# MÉMOIRE
## *sur l'application du Calcul des probabilités la valeur du témoignage.*[*]

MM. PREVOST and LHUILIER

*Mémoires de l'Académie royale des sciences et belles-lettres... Berlin.*[†]
1797 pp. 120–52.

The end of this memoir is rather to understand the actual state of this theory, than to add anything new. However in determining the points of view under which one can envision it, we believe to make a useful work, & we encounter even some consequences which are not without interest. In order to follow a clear order, we examine first the simplest principles of the application which we have planned to make, those which are, so to speak, only a translation of the general principles of stochastic into historic or judicial language:[1] next we will complicate a little this language in order to bring together for us some real things; & either by some partial abstractions, or by some alterations done by design to the general principles, we substitute in it others which become more easily applicable to our subject

### SECTION I.
*Rigorous & abstract principles.*

§ 1. Testimony is an argument. JAC. BERNOULLI has established some principles on the estimation of the value of arguments. He distinguishes the *mixed*, which prove always to determine some thing; namely true or false; the direct or the contradictory: & the *pure*, which prove in some cases the truth of an assertion, & in the other cases prove nothing. In order to estimate from these two kinds of combined arguments, he

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†Read 23 August 1797.

[1]On these principles we reference our two preceding memoirs inserted into those of the Academy for 1796. In these memoirs we have constantly used a common emblem in order to fix, by imagination itself, the attention of our readers on the abstractions which we had in heart to expose them. This emblem is the one of the polyhedron. We have constantly supposed that this die could bear faces of two kinds, ace & non-ace. It is as if we had said some faces marked & some faces non-marked, some faces black & some faces white. In a word, two contradictory cases, which are mutually exclusive, & between which in the abstract theory, the mind has no reason to choose. Now quitting this abstract theory, & supposing it clarified; we attempt to make application of it to a particular & useful object. Thus in order to verify our assertions, it will suffice always to restore them to the emblem of the die. A witness can be conceived under this form. The true or false depositions will be aces & non-aces. And we will make, so to speak, only the translation of our principles.

gives a formula

$$\left(1 - \frac{cfi}{adg} \times \frac{ru}{qt+ru}\right),$$

which LAMBERT has subjected to a quite simple proof, to which it has not at all resisted.[2]

If one of the arguments which proves the contradictory of the proposition which one argues is supposed certain, (that is to say if one makes $q$ or $t = 0$), the formula of BERNOULLI conserves a positive value $\left(1 - \frac{cfi}{adg}\right)$; while it must give a null probability. Indeed this formula expresses the value of the arguments for the direct proposition, it is clear that the supposed certitude of the contradictory annihilates it.

§ 2. LAMBERT indicates next the cause of this error. JAC. BERNOULLI has counted as valid all the cases in which the pure arguments, envisioned in themselves, prove the truth, either as the cases of the mixed arguments combined with them prove the truth, or that they prove its contradictory. But this combination, made without having regard to the particular of which there is question, presents some cases absolutely inconceivable, & impossible to realize; which consequently must be excluded, & can not be enumerated in the totality of the equally possible cases. The arguments conspiring, & proving at the same time one same proposition, are totally in the true, or totally in the false; but they can not be supposed at the same time one in the true & the other in the false. This case, which is considered in the abstract theory of combination, is not at all in the number of those which one must count in the estimation of the arguments.

§ 3. The error committed by JAC. BERNOULLI has been repeated many times after him, before LAMBERT had addressed it. In particular, in the application of this theory to the object of our actual researches, one estimated the value of the argument drawn from the simultaneous & uniform depositions of many witnesses, by taking the complement of the formula by which one estimated the probability of their successive lies: this which supposes the same error which JAC. BERNOULLI had committed in his general formula.[3]

[2] *Organon*, T. II. *Phaenomenol*. § 239.

Here is the signification of the letters employed in this symbolic argument. They designate the cases of each kind which result from diverse arguments.

| Argument | 1st | 2nd | 3rd | 4th | 5th | &c, |
|---|---|---|---|---|---|---|
| All possible cases | $a$ | $d$ | $g$ | $p$ | $f$ | &c, |
| Provable cases | $b$ | $e$ | $h$ | $q$ | $t$ | &c, |
| Non provable cases or proving the contrary | $c$ | $f$ | $i$ | $r$ | $u$ | &c, |

The formulas of this author, & in particular the formula cited in the text, tending to determine the probability which results from the combination of these diverse arguments. This is relative to the mixture of the mixed & pure arguments. The first three arguments are supposed pure, & the last two (the 4th & the 5th) are supposed mixed. See *Art. conject.* Pars IV. Cap. III. §§ 4 & 6.

[3] See *Trans. phil.* n. 256 p. 359, & many other Memoirs & Courses where this subject is found treated. (Translator's note: The reference is here to "A Calculation of the Credibility of Human Testimony" by an anonymous author.)

§ 4. We see now how LAMBERT corrects this theory. He remarks that beyond the mixed & pure arguments, one can imagine, & one must admit a third sort of argument, of which BERNOULLI has not given the analysis. This argument is such that the cases where he does not prove the truth are of two kinds: some of these cases prove the contrary of the truth, & the others prove nothing of the whole.[4]

Such is precisely the argument drawn from testimony. In a deposition, one imagines some cases where it establishes the truth, some cases where it establishes the contrary, & some cases where it signifies nothing. Men say true, or they tell lies, or they speak carelessly of that of which they are poorly cognizant. LAMBERT has had regard to this last circumstance, & has made it enter into the symbolic expression of the credibility of a witness. He observes besides that one can easily eliminate this element in all the particular cases where this can be necessary, without his formulas suffer from it.

Let there be a testimony of which such that, out of a number of cases $= v + i + m$, one must believe $v$ times its deposition true, $i$ times insignificant, & $m$ times false, or decidedly false, & consequently establishing the contradictory.

Let next there be a second testimony of which the credibility is represented in an analogous manner by $v'$, $i'$, $m'$.

If these witnesses depose uniformly, LAMBERT finds the credibility of their testimony, that is to say the ratio of the number of cases of truth to all the cases

$$= \frac{vv' + vi' + v'i}{vv' + vi' + v'i + ii' + mm' + mi' + m'i};$$

the probability of the contradictory

$$= mm' + mi' + m'i$$

divided by the denominator of the preceding fraction; & finally the probability of an insignificant deposition $= ii'$ divided by the same. It is to this result that the expressions are reduced a little different than LAMBERT employs.[5]

§ 5. This formula is so general that it comprehends also the case of two discordant witnesses, who affirm the two contradictories. In changing only one into the other the expressions of the probabilities of the true & of the false for one of the witnesses, the formula gives the probability that the other says the truth.[6]

If there are many witnesses, of whom some discordant, one could always resolve the questions relative to the probability of their testimony by these formulas. Because as one supposes that all the depositions are in favor of both of the contradictories; one will divide them into two classes, & one will determine separately the probability of each class by the formula of the concordant witnesses. This operation will reduce the depositions to two composite discordant witnesses. And the question will be resolved. This solution follows naturally from that which precedes.

