

Note sur le calcul des probabilités*

M. Poisson

Comptes Rendus II, 1836, pp. 395–400

“I demand to the Academy the permission to expose briefly some remarks proper to fix the ideas on the discussion which is raised within its last session, relatively to the results of the calculus of probabilities.

“No part of mathematics has more certitude than this calculus; its rules are demonstrated as rigorously as the properties of numbers; and I have no knowledge that some geometers are deceived by applying it. If I differ with Laplace in that which concerns the decisions of jurors, it is that I am departing from different data, and that the problem which he has resolved is not really the same as the one of which I myself am occupied. The sense that one seems to attach to the phrase underlined in the *Compte rendu* of the last session, will be the contrary of his thought.¹ No person more than he had an idea impeded on account of the utility of the calculus of probabilities in a great number of questions relative to some physical things, moral or of public administration, or physics and morals at the same time, as the errors of observations, which depend on the construction of the instrument and on the aptitude of the observer. If his opinion were not generally known, I would be able to cite numerous examples of them extracted from his works: in speaking of a calculation, even approximate, on the probability of testimony, he says² that *it is always preferable to the most specious reasonings*; on the occasion of decisions by jurors, he says again *I am going to try to apply the calculus to this subject; persuaded that it is always the better guide, when it is applied on the data that good sense suggests to us, etc. etc.* It is Jakob Bernoulli who has given first to the applications of the calculus of probabilities the importance that they merit, by means of the theorem of which the science is indebted to him; before, this calculus had been employed only in the determination of chances in the different games of chance. Condorcet, who we have cited, has written his work on the views of a practical utility, and at the invitation of minister Turgot, of whom the name will be always an authority in the matters of public administration.

“Without doubt Laplace is shown a man of genius in the *Mécanique Céleste*; it is he who has given proof of the most penetrating sagacity in order to discover the causes of phenomena; and it is thus that he has found the cause of the acceleration of the movement of the Moon and that of the great inequalities of Saturn and of Jupiter, that Euler and Lagrange had sought fruitlessly. But one can say that it is yet rather

*Translated by Richard J. Pulskamp, Department of Mathematics & Computer Science, Xavier University, Cincinnati, OH. January 20, 2010

¹*Essai philosophique sur les Probabilités*, page 12.

²*Ibidem*, page 138.

in the calculus of probabilities that he has been a great geometer; because these are the numerous applications that he has made of this calculus which have given birth in the calculus to the finite partial differences, to his method for the reduction of certain integrals in series, and to that which he names the *theory of generating functions*. One of the most beautiful works of Laplace, his *Memoire of 1775*, has also for occasion, and in part for object, the calculus of probabilities. We believe that a subject which has fixed the attention of similar men is worthy of ours, and we try, if that is possible to us, to add some thing to that which they have found in a matter so difficult and so interesting.

“The invariability of the ratios between the very great numbers of events of a like nature, as much as their general causes remain the same, is now demonstrated *à priori*. I swear that I fasten a great importance to this demonstration; but, important or not, again it is necessary to know it in order to judge if it is exact. However, in expecting that it had been published in the work of which I am actually occupied, a great number of examples of each nature can suffice in order to put beyond doubt the constancy of its ratios, either in the physical things, or in the moral things: and, in effect, one does not know what difference there would be able to exist in this regard between these two kinds of things, since there is no question here of the same nature of the causes, but only of the chances known or unknown that they give to the events, and which are sometimes again more variable and more irregular for the physical things than for the moral things. Thus, in my work on the *proportion des naissances annuelles des deux sexes*,³ I myself was supported by the results of ten years of observations, during which this proportion had not varied by a half-hundredth of its mean value. Since the publication of this memoir seven new years of observations have been added to the first, and the proportion of which there is question has not varied more than a half-hundredth. However, this proportion varies much in the different households, and consequently also the unknown cause which renders, in human kind, the masculine births preponderance; it varies also much from one department to another, and for one same department, from one year to the following; and there is even sometimes one or two departments where the number of feminine births from one year exceeds the one of the masculine births.⁴ I will make note that this difference being slight, it is necessary a very great number of observations in order to determine it, and that the calculus of probabilities is indispensable in order to show that it is not the effect of chance, but that it has very probably a special cause, by virtue of which the same difference will be reproduced constantly by the sequence. In moral things, the same permanence of the ratios among the great numbers are observed equally; and in this regard the most striking example is the one that I have cited already many times, of the very nearly constant ratio between the numbers of the convictions pronounced annually in France by the jurors, and the total number of the accused: during six consecutive years that the legislation on the jury has not changed, this ratio has not varied more sensibly, and it is also found very nearly the same in Belgium, as it results from the count rendered by the administration of justice in this realm, which has been published by the government. This ratio has changed with the diverse legislations on the jury which are succeeded in

³*Mémoires de l'Académie*, Tome IX.

