OBSERVATIONS
sur les calculs relatifs à la durée des mariages & au nombre des époux subsistans*

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This subject has been treated by some very able geometers with whom I intend not at all to enter into comparison. But as it holds by its nature to some political establishments, one has often obscured some quite simple calculations with some absolutely strange considerations; the end of this little Memoir is to present below some observations, & to indicate some researches that I myself propose to develop besides.

§ 1. Let there be a number n of couples of men & women of the same age, so that there is in all $2n$ persons, one demands when there will be deceased a certain number of them, what will be probably the number of couples who will subsist. If one person dies, the probability that this will be a male is $\frac{n}{2n}$, the probability that this will be a female is $\frac{n}{2n}$. In the two cases the number of couples will be reduced to $n-1$. If two persons die, the probability that this will be two males or two females will be $\frac{n(n-1)}{2n(2n-1)}$ & in each of these cases, the number of couples will be reduced to $n-2$; the probability that this will be a male & a female in a determined order will be $\frac{nn}{2n(2n-1)}$, a probability that it is necessary to multiply by the number of combinations of two things taken one by one, namely by 2, this which gives $\frac{2nn}{2n(2n-1)}$; but in this case a man being dead, there will be $n-1$ cases in order that the woman who will die is not his wife, this which will reduce the number of couples to $n-2$, & 1 case in order that this is his wife, this which will give $n-1$ for the number of couples; the number of couples in this case will be therefore $\frac{(n-1)(n-2)+n-1}{n} = \frac{(n-1)^2}{n}$. The number of the remaining couples will be therefore

$$= \frac{n(n-1)}{2n(2n-1)}(n-2) + \frac{n(n-1)}{2n(2n-1)}(n-2) + \frac{2nn}{2n(2n-1)}\frac{(n-1)^2}{n} = \frac{(2n-2)(2n-3)}{2(2n-1)}.$$

§ 2. If three persons die, the probability that this will be three males or three females will be $\frac{n(n-1)(n-2)}{2n(2n-1)(2n-2)}$ & in these two cases the number of couples will be $n-3$; the

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probability that this will be two males & one female or two females & one male will be $\frac{3n(n-1)n}{2n(2n-1)(2n-2)}$ for each case. Under these assumptions, two men being dead, there will be $n-2$ cases in order that the female who will die is not his wife, this which gives $n-3$ for the number of couples, & 2 cases in order that it is his wife, this which will give $n-2$ for the number of couples; one has therefore $\frac{(n-2)(n-3)+2(n-3)}{n} = \frac{(n-2)(n-1)}{n}$. The number of remaining couples will be therefore

$$\frac{1}{2n(2n-1)(2n-2)}\big(n(n-1)(n-2)(n-3)+3n(n-1)^2(n-2)$$
$$+3n(n-1)^2(n-2)+n(n-1)(n-2)(n-3)\big) = \frac{(2n-3)(2n-4)}{2(2n-1)}$$

§ 3. If four persons die, the probability that they will be four males or four females will be $\frac{n(n-1)(n-2)(n-3)}{2n(2n-1)(2n-2)(2n-3)}$, & in these two cases the number of couples will be $n-4$; the probability that there will be three males & one female or three females & one male will be $\frac{4n(n-1)(n-2)n}{2n(2n-1)(2n-2)(2n-3)}$. Under these assumptions, three men being dead, there will be $n-3$ cases in order that the wife who will die not belong to them, this which gives $n-4$ couples, & 3 cases in order that she belong to them, this which gives $n-3$ couples, one has therefore $\frac{(n-3)(n-4)+3(n-3)}{n} = \frac{(n-3)(n-1)}{n}$. The probability that this will be two men & 2 women is $\frac{6n(n-1)(n(n-1)}{2n(2n-1)(2n-2)(2n-3)}$. Under this assumption, two men being dead, one will consider for the two women who remain to die, that the number of possible cases is the one of the combinations of n things taken two-by-two, that is to say, $\frac{n(n-1)}{2}$. From this case, 1 gives $n-2$ couples, $2(n-2)$ gives $n-3$ couples, & $\frac{(n-2)(n-3)}{1.2}$ gives $n-4$ couples, one has therefore

$$\frac{1.(n-2)+2(n-2)(n-3)+\frac{(n-2)(n-3)(n-4)}{2}}{\frac{n(n-1)}{2}} = \frac{(n-2)^2}{n}.$$

The number of remaining couples will be therefore

$$\begin{aligned}&= \frac{1}{2n(2n-1)(2n-2)(2n-3)}\big(n(n-1)(n-2)(n-3)(n-4)\\&+ 4n(n-1)^2(n-2)(n-3)+6n(n-1)^2(n-2)^3\\&+ 4n(n-1)^2(n-2)(n-3)+n(n-1)(n-2)(n-3)(n-4)\big)\\&= \frac{(2n-4)(2n-5)}{2(2n-1)}.\end{aligned}$$

§ 4. If five persons die, the probability that they will be five men or five women will be $\frac{n(n-1)(n-2)(n-3)(n-4)}{2n(2n-1)(2n-2)(2n-3)(2n-4)}$, this which gives $n-5$ couples; the probability that this will be four men & one woman or five women & one man will be $\frac{5n(n-1)(n-2)(n-3)n}{2n(2n-1)(2n-2)(2n-3)(2n-4)}$. Under these assumptions, the four men being dead, there will be $n-4$ cases in order that the woman does not belong to them, this which gives $n-5$ couples, & 4 cases in order that she belong to them, this which gives