Here concludes the theory of LAMBERT.

[4] *L. c.*
[5] *Ibid.* § 237.
[6] § 238.

§ 6. If one wishes for a moment to set aside insignificant testimonies, & to admit that all the testimonies are of the class of mixed arguments, one will note a simplification of the operation that we just indicated for the case of discordant witnesses, into a class of rather numerous cases. We suppose many witnesses of whom the credibility must be estimated in a similar manner (as $\frac{5}{6}$, for example). One could neglect all the couples or pairs of witnesses who contradict themselves. Common sense says that one being worth the other, this deduction from arguments equal on all sides alters not at all the conclusion. And the calculus gives the same result, as we will show it presently. (§ 8).

§ 7. The formula determined by LAMBERT, after the consideration of each particular case of combination, is derived easily from principles.[7] This formula (by neglecting the case of insignificance, that is to say the terms affected with the factor $i$), becomes for the concordant witnesses this here,

$$\frac{vv'}{vv' + mm'}.$$

Now here is how we will have found it.

The accord of the depositions is manifestly posterior to the reasons which have engaged the witnesses to depose as they have done, or to the cause of this accord among the depositions. Now this cause is that which we wish to determine. There is offered only two possible causes of one such effect. The accord results either from the simultaneous truth, or from the simultaneous lying of the witnesses. And since the respective credibilities of each witness taken apart are $\frac{v}{v+m}$, $\frac{v'}{v'+m'}$; there results from it that the cases of simultaneous truth are

$$\frac{vv'}{(v+m)(v'+m')};$$

& of simultaneous lying

$$\frac{mm'}{(v+m)(v'+m')};$$

& that consequently the probabilities of the accord of the two witnesses by either of these causes are between them $= vv' : mm'$. Therefore (by the etiological principle) such is also the ratio of the probabilities that this accord has been produced by the causes. And since there is not another conceivable cause, one has the absolute probabilities of these causes, (by an immediate consequence of the etiological principle) equal to $\frac{vv'}{vv'+mm'}$, $\frac{mm'}{vv'+mm'}$. And in particular the probability of the truth of the witnesses $= \frac{vv'}{vv'+mm'}$.[8]

[7] Exposed & discussed in our preceding Memoirs cited above.

[8] It is perhaps apropos to prevent here an objection which could hit at first glance. Neither the time, nor the space influence on the probability. Consequently all agreement, which is returned to this quantity, must be able to be transported & multiplied indifferently under these two relations. Why therefore does one treat by two diverse theories (*a priori* & *a posteriori*) the composite probability of the successive & simultaneous testimonies? I respond that one could without doubt treat them by one same theory: but that it would be necessary to employ some strange hypotheses. Now in the applications one has not in view the uniformity, but the reality. The case of many simultaneous witnesses, testifying all at once to a single judge, is totally natural. The same hypothesis transferred from space to time, will be little applicable.

In general when $n$ witnesses of whom the truths are $v$, $v'$, $v''$, $\dots v^N$; & the falsities $m$, $m'$, $m''$, $\dots m^N$; are agreed to depose on a simple fact, the probability that the fact is conformed to their testimony, is to the probability of the contrary, in the ratio of $vv'v'' \cdots v^N$ to $mm'm'' \cdots m^N$; & hence these probabilities are respectively

$$\frac{vv'v'' \cdots v^N}{vv'v'' \cdots v^N + m,\, m',\, m'',\, \dots m^N} \;\&\; \frac{m,\, m',\, m'',\, \dots m^N}{vv'v'' \cdots v^N + m,\, m',\, m'',\, \dots m^N}.$$

In particular let $\begin{array}{c} v = v' = v'' \cdots v^N \\ m = m' = m'' \cdots = m^N \end{array}$. These probabilities are $\frac{v^n}{v^n+m^n}$ & $\frac{m^n}{v^n+m^n}$.

§ 8. Now if out of $n + n'$ witnesses, there are $n$ concordant on one part & $n'$ concordant on the other; such that these two classes disagree between them & affirm the two contradictories. The probability that the fact is conformed to the concordant testimonies of the first witnesses, is to the probability that it is conformed to the testimonies of the second, in the ratio of $v^n \times m^{n'}$ to $m^n \times v^{n'}$; & hence these probabilities are

$$\frac{v^n \times m^{n'}}{v^n \times m^{n'} + m^n \times v^{n'}} \;\&\; \frac{m^n \times v^{n'}}{v^n \times m^{n'} + m^n \times v^{n'}}.$$

Let e.g. $n > n'$; the ratio of these probabilities is the one of $v^{n-n'}$ to $m^{n-n'}$; & hence these absolute probabilities are respectively

$$\frac{v^{n-n'}}{v^{n-n'} + m^{n-n'}} \;\&\; \frac{m^{n-n'}}{v^{n-n'} + m^{n-n'}};$$

the same as if $n - n'$ witnesses had deposed in the sense of the witnesses of the first class.

§ 9. The abstract & rigorous principles of evaluation of testimony have the inconvenience to offer few applications. One is scarcely invited to reason with such precision in the cases where one could employ it. A judge can nearly never determine for each case the veracity of the testimony which he hears. However one senses that the judgment which he carries on the value of each testimony is the effect of some implicit calculation of his credibility. It appears to me that he divides tacitly the testimonies into many classes, & that he recall vaguely the diverse depositions which he receives.

A historian uses it likewise; often also in the judgments relative to the objects of his researches, he is uniquely directed by his strength of confidence common to the testimony of men, without which it is in his power to distinguish, as the judge, from the classes of credibility. It happens all the time that a fact is principally founded on a tradition of which the origin is obscure.

