shown that in 1832 the proportion of acquittals is lowered from 0.46 to 0.41; it has stayed the same, to nearly a thousandth, in the year 1833, for which the legislation had not changed: the ratio of the number of convictions to that of the accused, before, during and after 1831, has been therefore successively 0.61, 0.54, 0.59, so that the influence of the question of extenuating circumstances on the mind of the jurors is found less, in the ratio of 0.2 to 0.7, or 2 to 7, than the effect of one more vote required in the majority.

“During the two years 1832 and 1833, the number of political trials submitted to the Assize Courts has been considerable; one has subtracted from the total number of the criminal trials, in the evaluation which has given 0.41 for the proportion of acquittals; by having regard of it, one finds that this proportion would be raised to nearly 0.43; which shows already the influence of the kind of affairs on the number of acquittals pronounced by the juries. This influence is rendered quite evident in the *comptes généraux*; the criminal trials are classified into two principal divisions: those which have for object some thefts or attempts against properties; those which are reported in some attempts against persons, and of which the number is generally the third of those of the first, or the fourth of the total number of affairs. In the first division, the ratio of the number of acquittals to that of the accused has been only 0.34; in the second, it is raised to 0.52, that is that the number of acquittals has even surpassed by 0.04, that of the convictions. From 1825 to 1830, the annual values of each of these two ratios has varied only around 0.02 on both sides of these fractions 0.34 and 0.52. The difference that they present seems to indicate a greater severity on the part of the jurors for the thefts than for the attempts against persons, either because they believe the first more dangerous for society, because they are more frequent, or because, in the case of thefts, the penalty are less grave in general. But a different manner of judging in these two kinds of crimes would not suffice, as one sees a little while ago, in order to produce the great inequality in the number of acquittals that experience has made known.

“The *comptes généraux* puts again in evidence other ratios that the great numbers have rendered nearly invariable, and that I will cite, although I have not had to make use of them. Thus, for example, from 1825 to 1833, the ratio of the number of women put to judgment to the total number of the accused has been annually from 0.18 to 0.19; a single time it is raised to 0.20, and a single time it is descended to about 0.16. It is consistently greater in the affairs of theft than in the case of attempts against persons;

the proportion of acquittals is also more considerable for women than for men, and it was raised for them to 0.46, in the epoch where it was only 0.39 for the accused of the two kinds.

“But the constancy of these diverse proportions, which is observed each year in the entire France, no longer takes place when one considers the isolated Assize Court. The proportion of acquittals varies notably from one year to another for a same department, and under one same legislation; which shows that in the jurisdiction of an Assize Court, the annual number of criminal affairs is not great enough that the irregularities of the votes of the jurors is balanced, and that the ratio of the number of acquittals to that of the accused attains permanence. This ratio varies yet more from one department to another; and the number of trials in each jurisdiction of Assize Courts, even when as one reunites the known results of many years, it is not considerable enough either that one can decide, with a sufficient probability, what are the parts of France where the juries have more or less tendency to severity. There is scarcely only the department of the Seine where the criminal trials are numerous enough so that the annual ratio which is observed between the number of acquittals and that of the accused is not very variable, and can be compared to the one which takes place in all France. The number of individuals led each year before the Assize Court of Paris is around 800, or nearly the ninth of the number corresponding to all of France. From 1825 to 1830, the proportion of acquittals has varied from 0.27 to 0.40, and its mean value has been only 0.35, while it was raised to 0.39, or to 0.04 more, for all France. But the ratio of the number of convictions to the number of accused, rendered by the smallest majority of seven votes against five, is very little different for Paris from that which it was for the whole of all the Assize Courts.

“Such are the data that experience has furnished to the present on the decisions of juries. The precise object of the theory is to calculate according to these data, when they will be complete, for a jury composed of any number of persons, judging to any majority whatsoever, and for a very great number of judgments, the proportion of acquittals and of convictions which will take place very probably, and the chance of the error in a judgment, either that it convicts, or that it absolves. The product of the probabilities of the error in a judgment of conviction, multiplied by the chance that it will take place, is the true measure of the danger to which society exposes a non-guilty accused; the product of the chance of error of an acquittal and of the probability that it will be pronounced, is that of the danger that society itself courts, and that it matters to it equally to know, since it is the imminence of this danger which can only justify the eventuality of an unjust conviction. In this important question of humanity and of public order, nothing could replace the analytical formulas which express these diverse probabilities. Without their aid, if there were the question to change the number of jurors, or to compare two countries where it was different, how would one know that a jury composed of twelve persons, and judging with the majority of at least eight votes to four, offers more or less guarantee to the accused and to society, that a jury composed of nine persons, for example, taken on the same list as before, and judging at such or such majority? How would one decide if the combination which existed before 1831, of a majority of at least seven votes against five with one intervention of judges in the case of *minimum*, is more advantageous or less favorable than that which is going to take place now, of the same majority with the influence of the question of

extenuated circumstances? We will say in a short while in which manner the calculus decides between these two combinations.