$n-4$ couples; one has therefore $\frac{(n-4)(n-5)+4(n-4)}{n} = \frac{(n-4)(n-1)}{n}$; the probability that this will be three men & two women or three women & two men will be $\frac{10n(n-1)(n-2)n(n-1)}{2n(2n-1)(2n-2)(2n-3)(2n-4)}$. Under these assumptions, three men being dead, one will consider that for the two women who remain, the number of possible cases is $\frac{n(n-1)}{2}$; from these cases, 3 give $n-3$ couples, $3(n-3)$ give $n-4$ couples, & $\frac{(n-3)(n-4)}{2}$ give $n-5$ couples. One has therefore

$$\begin{aligned}&\frac{3(n-3)+3(n-3)(n-4)+\frac{(n-3)(n-4)}{2}(n-5)}{\frac{2(n-1)}{2}}\\ &=\frac{(n-3)(n-2)(n-1)}{n(n-1)}=\frac{(n-3)(n-2)}{n}.\end{aligned}$$

The number of remaining couples will be therefore

$$\begin{aligned}&\frac{1}{2n(2n-1)(2n-2)(2n-3)(2n-4)}\times\\ &\big(n(n-1)(n-2)(n-3)(n-4)(n-5)+5n(n-1)^2(n-2)(n-3)(n-4)\\ &+10n(n-1)^2(n-2)^2(n-3)+5n(n-1)^2(n-2)(n-3)(n-4)\\ &+n(n-1)(n-2)(n-3)(n-4)(n-5)\big)=\frac{(2n-5)(2n-6)}{2(2n-1)}.\end{aligned}$$

§ 5. The analogy is evident actually, & one sees that if i persons die the number of remaining couples will be $\frac{(2n-i)(2n-i-1)}{4n-2}$. We make now with Mr. Daniel Bernoulli (*Mémoires de Petersburg* 1766 & 1767)[1] $i=2n-r$, the number of remaining couples will be $\frac{r(r-1)}{4n-2}$, as this great geometer finds § 5. in the memoir on the duration of marriages. If n & r are infinite, one will have $\frac{rr}{4n}$, as Mr. Bernoulli finds it § 6.

§ 6. In general, if there are dead p men & q women, one will have for the number of possible cases, $\frac{n(n-1)\ldots(n-q+1)}{1.2\ldots q}$. From these cases

$\frac{p(p-1)\ldots(p-q+1)}{1.2\ldots q}$	give	$n-p$ couples,
$\frac{p(p-1)\ldots(p-q+2)}{1.2\ldots q-1}(n-p)$	give	$n-p-1$ couples,
$\frac{p(p-1)\ldots(p-q+3)}{1.2\ldots(q-2)}\frac{(n-p)(n-p-1)}{1.2}$	give	$n-p-2$ couples,
$\frac{p(p-1)\ldots(p-q+4)}{1.2\ldots(q-3)}\frac{(n-p)(n-p-1)(n-p-2)}{1.2.3}$	give	$n-p-3$ couples,

& thus in sequence; finally

$$\frac{(n-p)(n-p-1)\ldots(n-p-q+1)}{1.2\ldots q}\text{ give } n-p-q \text{ couples.}$$

[1] *Translator's note*: "De duratione media matriminiorum, pro quacumque coniugum aetate, aliisque quaestionibus affinibus," *Novi Commentarii Academiae Scientiarum Imperialis Petropolianae* XII, 1766-67, pp. 99-126.

Now

$$\frac{p(p-1)\dots(p-q+1)}{1.2\dots q}(n-p)+\frac{p(p-1)\dots(p-q+2)}{1.2\dots(q-1)}\frac{(n-p)(n-p-1)}{1}$$
$$+\frac{p(p-1)\dots(p-q+3)}{1.2\dots(q-2)}\frac{(n-p)(n-p-1)(n-p-2)}{1.2}$$
$$+\frac{p(p-1)\dots(p-q+4)}{1.2\dots(q-3)}\frac{(n-p)(n-p-1)(n-p-2)(n-p-3)}{1.2.3}\dots$$
$$+\frac{(n-p)(n-p-1)\dots(n-p-q+1)}{1.2\dots q}(n-p-q)$$
$$=\frac{(n-1)(n-2)\dots(n-q)}{1.2\dots q}(n-p).$$

One will have therefore by dividing by the number of possible cases, the number of couples $=\frac{(n-q)(n-p)}{n}$. Therefore there remain s men & t women of n couples, the number of couples will probably be $\frac{st}{n}$ as Mr. Bernoulli finds it § 16. Thus the conclusions that this great geometer has deduced from the differential calculus result naturally, as one sees, from the doctrine of combinations.

§ 7. Until here, we have supposed that the men & the women have an equal facility to die, this which is a too restrictive assumption. If one supposes now that the men & the women have not an equal facility to die, in such as way that α is the chance of the men, & β that of the women, one will be able to proceed in the same manner, but the formulas will become so complicated that they will be of little use. I will only indicate the calculation.