§ 10. But it is chiefly the complication of facts which are the object of testimony, which renders very difficult the application of the abstract theory. One comes to see that this theory supposes a very exact analysis of the fact, & of all the elements of credibility which found it. But 1° for that which concerns the traditional facts, there are not such of them, which are simple. Because suppose one of them, so little composite, that it is reduced to a single question addressed to the witness, of which the response is *yes* or *no*, without any accessory. I say further that the testimonial argument will offer always a double question; because each witness of the traditional chain, affirming or denying touching a fact which he has not seen, says two things. *One has said to me.* And: *That which one has said to me is quite that which I repeat.* One demands of him: *Has one said to you this fact?* Whatever be his response, the doubt on his truth can carry on two points. *Perhaps one has nothing said* to the witness which responds affirmatively. Or: *Perhaps* to him *one has denied* that which he affirms. (Reciprocally for the inverse case.) 2° The ordinary facts on which one deposes are extremely complicated. The least reason supposes nearly inevitably a tacit or expressed affirmation on a multitude of circumstances of time, of place, & of each other kind. We know that one will respond to that which there is means to pass, by the theory, from the simple to the composed: & that one can determine the formulas which, for each degree of composition of the facts, will give the probability of the testimony. We do not deny it. But we say that if one wishes to reflect in the least on the enormous complication that these formulas will offer as soon as the facts will be complicated, & that one will have some varied dispositions on all these circumstances, one will renounce the expectation to render them applicable. 3° Not only the facts are complicated among them, but the arguments which ground the truth of each fact are mixed & of difficult analysis. In a way that each circumstance being envisioned as a fact apart, offers in the applications a new estimation to make of the credibility of the witness relative to the nature of the fact, this which renders variable this quantity, & forces the calculator to change his givens in each case, & many times in a like case, if it is complicated in the least. 4° Finally it is quite difficult to have studied a man in a manner to take count in part of the cases where his disposition is insignificant, by opposition to those where it is decidedly false.

Such are the difficulties which the exact application of the rigorous principles of the calculus of testimony present: Principles of which LAMBERT has first traced the true theory, & of which it is necessary always to depart more or less directly.

## SECTION II.
### *Method of hypothesis.*

§ 11. There results from the preceding reflections, that the matter of testimony is one of those extremely complicated matters, of which rigorous analysis is nearly impossible, & to which in consequence one can not at all with advantage apply directly the principles of an abstract theory. We infer from it this consequence, it is that it is necessary to have recourse to a method of indirect application. And the one which appears to us to be acceptable to this subject is that which is frequently & usefully employed in the matters of physics which offers the same difficulties. This method is that of hypothesis, which consists to pretend some general principle, such that it is fecund in consequences, & such at the same time that it be a representation more or less faithful to reality. The examples of this method in natural philosophy are so frequent that it is little necessary to recall them. And the fruits which one has drawn from it are very evident. In the use of this method, one must be attentive to compare without ceasing the consequences deduced from the fictive principle with the truth of experience. And one must expect (as indeed one experiences it) that the more the consequences diverge, the more there must be separation between the hypothetical truths which the calculus gives, & the natural truths which observation gives. It is thus, for example, that one has treated with so much success hydrodynamics by reducing it by hypothesis to some simple principle, which one is quite removed to envision as the exact expression of reality. Also one is not at all surprised to find in its complicated results, & removed from their origin, some aberrations by which they are separated from observed phenomena. But it is not necessary to conceive the expectation to arrive in the matter which occupies us, to some results so precise as those which one obtains in natural philosophy. It will be necessary therefore to content ourselves in some more extended approximations, & for this effect to make more of a tentative.

§ 12. We observe first that, in the method which we have in view, there are some hypotheses of two kinds. Ones are of simple partial abstractions, by which one dismisses some element which too complicates the results. (as when one supposes a machine without friction). The others are of fictions where not only one omits some elements, but where one introduces arbitraries into it. One can well say, for example, that the principle of the conservation of sheer forces, introduced so usefully by DAN. BERNOULLI in the theory of hydrodynamics, is of this last category; since there is really no place at all to believe in the perfect elasticity of the elements of fluids. It is not less true than the first results of this principle, & many results rather extended, are conformed to the observed phenomena; in a way that this principle is a kind of faithful representation of nature until a certain limit of separation. And if the principle of equilibrium, which has been substituted for it, differs from it only in appearance, it gives place to the same remark.

We try therefore diverse hypotheses on testimony, either of those which consist only in partial abstractions, or of those which introduce some arbitrary elements; that is to say which modify those which influence really onto the phenomenon which we wish to evaluate, in a manner to simplify without too much deviation.

## CHAPTER I.
### *Hypotheses of simple partial abstraction.*
### *Principal assumption.*

Let a simple fact be exposed by a question to which one responds by *yes* or *no*.

### *First secondary assumption.*

§ 13. We suppose that a testimony is never insignificant; that is to say, that each deposition is a mixed argument; such that according to the truth or falsity of the one giving evidence, the fact is decidedly true or decidedly false: in a way that, consequently, the contrary of the false assertion is decidedly true. We examine the consequences of this hypothesis.

*First consequence.* In traditional testimonies, the double lie gives the truth.[9]

[9] This consequence has not been indicated, to our knowledge, besides that in the small abridgment that M. PREVOST published in 1794 for the usage of his disciples, & where is found this remark. Certum est recte processisse nostram argumentationem, si modo ille qui excipit testimonium decreverit ei fidere nonnisi eo casu quo a merentibus fuerit ad se usque delatum. Caeteroquin fieri posse condipitur ut plures testes in serie mentiantur, et tamen verum proseratur fine propter alternationes veri falsique in testando. Cresceret

Indeed, the ear-witness lying on the allegation of the first, returning *yes* when he has heard *no* (& the contrary); & hence, if the testimony *no* of the first witness was a lie, the second witness, lying on this lie, restores the *yes* which is conformed to the fact.

A witness lying on a lie restores the truth; very nearly the same as in algebra the product of two negative quantities is positive; & precisely as in grammar one observes that two negations are worth an affirmation.

*Second consequence.* One can thence estimate by the calculus the probability of the composite testimony resulting from a chain of two witnesses, by knowing the probabilities of their simple testimonies.

Indeed: let there be two witnesses of whom one, eye-witness, out of $v + m$ utterances says $v$ truths & $m$ lies; & of whom the other, ear-witness, out of $v' + m'$ utterances says $v'$ truths & $m'$ lies.

The *yes* pronounced by the ear-witness, has been preceded by the *yes* or by the *no* of the eye-witness: the sequence of testimonies is therefore one of the following two

| | |
|---|---|
| *yes* | *no* |
| *yes,* | *yes*. |

Let the sequence be $\begin{matrix}\textit{yes}\\ \textit{yes}\end{matrix}$. The ear-witness has said true (on that which he has heard.) The fact is conformed to the *yes* of the ear-witness, if the eye-witness has said true; & it is contrary to him, if the eye-witness has lied.

Let the sequence be $\begin{matrix}\textit{no}\\ \textit{yes}\end{matrix}$. *The ear-witness has said false (on that which he has heard.) The fact is contrary to the yes of the eye-witness, if the eye-witness has said true; & it is conformed to him, if the eye-witness has lied.*

The judge who hears only the last witness, & who is ignorant of the composition of the chain which forms the composite testimony, must estimate the probability of this testimony as follows.