“The formulas of probability by which one just defined the object, and which one will find in this work, have been deduced, without any particular hypothesis, from the general and known rules. They contain two special quantities which depend on the moral state of the country, of the mode of criminal procedure actually in use, and on the skill of the magistrates charged with supervising it. The one expresses the probability that a juror taken at random will not be deceived in his vote; the other is the probability before the opening of the arguments, of the culpability of an accused taken equally at random. These are the two essential elements of the question of the criminal judgments; the first is independent of the second, but the one here can depend on the other. Their numerical values must be concluded from the data of experience, just as the constants contained in the formulas of astronomy are deduced from observations. The entire solution of the problem which one has proposed in these researches required therefore the concurrence of theory and experience. This leaves yet much to desire, we have determined the actual values of the two elements as well as it has been possible, according to the preceding data, which would be able to be completed by the following. The new law requires that the jury make known the cases where its decision has been rendered at the majority of seven votes against five; we would be able therefore to mention in the *comptes généraux*, the number of times where this circumstance will have taken place, separately in the cases of thefts and in the cases of attempts against persons, and separately also for the accused of the two sexes. It will be possible then to calculate, for these diverse cases, the values of the two elements in question, when those which have been able to be determined, are returned, without distinction of sex of the accused, to some affairs of every nature, in the proportion where they are presented annually. However we have set aside the political affairs, of which the number, in 1832 and 1833, have been rather considerable in order to induce in error on the relative consequences in these two years. Under the control of one same legislation, each of these two elements, by its nature, would be able to vary progressively, and not have the same value either in the different parts of France. It is this that a long series of observations, and the calculations on which they will be based, will be able to teach to us or to our successors.

“Here is actually the announcement of the numerical results that we will find in this work, and which is reported in the four recent epochs where the legislation has been different: in the years anterior to 1831, in that year 1831, in the years posterior, and finally in the actual epoch.

“Before 1831, the probability that a juror was not deceived in his vote was a little below  $\frac{3}{4}$  for all France, and a little higher than this fraction for the department of the Seine in particular. The precise sense of this result of the calculation and of the observation is that, if one had taken at random a very great number of jurors, 10000 for example, and if one had submitted to them the criminal affairs of all natures which are judged in France, during many years, to the number of 500000 in order to fix the ideas, the three-fourths of the product of 10000 and of 50000 would express, very nearly, the number of votes of conviction or acquittal that one must regard as true, that is, convicting of the guilty accused, or absolving of the accused of whom the guilt was not probable enough in order to render their necessary conviction. Despite a greater ex-

perience of the criminal trials that the judges have without doubt, their chance of not being deceived in their votes is however little different from that of the jurors, at least in the most doubtful cases where the majority of the jury is formed only by seven votes against five. Indeed, these cases themselves are presented in the number of 1911 during the five years elapsed from 1826 to 1830; the Assize Court, composed then of five advisors, and called, in the case, to intervene, is joined 314 times with the majority of the jury; now, it would have to be joined only 291 times, by supposing the probability of not being misled equal for the jurors and for the judges; and while these two numbers 314 and 291 are not considerable enough to decide at what point this hypothesis can stray from the truth, their slight difference suffices to prove that there must exist also quite little between the chances of error of the judges and the jurors. The chance of error of these does not proceed therefore, as we would believe it, from their defect customarily: there is place to think that its principal cause is the arbitrariness which remains in the conscience of each, judge or juror, on the degree of probability necessary in order that a man convict another.

“The calculus combined with observation, proves also that in France the probability that an accused is guilty, when he appears before the Assizes Courts, for an affair of any nature, has 0.64 for the value; in Paris it is notably greater, and is raised to 0.68; this which we must regard as a fact, which would be able to depend on the cleverness of the magistrates, or to other causes. If the accused were judged uniquely after the information anterior to the opening of the arguments, one should therefore, in a very great number of affairs, annually for example, convict 64 or 68 out of 100 of them; in the six years which have preceded 1831, the proportion of the convictions differed little from those, since it was 0.61 in France and 0.65 in Paris; but a conviction, when it was pronounced, increased, in a very great ratio, the probability of guilt which took place before.