§ 8. If only one person dies, the number of couples remaining is always $n-1$. Indeed one has $\frac{\alpha n}{(\alpha+\beta)n}(n-1)+\frac{\beta n}{(\alpha+\beta)n}(n-1)=n-1$. If two of them die, this will be two men or two women, or else one man & one woman. The probability that a man dies first is $\frac{\alpha n}{(\alpha+\beta)n}$, the probability that one of them dies again is found in the following manner. Let P be the probability that a man will die & p the one that a woman will die, since there remain $(n-1)$ men & n women, one will have $P:p=\frac{\alpha:\beta}{n-1:n}$ therefore $P:P+p$, or $P:1=\alpha(n-1):(\alpha+\beta)n-\alpha$. The probability that there die two men is therefore $\frac{\alpha n.\alpha(n-1)}{(\alpha+\beta)\alpha((\alpha+\beta)n-\alpha)}$, this case gives $n-2$ couples; one will have for the case where there die two women by exchanging α & β, therefore $\frac{\beta n.\beta(n-1)}{(\alpha+\beta)n((\alpha+\beta)n-\beta)}$, this case gives also $n-2$ couples. If there die first one man, next a woman, one has for the first event $\frac{\alpha n}{(\alpha+\beta)n}$ & for the second $p:P=\frac{\beta:\alpha}{n:n-1}$ therefore $p=\frac{\beta n}{(\alpha+\beta)n-\alpha}$, one has therefore for this case $\frac{\alpha n.\beta n}{((\alpha+\beta)n-\alpha)\alpha+\beta)n}$. If a woman dies, next a man, one has for the first event $\frac{\beta n}{(\alpha+\beta)n}$, & for the second $P:p=\frac{\alpha:\beta}{n-1:n}$, therefore $P=\frac{\alpha n}{(\alpha+\beta)n-\beta}$, one has therefore for this case $\frac{\alpha n.\beta n}{(\alpha+\beta)n((\alpha+\beta)n-\beta)}$, these two cases give $\frac{(n-1)^2}{n}$, as we have proved above. One has therefore by reuniting the terms,

$$\begin{aligned}
&\frac{\alpha n.\alpha(n-1)}{(\alpha+\beta)n((\alpha+\beta)n-\alpha)}(n-2)+\frac{\alpha n.\beta n}{(\alpha+\beta)n((\alpha+\beta)n-\alpha)}\frac{(n-1)^2}{n}\\
&+\frac{\beta n.\alpha n}{(\alpha+\beta)n((\alpha+\beta)n-\beta)}\frac{(n-1)^2}{n}+\frac{\beta n.\beta(n-1)}{(\alpha+\beta)n((\alpha+\beta)n-\beta)}(n-2)\\
&=\frac{\alpha n(n-1)(\alpha(n-2)+\beta(n-1))}{(\alpha+\beta)n((\alpha+\beta)n-\alpha)}+\frac{\beta n(n-1)(\alpha(n-1)+\beta(n-2))}{(\alpha+\beta)n((\alpha+\beta)n-\beta)}\\
&=\frac{((\alpha+\beta)^2n(n-2)+3\alpha\beta)(n-1)}{((\alpha+\beta)n-\alpha)((\alpha+\beta)n-\beta)}.
\end{aligned}$$

§ 9. If three persons die, these will be 1° three men, a case of which the probability is $\frac{\alpha n}{(\alpha+\beta)n}\cdot\frac{\alpha(n-1)}{(\alpha+\beta)n-\alpha}\cdot\frac{\alpha(n-2)}{(\alpha+\beta)n-2\alpha}$, or 2° three women, a case of which the probability is $\frac{\beta n}{(\alpha+\beta)n}\cdot\frac{\beta(n-1)}{(\alpha+\beta)n-\beta}\cdot\frac{\beta(n-2)}{(\alpha+\beta)n-2\beta}$, or 3° two men & one woman, this which gives three cases

I. 1 man, 1 man, 1 woman; probability $=\frac{\alpha n}{(\alpha+\beta)n}\cdot\frac{\alpha(n-1)}{(\alpha+\beta)n-\alpha}\cdot\frac{\beta n}{(\alpha+\beta)n-\alpha-\beta}$

II. 1 man, 1 woman, 1 man; probability $=\frac{\alpha n}{(\alpha+\beta)n}\cdot\frac{\beta n}{(\alpha+\beta)n-\alpha}\cdot\frac{\alpha(n-1)}{(\alpha+\beta)n-\alpha-\beta}$

III. 1 woman, 1 man, 1 man; probability $=\frac{\beta n}{(\alpha+\beta)n}\cdot\frac{\alpha n}{(\alpha+\beta)n-\alpha}\cdot\frac{\alpha(n-1)}{(\alpha+\beta)n-\alpha-\beta}$

or 4° two women & one man, this which gives three cases

I. 1 man, 1 woman, 1 woman; probability $=\frac{\alpha n}{(\alpha+\beta)n}\cdot\frac{\beta n}{(\alpha+\beta)n-\alpha}\cdot\frac{\beta(n-1)}{(\alpha+\beta)n-\alpha-\beta}$

II. 1 woman, 1 man, 1 woman; probability $=\frac{\beta n}{(\alpha+\beta)n}\cdot\frac{\alpha n}{(\alpha+\beta)n-\alpha}\cdot\frac{\beta(n-1)}{(\alpha+\beta)n-\alpha-\beta}$

III. 1 woman, 1 woman, 1 man; probability $=\frac{\beta n}{(\alpha+\beta)n}\cdot\frac{\beta(n-1)}{(\alpha+\beta)n-\alpha}\cdot\frac{\alpha n}{(\alpha+\beta)n-2\beta}$