The probability that the fact is true (conformed to the testimony of the ear-witness) is to the probability of the contrary, in the ratio of $vv' + mm'$ to $vm' + mv'$; therefore these two absolute probabilities are

$$\frac{vv' + mm'}{vv' + mm' + vm' + mv'} \ \& \ \frac{vm' + mv'}{vv' + mm' + vm' + mv'}$$

or

$$\frac{vv' + mm'}{(v+m)(v'+m')} \ \& \ \frac{vm' + mv'}{(v+m)(v'+m')}.$$

*Alternately.* Out of $(v+m)(v'+m')$ testimonies of these two witnesses.

There are $vv'$ which give the truth by the succession of two true testimonies.

There are $vm' + mv'$ which give the false; because one of the witnesses has said true & the other false.

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probabilitas veri, si haec consideratio foret admittenda; nam prodit novos casus nondum numeratos quibus testimonium verum excipitur. Pendet scilicet haec observatio ab hypothesi sub qua de testimonio loquimur in praesentia. Agi ponitur de facto quodam unico, et de quaestione singulo verbo affirmandi negandive in responso excipienda. Sed sic res vix ac ne vix quidem in traditione exhibetur. *De probabilite*, § 60.

There are $mm'$ which give the truth by the succession of two false testimonies.

Hence, out of $(v+m)(v'+m')$ cases, there are $vv'+mm'$ which give the truth, & $vm'+mv'$ which give the falsity.

Therefore the probability that the fact is true, is $\frac{vv'+mm'}{vv'+mm'+vm'+mv'}$ & the probability that the fact is false, is $\frac{vm'+mv'}{vv'+mm'+vm'+mv'}$.

*Example*. Let there be two equi-truthful witnesses; these two probabilities are $\frac{vv+mm}{(v+m)^2}$ & $\frac{2vm}{(v+m)^2}$.

*Third consequence.* Let there be two composite chains of two traditional witnesses. Let the veracity of the eye-witness of each chain be the same as the veracity of the ear-witness of the other; the probabilities of the two composite testimonies are the same.

*Fourth consequence.* Let there be two composite chains each of two traditional witnesses. Let the witnesses who compose one of the chains alternate with the witnesses who compose the other chain as to truth & falsity; the probabilities of the two composite testimonies are the same. This remarkable consequence results immediately from the formulas of the second.

*Fifth consequence.* The probability of the composite testimony is doubt, when each simple testimony is doubtful.

But the two witnesses being supposed equi-truthful, the probability of the compose testimony is greater than doubt, whatever be the probability of the simple testimony different from doubt.

*Sixth consequence.* When the two witnesses are, either both more truthful than they are liars, or both more liars than they are truthful; the probability of the composite testimony is greater than doubt.

But, if one of the witnesses is more truthful than he is a liar, while the other witness is more liar than he is truthful; the probability of the composite testimony is smaller than doubt.

The preceding consequences are relative to a chain composed solely of two simple testimonies. I am going to extend them to a chain composed of any number of witnesses, by supposing them (for greater facility) equi-truthful.

*Seventh consequence.* Let a chain be composed of $n$ successive witnesses, of whom the first is eye-witness, & let the probability of the testimony of each of them be $\frac{v}{v+m}$. The probability of the composite testimony if

$$\frac{1}{2}\left(1+\left(\frac{v-m}{v+m}\right)^n\right);$$

& the probability of the contrary is

$$\frac{1}{2}\left(1-\left(\frac{v-m}{v+m}\right)^n\right);$$

*Eighth consequence.* The probability of the composite testimony is doubt, when each simple testimony is doubtful.

*Ninth consequence.* Whatever be the truth of each witness in particular: the probability of the composite testimony approaches doubt, so much more as the number of successive witnesses is greater.

*Tenth consequence.* The number of successive witnesses being *even*, & the probability of each simple testimony being different from doubt, the probability of the composite testimony is greater than doubt, whatever be the truth of each witness in particular.

But the number of successive testimonies being *odd*, the probability of the composite testimony is greater than doubt, only when each successive witness is more truthful than he is a liar; & in the contrary case, the probability of the composite testimony is smaller than doubt.

*Eleventh consequence.* The probability of the composite testimony is independent of the order following which the truth & the lie can succeed themselves in a traditional chain.

### *Second secondary hypothesis.*

§ 14. Each simple testimony is in part true, in part insignificant, & in part false.

*Evident principle.* When in a chain of traditional witnesses, there is slipped past some insignificant testimony, the composite testimony is insignificant.

*First consequence.* The assumption that testimony can be insignificant, diminishes each of the probabilities that the composite testimony is true or false; but it alters not the ratio of these probabilities.

*Second consequence.* The more the number of traditional witnesses is considerable, the more the probability that the composite testimony is insignificant, approaches being unity or certitude: in a manner that there is no limit to the grandeur of this probability, by the increase of the number of witnesses.

This absolutely new theory can find more than one application. But it is difficult to employ it in some cases where the fact is complicated, because then the truth arising from double falsity is only a very small portion of the field of possibilities. Thus a judge, a historian will lend themselves with difficulty to admit it, although it is quite rigorous & sure by means of the hypothesis.

## CHAPTER II.
## *Fictive hypotheses.*

We will give in this chapter only two examples. We will recall first the hypothesis of CRAIG. Next we will propose another test tending very nearly to the same end.

### ARTICLE I.
### *Hypothesis of* CRAIG.

§ 15. CRAIG[10] poses a principle that (independently of the number of successive witnesses) the time & the space have a constant influence on the faith due to the testimony: that this influence tends to destroy this faith: & that it acts, in each element of time & of space very small (infinitely small), by some equal elementary impulsions; in

[10] *Theologiae christianae principia mathematica. Lond. 1699. et Lipsig 1755.* These assumptions are in part implicit. But one reads (*Prop. IV.*): *Causa suspicionis supponitur esse vis uniformis.* And Prop. V and VI suppose this force *perpetual* as that of gravity.

a way that its action is absolutely comparable to that of the perpetual forces (such as gravity).

§ 16. This hypothesis has the advantage to offer a great facility to deduce from it consequences. Because one must envision as such all the theorems of dynamics deduced since a long time from this principle. And it is no longer a question that to interpret from it the terms in the sense of this new application. Thus, for example, the law of diminution of faith will be that of the squares of the time & of the distances, as the author established it since the first propositions of his work.