“I just remarked that this probability anterior to the arguments, surpasses the ratio of the number of convictions to that of the accused. As soon as the first of these two fractions is greater than  $\frac{1}{2}$ , one demonstrates, in fact, that the second must always be inferior to the other, or at least contrary, if it is not impossible, it will be beyond all feasibility, for a number of affairs that one supposes very great. It is a point of this theory on which it matters most to call attention. There results from it that, whatever be the modifications that one makes a jury to undergo, and the majority that one will require for its decisions, as long as nothing will be changed in the mode of the criminal procedure, one must regard the fraction 0.64 as a limit which could never overtake the proportion of the annual convictions in all France, that it could attain if there were no chance of error in the vote of the jurors, and of which it will be approached more and more in measure as this chance will diminish more. However, this probability of 0.64, of guilt before the opening of the arguments, is relative to any affair whatever which will be taken at random among those which the Assizes Courts have to judge; it can be different, when one considers separately a special class of affair; and it is necessary that it be greater in the accusations of theft, since the proportion of the convictions, which it must always surpass, is raised then to 0.66. Reciprocally, if this particular proportion surpasses its general value, this does not keep uniquely to a greater severity of the jurors when the question is of crimes against property. This difference results also from that which, by the nature of these crimes, the magistrates charged of the

information anterior, arrive to establish a greater probability that the accused is guilty.

“All things equal besides, it is evident that the proportion of convictions would diminish in measure as one would require of a jury a greater majority. If it were necessary, as in England, the unanimity of the twelve jurors, either for conviction, or in order to absolve, the probability of a conviction would be little different from a 50th, and that of an acquittal will be a little less than a half; this which would render the decisions very difficult, at least it was no more often a kind of arrangement between the jurors, and that a part of among them made the sacrifice of their opinion. One sees even that, without that, the unanimous acquittals would be more rare and more difficult in the ratio of the double to the simple.

“According to these values  $\frac{3}{4}$  and 0.64 of the two elements which contain the formulas of the probabilities, and which have been deduced from observations anterior to 1831, we find 0.06 for the probability of the error of a conviction rendered by the majority of seven votes against five; but, in this case, if the majority of the court is joined to that of the jury, that which was necessary for a definitive conviction, this probability was reduced to the 7th of its size. So much to this majority as to one greater, the proportion of the not guilty convicted must be, before 1831, a little over 15 ten-thousandths of the number of the accused, or of very nearly 10 or 11 per year in all France. At the same time, the proportion of the guilty and acquitted accused, should be raised to a little more than a 30th, that is around 250 each year. But one must not lose sight of the sense that we attach to these words *guilty* and *not guilty*, which have been explained above, and of which there results that the first proportion is only an upper limit of the number of the really innocent convicted, while the second is, on the contrary, a lower limit of that of the individuals acquitted, although they were not innocent. It is not necessary either to forget that the probability of the error of judgments, that we conclude from the calculus and from observation, is reported for the whole of the trials which are subject during one or many years in the Assize Courts: to determine the chance of error of a judgment rendered in a known and isolated affair, is impossible, according to me, at least to found the calculation on some completely precarious hypothesis, which would lead to some very different results, and, nearly, to those which one could wish, according to the assumptions which one would have adopted.

“In 1831, the necessity of at least eight votes against four for conviction, must have, according to the calculus, lowered the proportion of the not guilty convicted by a thousandths of the total number of the accused, and raised that of the acquitted guilty a little more than a tenth, that is, nearly a fourth of the pronounced acquittals.

“In the following years, where the question of the extenuating circumstances has been introduced, the number of convictions is increased, and it has become 0.59 of the number of the accused, instead of 0.54 which it was in 1831. This proportion 0.59 itself was approaching the probability 0.64, anterior to the arguments, that the accused is guilty, it is followed from it that that of the error of the vote of the jurors is diminished; and the probability that a juror is not misled in his vote, which was previously a little under  $\frac{3}{4}$  has become nearly equal to  $\frac{4}{5}$ . In this state of things, the proportion of the not guilty convicted was no more than a 4000th of the number of the accused, or less than two convictions per year, for all France; and as for that of the accused guilty and convicted, its value ought be 0.035, or very nearly 250 individuals each year, as before 1831.