The first two cases give $n-3$ couples, the third & the fourth cases give $\frac{(n-2)(n-1)}{n}$. One has therefore, by reuniting all these terms,

$$\frac{\alpha n.\alpha(n-1).\alpha(n-2)(n-3)+\alpha n.\alpha(n-1).\beta(n-1)(n-2)}{(\alpha+\beta)n((\alpha+\beta)n-\alpha)((\alpha+\beta)n-2\alpha)}$$
$$+\frac{\alpha n.\beta\alpha(n-1)(n-1)(n-2)+\alpha n\beta.\alpha(n-1)(n-1)(n-2)}{(\alpha+\beta)n((\alpha+\beta)n-\alpha)((\alpha+\beta)n-\alpha-\beta)}$$
$$+\frac{\beta n.\alpha.\alpha(n-1)(n-1)(n-2)+\beta n.\alpha\beta(n-1)(n-1)(n-2)}{(\alpha+\beta)n((\alpha+\beta)n-\beta)((\alpha+\beta)n-\alpha-\beta)}$$
$$+\frac{\beta n.\beta(n-1)\beta(n-2)(n-3)+\beta n.\beta(n-1)\alpha(n-1)(n-2)}{(\alpha+\beta)n((\alpha+\beta)n-\beta)((\alpha+\beta)n-2\beta)}$$
$$=\frac{\alpha^3 n(n-1)(n-2)n-3)}{(\alpha+\beta)n((\alpha+\beta)n-\alpha)((\alpha+\beta)n-2\alpha)}+$$
$$\alpha^2\beta n(n-1)^2(n-2)\left[\frac{1}{(\alpha+\beta)n((\alpha+\beta)n-\alpha)((\alpha+\beta)n-2\alpha)}\right.$$
$$+\frac{1}{(\alpha+\beta)n((\alpha+\beta)n-\alpha)((\alpha+\beta)n-\alpha-\beta)}$$
$$\left.+\frac{1}{(\alpha+\beta)n((\alpha+\beta)n-\beta)((\alpha+\beta)n-\alpha-\beta)}\right]$$
$$+\alpha\beta^2 n(n-1)^2(n-2)\left[\frac{1}{(\alpha+\beta)n((\alpha+\beta)n-\alpha)((\alpha+\beta)n-\alpha\text{-}\beta)}\right.$$
$$+\frac{1}{(\alpha+\beta)n((\alpha+\beta)n-\beta)((\alpha+\beta)n-\alpha-\beta)}$$
$$\left.+\frac{1}{(\alpha+\beta)n((\alpha+\beta)n-\beta)((\alpha+\beta)n-\alpha-\beta)}\right]$$
$$+\frac{\beta^3 n(n-1)(n-2)(n-3)}{(\alpha+\beta)n((\alpha+\beta)n-\beta)((\alpha+\beta)n-2\beta)}=$$
$$\frac{(\alpha+\beta)^6 n^2(n-1)(n-2)(n-3)+11\alpha\beta(\alpha+\beta)^4 n(n-1)(n-2)-12\alpha^2\beta^2(\alpha+\beta)^2(n-1)(n-2)}{(\alpha+\beta)^6 n^2(n-1)(n-2)+\alpha\beta(\alpha+\beta)^4 n(5n-6)+4\alpha^2\beta^2(\alpha+\beta)^2}.$$

In the case of two deaths, the formula can be set under this form

$$\frac{(\alpha+\beta)^3 n^2(n-1)(n-2)+3\alpha\beta(\alpha+\beta)n(n-1)}{(\alpha+\beta)^3 n^2(n-1)+\alpha\beta(\alpha+\beta)n}.$$

One can proceed likewise as far as one will wish, & to find likewise the form of the general expressions to which I will not stop myself here because their complication renders the use of them nearly impractical.

§ 10. Mr. Daniel Bernoulli, in his Memoir on the use of the infinitesimal calculus in the art of conjecture (*Nova Act. Petrop*. T. 12.)[2], exposes a general method for this case here, but I do not know if this method is quite certain. He names in this Memoir black balls & white balls that which we name men & women, & the extraction of one ball

[2] *Translator's note*: "De usu algorithmi infinitesimali in arte coniectandi specimen," *Novi Commentarii Academiae Scientiarum Imperialis Petropolianae* XII, 1766-67, pp. 87–98.