§ 17. On this basis he constructed a grand edifice; & (for some arbitrary determinations *Prop. XIV. Schol.*) he comes from it to discover the precise period where the Christian faith will perish, as much as it reposes on the belief in the facts of the gospel. The credibility of the history of JESUS CHRIST, as much as founded on the oral tradition, perished towards the year 800 of the era. And as much as founded on the written history, it will perish in the year 3150. Whence the author, being authorized by a passage of St. Luke, fixed this epoch as that where JESUS CHRIST will come onto the earth.

§ 18. P. PETERSON working a little later on similar principles, arrived, by some law of decreasing a little different, to some results which differed from those there according to the date. He fixed the year 1789 for that fall of the faith, which, according to him, presaged the end of the world.[11]

§ 19. We examine the foundation of this edifice.

I suppose that a witness 1000 miles distant has the voice strong enough through which I receive immediately his deposition, or which he communicates to me by prompt & sure signals: the distance will not at all influence on my faith. It is therefore only because of the number of successive witnesses, that I sense my faith is weakened by the distance. There is therefore double usage to count on one side the distance. There is therefore double usage to count on one side the distance, on the other the number of successive witnesses.

As for time, we will consider its action in a single witness (as CRAIG makes in *Schol. on Prop. XIV. of Chap. I.*) Here there is a real foundation to the introduction of this element. The testimony reposing on memory, & this depending on time, one must in this regard count the one for something. However here also, it is very nearly impossible to pretend any law, which is brought back a little from the mean of the phenomenon. In a way that it is perhaps more useful to make the uncertainties of remembrance enter into the general appreciation of the truth, in a manner vague & conformed to the givens of experience in each particular case.

But we pass to the case of two or more successive witnesses. What gives to the memory of the second witness, the time that the first has allowed to pass between the fact & his deposition? If one would wish to compute the effect of the time on the memory, it would be necessary to recommence at each witness, & to renounce to each formula where one supposes a continuous action.

[11] The work of PETERSON (printed in London in 1701 under the title of *Animadversiones in* T. CRAIG *Princip. mathem.*) is known to us only by this indication, that M. LE SAGE furnished us.

These considerations appear to us to render inadmissible the principles of CRAIG & to explicate plainly the sentiment of repulsion which common sense sustains to the hearing of his results.

§ 20. The principles of this author on the simultaneous testimony appears to me no more admissible. He does not distinguish the estimation *a priori* & *a posteriori*, & contents himself to sum the faith due to each of the simultaneous witnesses in order to conclude the total faith. Whence we see result diverse consequences either inconceivables or contraries to the sentiment of each man who carries his attention on this object. Such are these: 1. That one can obtain some probabilities greater than certitude, by multiplying the number of witnesses, (a deduction made from that which the author names *suspicion*). 2. That the concordance of the witnesses adds no weight to the deposition of each of them; & that from such testimonies, envisioned one to one, have not a credibility superior to that of the testimonies which are not at all buttressed by them.

§ 21. In a way that, either for the successive testimony, or for the simultaneous, the hypotheses of CRAIG appear to offer no just & interesting application.

This judgment is rather similar to the one of KÄSTNER (*Programma quo gradus et mensuram probabil. dari defenditur, Lipsiae 1748, pag. 13.*), cited & approved by the editor of CRAIG (T. DAN. TITIUS, *Lips. 1755*)

It appeared therefore that it is worth much more to renounce to all exact appreciation of the testimony, than to cast oneself into this route.

## ARTICLE II.
### *Another attempt.*

### § 22. *HYPOTHESES.*

I. *Each testimony announces decidedly the truth or the contrary to the truth.*

Note: For brevity, we name the contrary of the truth falsehood.

II. *Null testimony founded on falsehood can not give truth.*

### *REMARKS.*

I. It follows from these hypotheses, that each tradition, where is found one or many falsehoods, enunciate decidedly falsehood.

II. Since, by the first hypothesis, the testimony which does not enunciate truth, enunciates decidedly its contrary; the testimony is supposed relative to a simple fact, & such that the witness being interrogated satisfies by responding purely *yes* or *no*. Because the least modification to this response, making to imagine more than two possible responses; it would ensue that in the narration of a determined fact, there are many ways to say decidedly all the truth, or that the truth has more of a contrary. This which implicates contradiction.

III. The second hypothesis applies to the simple facts the judgment which one carries commonly on the complicated facts. Because in those it is extremely difficult that a sequence of falsehoods gives the truth. The complicated facts are perverted & perish in some way by the falsehood.

§ 23. In consequence of the preceding remarks, it will not be useless to trace the march that one should follow in order to treat, under our hypotheses, with rigor & clarity, the diverse questions which could be posed.

1. One will pretend a simple fact, & such that the testimony on this fact is announced by *yes* or *no*.

2. By developing the diverse possible cases, one will characterize each testimony as enunciating decidedly the truth or the falsity on the proposed fact uniquely, & not on any subordinated fact.

3. Finally in each tradition mixed with falsehoods, one will characterize always the final testimony (or the last deposition) as enunciating the falsehood.

§ 24. One can judge now to what point our hypotheses are natural. Without seeking to defend them, & in proposing them as a simple test, we will indicate here the principal considerations that this question presents.

1. It is in truth little natural to say that each testimony enunciates decidedly the truth or the falsehood, since often the witnesses say some things mixed with true & of false, & of which also the contrary is neither absolutely true, nor absolutely false. But this vice of our first hypothesis is corrected by the second. Indeed, this vice holds to that which one supposes each fact simple. Now the second hypothesis, applying to the simple facts the judgment that one carries from the composite facts (§ 22. *Rem. III.*), makes these judgments return in the natural order.

2. And indeed the consequence enunciated in our first remark (§ 22.) appears conformed enough to the common judgment of men. Because in the historical or judiciary facts, one does not trust to one tradition that one knows to be mixed with false testimonies. This holds without doubt to this that these facts are complicated (§§ 10 & 22. *Rem. III.*) Under our hypotheses, we avoid introducing directly the complication, but we conserve the effect of it.

§ 25. Before employing these hypotheses, it is good to remark the following axiom:

*Each non-proven fact is of a null probability.*

This axiom prevents an objection which could be offered against each hypothesis from which follows that the tradition can diminish the credibility of testimony below each given quantity. In the greater part of testimonial cases, this axiom is not verified in appearance. But this comes from this that the attested facts are proved besides, & independently of the testimony which one evaluates.

§ 26. We attempt now to make use of our hypotheses.

In order to render this work useful, it would be agreeable to have a well determined plan, which gave place to some comparisons between the calculus & the observation. But this is very difficult in this matter. The only determination which we believe able to give to our plan, will consist in attempting rather to appreciate historical testimony, than the judiciary testimony. In consequence we will have rather in view (thus as CRAIG) the cases where the testimony can be estimated by a kind of mean, than the particular cases, in which it is necessary, by virtue of a delicate observation, to fix by way of experience the credibility of a witness.