⁴See on this point the *Annuaire du Bureau des Longitudes* of each year.

our country; and these changes present a circumstance well worthy to note and very proper to show the liaison of the most varied effects to their general causes. Before 1831, the jury was able to convict by a majority of at least seven votes against five, and the proportion of the convictions elevated to 0.61; when the decision was rendered to this *minima* majority, it was obliged to say it; and one knew from this manner that the proportion of the convictions pronounced by the majority of seven votes against five, was 0.07; by subtracting this fraction from 0.61, there remains 0.54 for the proportion of the convictions pronounced by at least eight votes against four. Now, in 1831, the law has required for conviction the majority of at least eight votes against four, instead of that of seven against five, and the ratio of the number of convictions to the one of the accused is effectively lowered to 0.54: the difference is found only in the thousands, of which I have set aside in these citations.

“This ratio is able also to be changed by the influence of other general causes. At Paris, it is a little greater than in the rest of France, this which is able to hold to that which the number of annual accusations is proportionally more considerable than in France entire; a circumstance which renders also the danger of the greater acquittals. The ratio of which there is question is no longer the same for the accused of the two sexes, nor for all the kinds of crimes. When one will have reunited a greater number of observations, one will be able to determine its very probable values, by having regard to these diverse circumstances, and for the different parts of our country. Until there, one is obliged to be content with a mean value of this ratio for France entire, which is no less a constant quantity, as this results from experience and from theory, and that it is always useful to know.

“Among the numerous results from the calculus of probability, some are able to be confirmed by experience, and have been constantly; the others, by their nature, are not susceptible to any verification, this which does not prevent that they merit the same confidence, since they derive from the same principles and are demonstrated by the same reasonings. This calculus has besides that in common with the other parts of mathematics. Thus, for example, the last return of the comet of Halley has been first calculated and next observed directly, and observation has confirmed, in a remarkable manner, the result of the calculus; but if some astronomer, by following the same methods, and by calculating equally well, informed today to determine the period of its next return, which will take place toward 1910, it will be reasonable to believe with the same confidence in this future result, although assuredly no one of us can hope to verify it.”

“After the reading of this note, M. Poincot took the podium, and raised with a new force against this doctrine of that calculus applied to moral things. I know very well, says he, that the calculus of the probabilities, considered in itself, is as exact as arithmetic; and that likewise is of pure definition, since the *probability* of each thing is regarded as a *number*. I imagine still that this calculus is applied rather naturally to games of chance, to lotteries, to annuities, to assurances, etc., in a word to all the questions where one can make an exact enumeration of diverse cases which are, or that one supposes equally possible. There is there nothing which is not conformed to the natural indications of good sense. But that which repels the mind, it is the application of this calculation to the things of moral order. It is, for example, to represent by

a *number* the *veracity* of one testimony; to liken thus men to as much beyond, of which each has many faces, some for error, the others for truth; to treat likewise other moral qualities, and to make so many *numerical fractions*, which one proposes next to a calculation often very long and very complicated; and to dare, at the end of these calculations, where the numbers correspond only to some such hypothesis, to draw some consequence which is able to determine a wise human to carry a judgment in a criminal affair, or only to take a decision, or to give a counsel on one thing of some importance. Here is that which appears to me a kind of aberration of the mind, a false application of science, and which would be proper only to discredit it.”

M. Poisson responds “that the first rules of the calculation of the probabilities only are near evidents, and that the object of this calculus is to restore, by some certain reasonings, to these simple cases, the most complicated cases; that the theory of Jakob Bernoulli, which appears so simple and so natural, is however quite difficult to demonstrate,⁵ and that its illustrious author had employed twenty years of meditations; that the calculus, far to alter the first perceptions of the senses, never lacks to confirm them when they are exact, and that it rectifies them always when they would contain some illusion; that in some cases, even quite simple, good sense would not suffice in order to determine the probability of events; that in the game of *heads* and *tails*, for example, there is evidently odds one against one for the arrival of one or the other of the two faces of the coin in a first trial, but that in two consecutive trials, there is odds more than one against one for the similitude of the results, that which the calculus alone can teach us; and that finally it repeats that if he is deceived, it is not by some general considerations that one must satisfy it, it is by showing, in a precise manner, where is the defect of his calculation and of his reasoning, and their discordance with observations.”

⁵*Traité élémentaire du Calcul des Probabilités* of M. Lacroix, page 53.