outside of the urn corresponds to him to that which we call the death of a person. He calls $\phi : 1$ the ratio of the facility of extraction of the black & white balls, so that $\frac{\phi}{1}$ is that which we have called $\frac{\alpha}{\beta}$, that is to say the ratio of the mortality of the men to that of the women; he calls n the original number of couples, r the number of balls remaining in the urn, s & t the number of remaining black & white balls; finally he calls x the number of remaining couples. In § 7 he supposes s & t known, & finds $x = \frac{st}{n}$: we have found the same thing § 6. But he remarks quite well § 12 that the calculation is quite different when s & t is unknown, when one knows only their sum r, & when one must draw their values from the supposed known facility ϕ. Indeed, if one supposes s & t known, the calculation is reduced to the one of the probability of causes deduced from the events, instead that if one supposes ϕ known, the question is to estimate the probability of the events by deducing it from their causes. Now by calling ds & dt the diminutions of the black & white balls remaining in the urn, Mr. Bernoulli makes this composed proportion $ds : dt = \frac{s:t}{\phi:1}$, whence he deduces $\frac{ds}{s} = \frac{\phi\, dt}{t}$. He concludes from it that if $\phi = 1$, one has $s = t$. It seems to me that this conclusion should have inspired to Mr. Bernoulli some mistrust of the goodness of his solution, because the facilities can be equals, without that the number of the black & white balls remaining in the urn be the same. This probability is, to say truly, greater than any other, but the probability of the other cases must not be neglected for this, & the assumption $s = t \pm 1$, for example is a probability which differs little from the first. The problem consists in enumerating the diverse possible assumptions, & to estimate the degree of probability of each; now this is that which the differential equation of Mr. Bernoulli makes no point at all. The ratio of s to t does not appear therefore to be able to be drawn from the equation of Mr. Bernoulli. In the example which he gives § 13, he supposes $\phi = 2$, this which gives to him $x = \frac{(-\frac{1}{2}n + \sqrt{nr + \frac{1}{4}nn})^3}{nn}$. Let now $r = 2n - 1$, so that there is drawn only one ball, the number of couples will be necessarily $n - 1$, & one finds it such by our method. But if one makes $r = 2n-1$ in the formula of Mr. Bernoulli, one finds not $x = n - 1$, as this must be. The fault of this solution seems to me to consist in this that by the same assumption s & t depending on ϕ, it would be necessary to substitute their values in ϕ into the differential equation, then to integrate it, instead to give to ϕ some arbitrary values, & to draw next from the differential equation the values of s & t. It is to the geometers to see if I deceive myself, or if really the solution of Mr. Bernoulli is not legitimate. In the first case, there will result from it that this solution merits at least to be clarified & developed with more care. In the second, there will ensue from it that it is necessary to proceed in these matters with an extreme circumspection, & that the direct & rigorous calculation is necessary in order to discover the errors which slip so easily into the indirect & metaphysical solutions, if I am able to express myself so.

§ 11. If under the hypothesis of an equal facility, one wishes to have the probability that after that there will be dead any number whatsoever of persons, each death will have broken a marriage, there will be only to sum in the preceding calculations all the terms which are multiplied by the least number of couples remaining, one will have for the case of 1 death, 1, for the case of 2 deaths, $\frac{2^2.n(n-1)}{2n(2n-1)}$, for the case of 3 deaths $\frac{2^3.n(n-1)(n-2)}{2n(2n-1)(2n-2)}$, for the case of 4 deaths $\frac{2^4.n(n-1)(n-2)(n-3)}{2n(2n-1)(2n-2)(2n-3)} \cdots$ for the case of m deaths $\frac{2^m.n(n-1)...(n-m+1)}{2n(2n-1)...(2n-m+1)}$. If one supposes that the half of the married persons have perished, one will have $m = n$, this which will give $\frac{2^n.n(n-1)...1}{2n(2n-1)...(n+1)}$ for the probability that all the marriages are broken. This is that which Mr. Jean Bernoulli finds in a Memoir on the theory of chances § 7. (*Mém. de Berlin* 1768)[3]. One will observe that as long as $m < n$ all the marriages can not be broken, & our formulas give, as I have said, only the probability that each death breaks a marriage. If $m > n$, it is impossible that each death breaks a marriage, as our formulas give this probability $= 0$.

§ 12. The proof of the formulas of the previous § is quite simple. The case of 1 death is evident. For the case of 2 deaths, we have seen § 1 that the probability that this will be two males & two females is $\frac{n(n-1)}{2n(2n-1)}$, that the probability that this will be one male & one female is $\frac{2nn}{2n(2n-1)}$, & that in this case there is $\frac{n-1}{n}$ probability that the woman who will die will not be the wife of the man who dies. One has therefore for the probability that there will be 2 broken marriages, $\frac{n(n-1)}{2n(2n-1)} + \frac{2n(n-1)}{2n(2n-1)} + \frac{n(n-1)}{2n(2n-1)} = \frac{2^2 n(n-1)}{2n(2n-1)}$ as we have said preceding §. If 3 persons die, we have seen § 2 that the probability that these will be 3 men & 3 women is $\frac{n(n-1)(n-2)}{2n(2n-1)(2n-2)}$, that the probability that these will be two men & one woman, or two women & one man will be $\frac{3n(n-1)n}{2n(2n-1)(2n-2)}$ & that in these cases there are $\frac{n-2}{n}$ probability that the husband & the wife are not dead at the same time. One has therefore for the probability that there will be three broken marriages

$$\frac{1}{2n(2n-1)(2n-2)}(n(n-1)(n-2) + 3n(n-1)(n-2) + 3n(n-1)(n-2)$$
$$+ 3n(n-1)(n-2) + n(n-1)(n-2) = \frac{2^3.n(n-1)(n-2)}{2n(2n-1)(2n-2)}$$

as we have said above. If four persons die, we have seen § 3 that the probability that these will be four men or four women is $\frac{n(n-1)(n-2)(n-3)}{2n(2n-1)(2n-2)(2n-3)}$, that the probability that these will be three men & one woman, or three women & one man is $\frac{4n(n-1)(n-2)n}{2n(2n-1)(2n-2)(2n-3)}$, & that in this case there is probability $\frac{n-3}{n}$ that the husband & the wife will not be dead at the same time; that the probability that these will be two men & two women is $\frac{6n(n-1)n(n-1)}{2n(2n-1)(2n-2)(2n-3)}$ & that in this case there is $\frac{\frac{(n-2)(n-3)}{2}}{\frac{n(n-1)}{2}} = \frac{(n-2)(n-3)}{n(n-1)}$, probability that the husband & the wife will not die at the same time. One will have therefore for the probability that there will be three broken marriages

$$\begin{aligned}&\frac{1}{2n(2n-1)(2n-2)(2n-3)}(n(n-1)(n-2)(n-3)(n-4)\\ &\quad + 4n(n-1)(n-2)(n-3) + 6n(n-1)(n-2)(n-3)\\ &\quad + 4n(n-1)(n-2)(n-3) + 4n(n-1)(n-2)(n-3)(n-4)\\ &= \frac{2^4.n(n-1)(n-2)(n-3)(n-4)}{2n(2n-1)(2n-2)(2n-3)}.\end{aligned}$$

The analogy is now evident.