§ 27. The traditional testimony holds its principle force from its combination with the simultaneous. A single sequence of successive witnesses would lose very quickly its credibility. But many sequences or chains combined conserve it a long time. We are going to pretend a combination of such chains, regular enough in order to apply the calculus, & which however represent in some fashion the real combinations. And applying our hypotheses, we will recognize 1. if the tradition can give an increasing credibility? 2. what is the mean requisite credibility in order that, under the most favorable assumption, the credibility is deteriorated by the tradition?

§ 28. Let there be two sequences or chains of witnesses. Let each of them be formed by two successive witnesses. Let these two chains be independent the one another. Finally let these deposed witnesses agree on the same fact, & let a unique judge receive simultaneously the last testimony of both chains.

One demands the credibility of this tradition?

§ 29. Let $\frac{v}{v+m}$ be the mean credibility of any isolated witness.

The credibility of the second witness of any chain on the fact in question, will be $= \left(\frac{v}{v+m}\right)^2$ (§ 22. *Hypoth. II.*)

But the judge receiving at the same time two uniform depositions, must estimate the credibility of their testimony by the general formula of LAMBERT, by making vanish the terms where $i$ is found employed. (§ 22. *Hyp. I.*), thus as we have made above. (§ 7.)

Here we have instead of the two credibilities $\frac{v}{v+m}$, $\frac{v'}{v'+m'}$, one same credibility for each of the two witnesses, which is $= \left(\frac{v}{v+m}\right)^2$. And hence, the formula becomes

$$\frac{v^4}{v^4 + (2vm + m^2)^2}.$$

§ 30. Now if we wish that the tradition alter not at all the weight of the testimony, it is necessary that the credibility of this tradition, such as it arrives to the judge, equals the mean credibility of a single isolated witness. We have therefore to resolve this equation

$$\frac{v^4}{v^4 + (2vm + m^2)^2} = \frac{v}{v+m}$$

where making $m = 1$, we will find $v \begin{matrix} > 4.8642 \\ < 4.8643 \end{matrix}$; & hence

$$\frac{v}{v+m} = \frac{4.864}{5.864} = \frac{5}{6} \text{ very nearly.}$$

§ 31. Whence if follows that, if the mean credibility of an isolated witness is $> \frac{5}{6}$, this tradition will make increase the credibility of the testimony: & that if this credibility of an isolated witness is $< \frac{5}{6}$, the same form of tradition will reduce the weight of the testimony.

§ 32. We follow this hypothesis from regular tradition. We consider the double chain that we had in view, as a unique witness, that we will name *composite witness*.

Let there now be four similar composite witnesses, distributed in a manner to form two traditional chains, each of two composite witnesses, & independent from one another. And let the last two composite witnesses of each chain attest uniformly one same fact to a single judge.

And since the credibility of a composite isolated witness is known, one will arrive, by a process analogous to the preceding (§ 29.), to estimate the credibility resulting from the tradition by the two chains that form from the similar witnesses.

The credibility of a single composite witness has been found

$$= \frac{v^4}{v^4 + (2vm + m^2)^2}.$$

Let therefore now this quantity

$$\frac{v^4}{v^4 + (2vm + m^2)^2} = \frac{v'}{v' + m'}.$$

And the credibility of the two chains of composite witnesses (which one can name a composite testimony *of the second order*) will be

$$= \frac{(v')^4}{(v')^4 + (2v'm' + m'm')^2}.$$

Similarly one will find, by the same artifice, the credibility of a composite witness *of the third order*

$$= \frac{(v'')^4}{(v'')^4 + (2v''m'' + m''m'')^2}$$

And thus in sequence.

§ 33. And for each of these orders, the same form of equation (§ 30.) will be presented to resolve, in order to determine the mean credibility of the witness of the preceding order, which gives a credibility equal to those two orders; that is to say which gives a tradition at each epoch, such that the weight of the testimony is not at all altered.

§ 34. On which we observe that, since the credibility $= \frac{5}{6}$ for the elementary isolated witness (or *of the first order*), gives this result, the same value will satisfy all the following equations. In a way that in order that a composite witness of an order $n$ has the same credibility as the one of order $n-1$, it is necessary that the credibility of the one be $= \frac{5}{6}$.

§ 35. Consequently if the elementary witness (or *of the first order*) has a credibility $> \frac{5}{6}$, those of the witnesses of the subsequent orders will go always increasing. The tradition will fortify continually & indefinitely the testimony. And inversely, if the credibility of the simple witness $< \frac{5}{6}$, it will be weakened.

§ 36. It is quite evident that one such order of regular tradition, & this weaving of simultaneous & successive depositions, is an order completely fictive. It indicates only that (under our hypothesis) one can imagine as possible (although quite difficult) this singular effect of the tradition to increase the value of the testimony.

It is necessary moreover to observe that, even under these fictive hypotheses, it is not the successive transmission which produces this effect. It is the number always increasing from simultaneous depositions. This increasing outweighs, & even over-compensates (according to the cases) the decreasing of the value which the succession or transmission of the testimony by many different mouths operates.

§ 37. Having thus proceeded by way of fictive hypothesis, it would be necessary to examine to what point these consequences agree with experience. But this would suppose the compilations of observations of which we are deprived.

One could be able, in this which it appears, to make many classes of facts.

I. The striking facts; such as the one here: CAESAR *vanquished* POMPEY.

II. The common facts.

§ 38. As for the first, we presume that our hypotheses are applicable to them until, as one can imagine that from such facts they lose very little, or even lose not at all from their credibility by the lapse of time. One must perhaps believe the fact cited above in example (§ 37.I.) with as much confidence as the contemporareous.

If this is true & conformed to the sentiment of educated & reasonable men, it would ensue that that which is passed relatively to some such facts resemble rather well to our fictions. And one would conclude from it that, for that which concerns these facts there, the mean credibility of the witnesses approaches much to be represented by the fraction $\frac{5}{6}$ which gives the limit:[12] this mean comprehending without doubt by compensation the monuments & other extraordinary testimonies.[13]

§ 39. For the facts of the second class, as they strike attention little, the ratios are strongly altered & become very uncertain by the traditional way, this which makes that one is little disposed to believe them. One says that the thermometer of the observatory has marked yesterday the degree $d$ at noon. I know this by the 3rd or 4th hand. I have no full confidence at all, although the tradition is so short. And when likewise 3 or 4 persons would say to me the same thing, I would have always more doubt than if one eye-witness gave this ratio to me, & in any case, I think, I would not have less of it. It is necessary therefore that here the ear-witnesses pass for quite inattentive; have a mean credibility $< \frac{5}{6}$, & even much less.