“One is not able to know in advance if the secret imposed on the votes of the jurors by the new law, will increase or diminish their chance of not being misled. By supposing that it remains the same, and that it was for measure the fraction  $\frac{4}{5}$ , as before; the number of the convictions will increase, by reason of the majority of at least seven votes against five, substituted for this of at least eight against four; this number will be contained between 0.62 and 0.63 of that of all accused; at the same time the proportion of the erroneous convictions will be raised to 0.0016, or to the sextuple of that which it was in the preceding years; that is to around eleven per year in all France, as before 1831; but the number of acquitted guilty will be much less than in any other epoch, and will be lowered to 0.014 of the number of the accused. In relation to the chances of error of convictions, there is therefore very little difference between the legislation anterior to 1831 and that which rules us today; the condition of majority was the same in the two epochs, the effect of the intervention of the court which would be able to take place formerly is found balanced very nearly by that of the question of the extenuating circumstances; but relatively to the chances of errors of acquittals, the advantage is on the side of the actual legislation, in the ratio of five to two.

“The distinctive character of this new theory of the probability of judgments is therefore to determine first, according to the data of the observation in a very great number of affairs, the chance of error of the votes of the judges and that of the guilty of the accused before the opening of the arguments. It must agree with all the numerous kinds of judgments; to those of the correctional police,<sup>6</sup> of military justice, of civil justice, provided that one has, in each kind, the data sufficient for the determination of the two elements in question. It must also apply to the judgments, that I have no need to quantify, which have been rendered in very great number by the extraordinary tribunals, during the unhappy times of the revolution; but in this regard it is indispensable to enter into some explications so that there remains no doubt on the generality and the exactitude of the theory. The difficulty that this case of exception presents has not escaped to some persons who wished much to listen with interest the results of my work.

“An accused is able to be condemned, either because he is guilty, and that the jurors are not deceived, or because he is innocent, and that the jurors are deceived. The ratio of the number of convictions to the one of the accused does not vary when the probability that the accused is guilty before the judgment, and that of the error of the vote of each juror, by changing both into their complements to unity. It remains the same, for example, when these probabilities are  $\frac{3}{4}$  and 23, and when they are only 14 and 13. It is also one same value, when they differ both very little from certitude, or from unity, and when they are both near null; and in these extreme cases, the number of convictions deviates very little from the one of the accusations. For this reason the equation that it is necessary to resolve in order to determine the magnitude of these two probabilities are always susceptible of two solutions real and inverses of one another. However, each of these two solutions has a character which distinguishes it: by adopting the one, the probability that a convicted accused is guilty will be greater than that of his innocence; the contrary will take place by adopting the other. In the ordinary cases, it is therefore the first solution that one must choose, for it would not be reasonable to

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<sup>6</sup>Translator's note: The *Tribunal de police correctionnelle* is the court of petty sessions.

suppose that the tribunals were unjust or judge most often in the reverse of common sense. But it is not likewise when the judgments are rendered under the influence of the passions; it is no longer then the reasonable root of the equations, it is the other solution that it is necessary to employ, and which gives to the convictions a so great probability of injustice. It is thus this social anomaly, of tribunals of which the greater part of the judgments are erroneous, is found comprehended in the generality of the algebraic formulas. In this theory, the iniquity of the juror and the passion of the accuser are considered as some causes of error, as well as too much pity or an excess of indulgence; and the calculus is established on the result of the votes, whatever be the motives which have dictated them.

“In the tribunals of correctional police, the ratio of the number of the acquittals to the one of the accused is comprehended between 0.14 and 0.15, according to the mean of nine years and all France. This single datum does not suffice in order to determine the probability anterior to the judgment, of the guilt of the accused, and the chance of error of the vote of each juror; by supposing the judgments pronounced by three judges, this which appears to hold generally, it would be necessary also to know according to what proportion the condemnations have taken place by unanimity, or by the simple majority of two votes against one. But whatever be this proportion, which is not known, one is able at least to be assured that the probability that the judges of the tribunals of correctional police are not deceived in their votes, is superior to that which took place for the jurors before the question of the extenuating circumstances; and if one assimilates, I admit a little gratuitously, the vote of the judges to the one of the jurors since this question has been introduced, and since one takes  $\frac{4}{5}$  for the probability that it is not erroneous at all, one finds  $\frac{19}{20}$  for the probability, before the judgment, of the guilt of the accused. This probability, according to their condemnations, would be yet greater and would differ from unity only by 0.006; but the proportion of the acquitted guilty ones would be raised nearly by a 10<sup>th</sup>, or to the two-thirds of the total number of the acquittals.