[3]*Translator's note*: Mémoire sur un probleme de la doctrine du hazard," *Histoire de l'Académie Royale des Sciences et des Belles-Lettres de Berlin*. Vol. XXIV, 1768 pp. 384–408 & supplement.

§ 13. If one supposes with Mr. Jean Bernoulli that there die constantly an equal number of men & of women, it will be necessary to take in the preceding formulas only those which express the probability that in this case each death breaks a marriage; thus one sees by the preceding § that for two deaths there is probability $\frac{n-1}{n}$ that the husband & the wife are not dead at the same time, that for four deaths this probability is $\frac{(n-2)(n-3)}{n(n-1)}$. One will find likewise, for six deaths this probability equal to $\frac{(n-3)(n-4)(n-5)}{n(n-1)(n-2)}$, & in general for $2v$ deaths, this probability equal to $\frac{(n-v)(n-v+1)\ldots 1}{n(n-1)\ldots(n-v+1)}$. If there are dead the half of them, one will make $2v = n$; thus by supposing n even, one will have $\frac{\frac{n}{2}\left(\frac{n}{2}-1\right)\ldots 1}{n(n-1)\ldots\left(\frac{n}{2}+1\right)}$, as Mr. Bernoulli finds it § 17.

§ 14. One finds in the excellent Work of Mr. Karstens entitled *Theorie von Wittwencassen*, Halle 1784,[4] a Table of the marriages subsisting each year, by supposing the spouses of the same age; & of different ages. The calculations of this able mathematician accords perfectly with the formulas that we have found above according to Mr. Bernoulli. Mr. Karstens deduces from it the number of marriages which are destroyed each year, & multiplying for each year the number of these marriages by the sum paid reduced at the epoch of the commencement of the payment, he obtains the total sum paid by the husbands during the duration of the marriages; this sum divided by the number of marriages gives the mean sum paid during the duration of a marriage. Deducing from this sum the annuity deposit on the head of the wife, he has the value of the rent payable to the wife while she will be widowed. In order to reduce this method to formulas, let $a^{(1)}$, $a^{(2)}$, $a^{(3)}$ $\ldots a^{(\mu)}$ be the number of marriages destroyed during the 1^{st}, 2^{nd}, 3^{rd} $\ldots \mu^{\text{th}}$ year, let $r = 1 + \frac{m}{100}$, by calling 1 the capital & $\frac{m}{100}$ the interest; one will have for the mean sum $\frac{a^{(2)}}{n} \cdot \frac{1}{r} + \frac{a^{(3)}}{n}\left(\frac{1}{r} + \frac{1}{r^2}\right) + \frac{a^{(4)}}{n}\left(\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}\right) + \frac{a^{(\mu)}}{n}\left(\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} \cdots + \frac{1}{r^{\mu-1}}\right) =$ (by calling $b^{(1)}$, $b^{(2)}$, $b^{(3)}$ $\ldots b^{(\mu)}$ the number of the marriages subsisting at the end of the 1^{st}, 2^{nd} $\ldots \mu^{\text{th}}$ year) $\frac{b^{(1)}}{n} \cdot \frac{1}{r} + \frac{b^{(2)}}{n} \cdot \frac{1}{r^2} + \frac{b^{(3)}}{n} \cdot \frac{1}{r^3} + \cdots + \frac{b^{(\mu-1)}}{n} \cdot \frac{1}{r^{\mu-1}}$. Now there results from § 6 that by calling (s), $(s+1)$, $(s+2)$, $(s+3)$ &c. the men who remain in life the $(s)^{\text{th}}$, $(s+1)^{\text{st}}$, $(s+2)^{\text{nd}}$, $(s+3)^{\text{rd}}$ year &c. (t), $(t+1)$, $(t+2)$, $(t+3)$ &c. the number of women who remain alive the $(t)^{\text{th}}$, $(t+1)^{\text{st}}$, $(t+2)^{\text{nd}}$, $(t+3)^{\text{rd}}$ year, one has by calling n the initial number of marriages, $\frac{b^{(1)}}{n} = \frac{(s+1)(t+1)}{n(s)}$, $\frac{b^{(2)}}{n} = \frac{(s+2)(t+2)}{n(s)}$, $\frac{b^{(3)}}{n} = \frac{(s+3)(t+3)}{n(s)}$ &c. Thus the total sum will be

$$\frac{(s+1)(t+1)}{n(s)} \cdot \frac{1}{r} + \frac{(s+2)(t+2)}{n(s)} \cdot \frac{1}{r^2} + \frac{(s+3)(t+3)}{n(s)} \cdot \frac{1}{r^3} \text{ \&c.}$$