[12] It is this which had elected, by form of example, the anonymous author of the Memoir of the *Phil. Trans.* which we have cited in the note § 3.

[13] It will not be beyon proper to compare to our results, the reflections of a judicious man who is occupied not at all in the application of the calculus to testimony, & who reasons on the one conformably to the common principles of history. "Historical evidence depends first on the credit which one accords to the assertions of the contemporaneous writers, & of the manner in which they accord with the facts, the public monuments, thus as with the facts & the circumstances which the readers are to declare to observe. These accredited writers confirm, according to the same principles, the truth of those who have immediately preceded them. Thus the facts are traced & discussed by retrograding as far as the chain can regularly lead which links them, & to more ancient events of which the authenticity inspires some confidence. It is on some parallel bases which the belief of things is founded which have been able to hit immediately our sense. For example, we have no other means to judge that the Roman Republic has existed, that the battle of Actium has been given, & that a Norman conqueror has invaded England." Voyage de la Chine par MACARTNEY. T. III. p. 21 of the French translation.

§ 40. It would be to consider some other divisions of facts I. relatively to their nature more or less mixed with circumstances which influence on the truth of the witnesses[14] 2. to their complication, &c.

Finally it would be necessary to distinguish the spontaneous witnesses from those who are interrogated.—The oral tradition from the written condition &c.

One should also have regard to the possibility of *collusion* in the concordant testimonies. It introduces some unfavorable cases to the credibility, estimated independently from this circumstance.

§ 41. Such is the first imperfect test of a method of hypothesis, which could be followed with more fruit, if one would have a certain number of results of observation well determined, to which one was able to compare those of the theory. Perhaps this work is not impossible to execute. This memoir has for end to indicate it as useful to the observations in this genre.[15]

§ 42. For the rest, likewise the calculus of probabilities *a posteriori* supposes an *instinct of providence*: thus the calculus of the testimony appears to suppose to us an *instinct of confidence* which serves as basis of it. There are above some remarks to make which can not find place here.

## SECTION III.
### *Mathematical clarifications on the preceding sections.*

§ 43. *On* § 13.

*On the first secondary assumption.*

*On the fifth consequence.* The probability in favor of the composite testimony, is to the probability of the contrary, in the ratio of $vv + mm$ to $2vm$.

But, $v$ & $m$ being unequal:

$$vv - 2vm + mm \left( = \begin{matrix} (v-m)^2 \\ (m-v)^2 \end{matrix} \right) > 0 :$$

therefore $vv + mm > 2vm$; therefore the first probability is greater than the second.

*On the sixth consequence*. Let at the same time $\begin{matrix} v > m \\ v' > m' \end{matrix}$, or $\begin{matrix} v < m \\ v' < m' \end{matrix}$, I affirm that $vv' + mm' > vm' + mv'$.

1. Let at the same time, $\begin{matrix} v > m \\ v' > m' \end{matrix}$: one will have $v(v' - m') > m(v' - m')$; or $vv' - vm' > mv' - mm'$. Therefore, adding to each member $vm' + mm'$, $vv' + mm' > mv' + vm'$.

2. Let at the same time, $\begin{matrix} v < m \\ v' < m' \end{matrix}$: one will have $v(m' - v') < m(m' - v')$; or, $vm' - vv' < mm' - mv'$; adding to each member $vv' + mm'$, $vm' + mv' < vv' + mm'$; or, $vv' + mm' > vm' + mv'$.

[14] LAMBERT *Phaenomenol.* § 233. & following.

[15] All this theory would apply to some other arguments, with some modifications. One could therefore draw from many sources the results of experience to compare to those of calculation.

Let at the same time, $\begin{matrix} v > m \\ v' < m' \end{matrix}$ or $\begin{matrix} v < m \\ v' > m' \end{matrix}$; I affirm that $vv' + mm' < vm' + mv'$.

1. Let at the same time, $\begin{matrix} v > m \\ v' < m' \end{matrix}$; one obtains $v(m' - v') > m(m' - v')$; or $vm' - vv' > mm' - mv'$; therefore, adding $vv' + mv'$ to each member, $vm' + mv' > vv' + mm'$; or, $vv' + mm' < vm' + mv'$.

2. Let at the same time, $\begin{matrix} v < m \\ v' > m' \end{matrix}$; the demonstration is the same: namely, $v(v' - m') < m(v' - m')$; or, $vv' - vm' < mv' - mm'$ thence, $vv' + mm' < vm' + mv'$.

*On the seventh consequence. Known lemma.* Let the binomials $a + b$ & $a - b$ be raised to a like power of which the exponent is $n$. I affirm that the sum of the odd terms of the power $(a + b)^n$ by counting from the first, is the half of the sum $(a+b)^n + (a-b)^n$; & that the sum of the even terms by counting from the second, is the half of the difference $(a + b)^n - (a - b)^n$.

*Dem.*

$$(a+b)^n = a^n + \frac{n}{1}a^{n-2}b + \frac{n}{1} \cdot \frac{n-1}{2}a^{n-2}b^2 + \frac{n}{1} \cdots \frac{n-2}{3}a^{n-3}b^3 + \frac{n}{1} \cdots \frac{n-3}{4}a^{n-4}b^4 +$$

$$(a-b)^n = a^n - \frac{n}{1}a^{n-2}b + \frac{n}{1} \cdot \frac{n-1}{2}a^{n-2}b^2 - \frac{n}{1} \cdots \frac{n-2}{3}a^{n-3}b^3 + \frac{n}{1} \cdots \frac{n-3}{4}a^{n-4}b^4 \cdots$$

therefore, $(a+b)^n + (a-b)^n = 2(a^n + \frac{n}{1} \cdot \frac{n-1}{2}a^{n-2}b^2 \cdots + \frac{n}{1} \cdots \frac{n-3}{4}a^{n-4}b^4 + \cdots)$

$$(a+b)^n - (a-b)^n = 2(\frac{n}{1}a^{n-1}b \cdots + \frac{n}{1} \cdots \frac{n-2}{3}a^{n-3}b^3 + \cdots)$$

Hence, the sum of the alternate terms counting from the first is quite $\frac{(a+b)^n+(a-b)^n}{2}$.

And the sum of the alternate terms counting from the second, is $\frac{(a+b)^n-(a-b)^n}{2}$.

*Remark.* The alternate terms of the binomial $(a + b)^n$ counting from the first, contains the powers of $b$ with even exponents, & the alternate terms counting from the second contains the powers of $b$ with odd exponents.

*Application.* In the traditional testimonies the composite testimony is true, when in the traditional sequence there is no false testimony, or when their number is even; & the composite testimony is false, when in the traditional sequence there are some false testimonies in odd number.