“The military tribunals are composed of seven judges; the convictions are able to be pronounced only by the majority of at least five votes against two; one evaluates their number at two-thirds of the one of the accusations; this which is very nearly the same proportion as in the Assizes Court before 1831 and in the actual period. This single datum is not sufficing in order to determine the special values of the two quantities contained in the formulas of probability, one is therefore obliged to make a hypothesis, more or less possible, on one of these values: if one supposes that the probability of not being deceived is equal to  $\frac{4}{5}$  for a military judge, as for a juror, one finds 0.77 and 0.98, for the probabilities that the accused is guilty before the judgment and after he is convicted, and a little more than 0.011 for the proportion of the acquitted guilty ones. Although the probability 0.98 approaches much to certitude, however the proportion of the not guilty convicted would be elevated to  $\frac{1}{90}$  of the number of the accused; but it is convenient to observe that the court-martial comprehend in their attributions, the criminal process and those of simple correctional police; and there is place to believe that it is in those which are found the greatest number of erroneous convictions; this qualification being taken besides in the sense which has been explicated above. Although it is in it, by comparing the military justice to the juries on the actual legislation, there

results from the proportions cited previously that the chances of error would be eight or ten times less in the Assizes Courts, either for acquittals, or for convictions.

“When the concern is judgments in civil matters, the formulas of probabilities, instead of two special quantities, contain no more than one, this which expresses the probability that each judge is not deceived in his vote. In the tribunals of the first instance, the judgments are rendered by three judges, in general, according to the information which has been given to me; but one does not know the ratio of the number of cases where they pronounce by unanimity, to the number of cases where they decide by the simple majority of two votes against one; and, a defect of this datum, it is not possible to calculate directly the chance of error of their votes. For the judgments of which there is made a call before the Royal Courts, one is able to calculate this chance by comparing the number of those who are confirmed to the number of those which are not, and supposing that it is the same for the judges of the two successive degrees. Although this hypothesis deviates perhaps much from the truth, I have admitted however, finally to be able to give an example of the calculation of the error to fear in the judgments in a civil matter. The truth or the good right would result from the decision, necessarily unanimous, of judges who would have no chance of being deceived; in each affair *ce bon droit absolu* is an unknown thing: nonetheless, one understands by the votes and the *erroneous* judgments those which are contrary to it; and the question consists in determining their probabilities, or that which is the same thing, the proportions according to which they would occur, to very nearly and very probably, in the numbers of cases sufficiently great.

“One finds in the *Compte général de l’administration civile*, recently published, the number of judgments of first instance which have been confirmed by the Royal Courts, and the one of the judgments that they have broken, during the last three months of 1831, and the years 1832 and 1833. The ratio of the second of these two numbers to their sum, has a little less than 0.32 for the value in all France; it has not varied from one year to another by a 50<sup>th</sup> of this mean value; so that despite the diversity of the affairs which had to have been presented, and without doubt also the unequal instruction of the magistrates of all the realm, it has sufficed however around 8000 arrests pronounced annually in order that the ratio of which there is concern attained nearly a constant value; this which offers again an example quite remarkable of the universal law of large numbers.

“By means of the value 0.32 of this ratio, and by taking the number seven for the one of the advisors of the royal courts who pronounce the arrest of name in civil matter, one finds 0.68, or a little more than  $\frac{2}{3}$ , for the probability that one of these advisors, or a judge of first instance, taken at random in all France, is not deceived in speaking in an affair, taken also at random, among those who are submitted annually to the two degrees of jurisdictions. It is possible that this possibility is different in the affairs judged in first instance and of which the parties have not called at all. According to this fraction 0.68, the probability that an arrest of court of appeal is conformed to the good right has for value 0.646, when it agrees with the judgment of the 1<sup>st</sup> instance, and 0.203 only when it is contrary to it; in the first case the probability that an arrest is erroneous, is only 0.036; in the second case, it is elevated to 0.114. One concludes from it that out of a very great number of arrests of the royal courts, the proportion of those which are not conformed to the good right deviate very probably very little from

the sum of the last two fractions, or by 0.15. If one of these arrests, erroneous or not, were submitted to a revision of a second royal court, there would be the probability 0.754 that it would be confirmed, or odds a little more than three against one.

“The questions which are treated in this work being able, if I do not deceive myself, to interest some persons to which the mathematical analysis is not familiar, I have believed useful to expose first, with some development the results which it contains, and the principles which are the base of it. It will be divided into two sections: the first will contain the general rules and the most useful formulas of the probabilities; one will find in the second their application to the special question of the probability of judgments.”