Now the life annuity on the head of a woman of age t is as one knows

$$\frac{(t+1)}{(t)} \cdot \frac{1}{r} + \frac{(t+2)}{(t)} \cdot \frac{1}{r^2} + \frac{(t+3)}{(t)} \cdot \frac{1}{r^3} + \frac{(t+4)}{(t)} \cdot \frac{1}{r^4} \text{ \&c.}$$

[4] *Translator's note*: Wenceslaus Johann Gustav Karsten (1732–1787) Professor of Logic at Rostock (1758), Butzow (1760) and both Mathematics and Physics at Halle. The work cited is *Theorie von Wittwencassen ohne Gebrauch algebraischer Rechnung.*

& the life annuity deposit on the linked heads of two persons of whom one has age t, & the other age (s), so that the rent finishes as soon as one of the two persons dies, is

$$\frac{(t+1)(s+1)}{(s)(t)} \cdot \frac{1}{r} + \frac{(t+2)(s+2)}{(s)(t)} \cdot \frac{1}{r^2} + \frac{(t+3)(s+3)}{(s)(t)} \cdot \frac{1}{r^3} + \frac{(t+4)(s+4)}{(s)(t)} \cdot \frac{1}{r^3} \text{ \&c.}$$

Therefore the annuity deposit on the head of one person of age t payable after the death of a person of age s, or the rent of the widowed, if the person of age t is a woman, & the person of age s a man, is

$$\begin{aligned} &\frac{(t+1)}{(t)} \cdot \frac{1}{r} + \frac{(t+2)}{(t)} \cdot \frac{1}{r^2} + \frac{(t+3)}{(t)} \cdot \frac{1}{r^3} + \frac{(t+4)}{(t)} \cdot \frac{1}{r^4} \text{ \&c.} \\ &- - - \frac{(t+1)(s+1)}{(s)(t)} \cdot \frac{1}{r} - \frac{(t+2)(s+2)}{(s)(t)} \cdot \frac{1}{r^2} \\ &- \frac{(t+3)(s+3)}{(s)(t)} \cdot \frac{1}{r^3} - \frac{(t+4)(s+4)}{(s)(t)} \cdot \frac{1}{r^4} \text{ \&c.} \end{aligned}$$

This last series is precisely the same as that of Mr. Karstens which we have given above, since that which we have called n is the same thing as (t), the initial number of women being evidently equal to the number of marriages subsisting at the beginning. One can restore the expression of this series to that of Mr. Karstens in the following manner. Let $\frac{(t+1)(s+1)}{(s)} = b^{(1)}$, $\frac{(t+2)(s+2)}{(s)} = b^{(2)}$, $\frac{(t+3)(s+3)}{(s)} = b^{(3)}$ &c. our series will be

$$\frac{1}{(t)} \left(b^{(1)} \frac{1}{r} + b^{(2)} \frac{1}{r^2} + b^{(3)} \frac{1}{r^3} + b^{(4)} \frac{1}{r^4} \text{ \&c.} \right).$$

Now $b^{(1)} = a^{(2)} + a^{(3)} + a^{(4)} \dots a^{(\mu)}$, $b^{(2)} = a^{(3)} + a^{(4)} \dots a^{(\mu)}$, $b^{(3)} = a^{(4)} + a^{(5)} \dots$, $b^{(\mu-1)} = a^{(\mu)}$, $a^{(\mu)}, \dots b^{(\mu)} = 0$, the series will become therefore

$$\begin{aligned} &\frac{1}{(t)} \left(a^{(2)} \frac{1}{r} + a^{(3)} \left(\frac{1}{r} + \frac{1}{r^2} \right) + a^{(4)} \left(\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} \right) \cdots \right. \\ &\left. + a^{(\mu)} \left(\frac{1}{r} + \frac{1}{r^2} \cdots \frac{1}{r^{(\mu-1)}} \right) \right) = \frac{1}{(t)} \left(\frac{a^{(2)} \left(1 - \frac{1}{r}\right)}{r-1} \right. \\ &+ \frac{a^{(3)} \left(1 - \frac{1}{r^2}\right)}{r-1} + \frac{a^{(4)} \left(1 - \frac{1}{r^3}\right)}{r-1} + \frac{a^{(5)} \left(1 - \frac{1}{r^4}\right)}{r-1} \cdots \\ &\left. + \frac{a^{(\mu)} \left(1 - \frac{1}{r^{\mu-1}}\right)}{r-1} \right) = \frac{1}{(t)} \left(\frac{b^{(1)}}{r-1} - \frac{\left(a^{(2)} \frac{1}{r} + a^{(3)} \frac{1}{r^2} + a^{(4)} \frac{1}{r^3} + \cdots + a^{(\mu)} \frac{1}{r^{\mu-1}}\right)}{r-1} \right). \end{aligned}$$

§ 15. If one wishes to calculate directly the sum to pay according to the number of widowed who subsist each year, one will find again the same result, because in order to have for each year the number of widowed, it is necessary to subtract the number of marriages subsisting from the number of women subsisting; thus it will be for the first year

$$(t+1) - \frac{(t+1)(s+1)}{(s)} = (t+1) \left(\frac{(s) - (s+1)}{(s)} \right),$$

that is to say that the number of the widowed is equal to the number of women subsisting, multiplied by the number of dead men & divided by the initial number of men. The sum to pay will be therefore

$$(t{+}1)\left(\frac{(s)-(s+1)}{(s)}\right)\frac{1}{r}{+}(t{+}2)\left(\frac{(s)-(s+2)}{(s)}\right)\frac{1}{r^2}{+}(t{+}3)\left(\frac{(s)-(s+3)}{(s)}\right)\frac{1}{r^3}{+}\&\text{c.} =$$