Now the manners in which the traditional sequence can be composed from testimonies of $n$ equi-truthful witnesses, of which each out of $v + m$ words say $v$ true & $m$ false, are expressed by the terms of the binomial $(v + m)^n$; in a way that the alternate terms of this developed power, counting from the first, expresses the ways in which the simple testimonies are false in even number; & the remaining alternate terms express the ways in which the simple testimonies are false in odd number. Hence, the probabilities that the composite testimony is true or false are between them as the sums of the odd terms & of the even terms of the power $(v + m)^n$; that is to say as $(v + m)^n + (v - m)^n$ & $(v + m)^n - (v - m)^n$. Therefore these probabilities are

$$\frac{(v+m)^n + (v-m)^n}{2(v+m)^n} = \frac{1}{2}\left(1 + \left(\frac{v-m}{v+m}\right)^n\right)$$

&

$$\frac{(v+m)^n-(v-m)^n}{2(v+m)^n}=\frac{1}{2}\left(1-\left(\frac{v-m}{v+m}\right)^n\right).$$

*On the ninth consequence.* The probability of the composite testimony being

$$\frac{1}{2}\left(1+\left(\frac{v-m}{v+m}\right)^n\right);$$

the more $n$ is great, the more the $n^{\text{th}}$ power of the fraction properly called $\frac{v-m}{v+m}$ is small; & hence, the more the quantity

$$\frac{1}{2}\left(1+\left(\frac{v-m}{v+m}\right)^n\right)$$

approaches being $\frac{1}{2}$.

*On the tenth consequence.* When $n$ is even, the power $\left(\frac{v-m}{v+m}\right)^n$ is positive, whatever be the value of $v$ relative to $m$; namely, greater or smaller as it. Hence, the probability $\left(1+\left(\frac{v-m}{v+m}\right)^n\right)$ is greater than $\frac{1}{2}$.

When $n$ is odd, the power $\left(\frac{v-m}{v+m}\right)^n$ is positive or negative, according as $v$ is greater or smaller than $m$; & hence, this probability is greater or smaller than doubt, according as $v$ is greater or smaller than $m$.

*On* § 14. *On the second secondary assumption.*

Let there be $n$ successive equi-truthful witnesses, of whom each out of $v+i+m$ words, pronounce $v$ truths, $i$ insignificants, & $m$ falsehoods. The probability that the composite testimony is true or false, is $\frac{(v+m)^n}{(v+i+m)^n}$ the probability that this testimony is true, is $\frac{1}{2}\frac{(v+m)^n+(v-m)^n}{(v+i+m)^n}$; the probability that this testimony is false, is $\frac{1}{2}\frac{(v+m)^n-(v-m)^n}{(v+i+m)^n}$.

Hence, the ratio of these probabilities is the same as if all testimonies were significant.

The probability that the composite testimony is insignificant, is $1-\left(\frac{v+m}{v+i+m}\right)^n$. Now, the power $\left(\frac{v+m}{v+i+m}\right)^n$ of the fraction properly called $\frac{v+m}{v+i+m}$ is so much smaller as $n$ is greater; & there is no limit to its smallness (by the increase of $n$). Hence, the probability that the composite testimony is insignificant, approaches to certitude so much more as the number of successive witnesses is greater; & it can differ from it less than any assigned quantity.

§ 44. *Addition.* I am going to indicate by one or two examples, the process to follow in the applications of the calculus to the combination of the simultaneous & traditional testimonies, according to the first secondary assumption (§ 13.)

*First example.* Let there be a number $n$ of ear-witnesses $A$, testifying for *yes* on the testimony of a single eye-witness $O$ heard by them simultaneously. One demands the probability of this composite testimony.

The witnesses $A$ being in accord to testify by *yes* on the allegation of $O$;
the probability that $O$ has said *yes*, is $\frac{v^n}{v^n+m^n}$;
the probability that $O$ has said *no*, is $\frac{m^n}{v^n+m^n}$.

The fact is conformed to the testimony of the $A$; 1. if $O$ having said *yes* he has said *true*; 2. if $O$ having said *no* he has said *false*.

The fact is contrary to the testimony of the $A$; 1. if $O$ having said *yes* he has said *false*; 2. if $O$ having said *no* he has said *true*.

Thence, the probability that the fact is conformed to the testimony of $A$ is to the probability of the contrary, in the ratio of $v^n \times v + m^n \times m$ to $v^n \times m + m^n \times v$, or of $v^{n+1} + m^{n+1}$ to $mv(v^{n-1} + m^{n-1})$.

Hence, the probability that the fact is conformed to the testimony of $A$ is

$$\frac{v^{n+1} + m^{n+1}}{v^{n+1} + m^{n+1} + mv(v^{n-1} + m^{n-1})} = \frac{v^{n+1} + m^{n+1}}{(v+m)(v^n + m^n)};$$

& the probability of the contrary is $\frac{mv(v^{n-1}+m^{n-1})}{(v+m)(v^n+m^n)}$.

*Second example.* Let there be $r$ traditional chains, composed each of $n$ traditional witnesses. Let the last witnesses agree in their testimonies. One demands the probability of the composite testimony.

The probability of each last testimony is $\frac{(v+m)^n+(v-m)^n}{2(v+m)^n}$. The probability of the contrary is $\frac{(v+m)^n-(v-m)^n}{2(v+m)^n}$.

The probability in favor of the accord of the $r$ last witnesses, is to the probability of the contrary, in the ratio of $((v+m)^n + (v-m)^n)^r$ to $((v+m)^n - (v-m)^n)^r$.

Hence, the probability in favor of the composite testimony is

$$\frac{((v+m)^n + (v-m)^n)^r}{((v+m)^n + (v-m)^n)^r + ((v+m)^n - (v-m)^n)^r}.$$

Probability of the contrary

$$\frac{((v+m)^n - (v-m)^n)^r}{((v+m)^n + (v-m)^n)^r + ((v+m)^n - (v-m)^n)^r}.$$

*Example.* Let $\begin{matrix} n = 2 \\ v = 2 \end{matrix}$, or let there be two chains composed each of two witnesses.

Probability in favor of the composite testimony

$$\frac{(vv+mm)^2}{(vv+mm)^2 + 4vvmm} = 1 - \frac{4vvm}{(vv+mm)^2 + 4vvmm}.$$

Probability of the contrary $\frac{4vvm}{(vv+mm)^2+4vvmm}$.

In order that this last testimony has worth equal to a single simple testimony, the values successively approached to $\frac{v}{v+m}$ are, $\frac{3}{4}$, $\frac{7}{9}$, $\frac{67}{80}$.