(by decomposing this series into two & dividing by (t) in order to have a mean)

$$\begin{aligned}&\frac{(t+1)}{(t)}\frac{1}{r}+\frac{(t+2)}{(t)}\frac{1}{r^2}+\frac{(t+3)}{(t)}\frac{1}{r^3}+\frac{(t+4)}{(t)}\frac{1}{r^4}\ \&\text{c.}\\ &-\frac{(t+1)(s+1)}{(t)(s)}\frac{1}{r}-\frac{(t+2)(s+2)}{(t)(s)}\frac{1}{r^2}\\ &-\frac{(t+3)(s+3)}{(t)(s)}\frac{1}{r^3}-\frac{(t+4)(s+4)}{(t)(s)}\frac{1}{r^4}-\&\text{c.}\end{aligned}$$

this which if the formula found above. Thus all these methods to calculate the funds of the coffers of the widower return to the same, this which it is not unuseful to repeat, since one has sought to spread some doubts on this object.

§ 16. All depends therefore on calculating the series

$$a^{(2)}\cdot\frac{1}{r}+a^{(3)}\cdot\frac{1}{r^2}+a^{(4)}\cdot\frac{1}{r^3}+\cdots+a^{(\mu)}\cdot\frac{1}{r^{(\mu-1)}}.$$

The numbers $a^{(2)}$, $a^{(3)}$, $a^{(4)}\ldots a^{(\mu)}$ represent the marriages destroyed in the 1st, 2nd, 3rd $\ldots\mu^{\text{th}}$ year. Now as these numbers vary little, & are very nearly the same for a space of five years, one has belief with reason to be able to take a mean among these numbers from five to five years, & to put the formula under this form

$$a^{(2)}\left(\frac{1}{r}\cdots+\frac{1}{r^5}\right)+a^{(7)}\left(\frac{1}{r^6}\cdots+\frac{1}{r^{10}}\right)+a^{(12)}\left(\frac{1}{r^{11}}\cdots+\frac{1}{r^{15}}\right)\ \&\text{c.}$$

by taking instead of $a^{(2)}\ldots a^{(7)}\ldots a^{(12)}$ &c. the mean among the number $a^{(2)}\ldots a^{(6)}$, $a^{(7)}\ldots a^{(12)}$ &c.

§ 17. Let

$$\frac{(t+1)}{(t)}\frac{1}{r}+\frac{(t+2)}{(t)}\frac{1}{r^2}+\frac{(t+3)}{(t)}\frac{1}{r^3}\ \&\text{c.} = P,$$

one has

$$(t+1)+(t+2)\frac{1}{r}+(t+3)\frac{1}{r^2}+(t+4)\frac{1}{r^3}\ \&\text{c.} = P(t)r;$$

$$\frac{(t+2)}{(t+1)}\frac{1}{r}+\frac{(t+3)}{(t+1)}\frac{1}{r^2}+\frac{(t+4)}{(t+1)}\frac{1}{r^3}\ \&\text{c.} = \frac{P(t)r}{(t+1)}-1=P'.$$

One has likewise

$$\frac{(t+3)}{(t+2)}\frac{1}{r}+\frac{(t+4)}{(t+2)}\frac{1}{r^2}+\frac{(t+5)}{(t+2)}\frac{1}{r^3}\ \&\text{c.} = \frac{P'(t+1)r}{(t+2)}-1=P''.$$

$$\frac{(t+4)}{(t+3)}\frac{1}{r}+\frac{(t+5)}{(t+3)}\frac{1}{r^2}+\frac{(t+6)}{(t+3)}\frac{1}{r^3}\ \&\text{c.} = \frac{P''(t+2)r}{(t+3)}-1=P'''\ \&\text{c.}$$

Now P being an annuity on the head of one person of t years, P' is an annuity on the head of one person of $t+1$ years, P'' is an annuity on the head of one person of $t+2$ years, & thus in sequence. One has therefore

$$\begin{aligned}
P' &= \frac{P(t)r}{(t+1)} - 1,\\
P'' &= \frac{P'(t+1)r}{(t+2)} - 1 = \frac{P(t)r^2}{(t+2)} - \frac{(t+1)r}{(t+2)} - 1,\\
P''' &= \frac{P''(t+2)r}{(t+3)} - 1 = \frac{P(t)}{(t+3)}r^3 - \frac{(t+1)r^2}{(t+3)} - \frac{(t+2)r}{(t+3)} - 1,\\
P^{\text{iv}} &= \frac{P'''(t+3)r}{(t+4)} - 1 = \frac{P(t)}{(t+4)}r^4 - \frac{(t+1)}{(t+4)}r^3 - \frac{(t+2)}{(t+4)}r^2 - \frac{(t+3)}{(t+4)}r - 1 \text{ \&c.}
\end{aligned}$$

& in general

$$P^{(\mu)} = \frac{P(t)}{(t+\mu)}r^{\mu} - \frac{(t+1)}{(t+\mu)}r^{\mu-1} - \frac{(t+2)}{(t+\mu)}r^{\mu-2} \cdots - \frac{(t+\mu-1)r}{(t+\mu)} - 1.$$

§ 18. Now let

$$\frac{(t+1)(s+1)}{(s)(t)}\frac{1}{r} + \frac{(t+2)(s+2)}{(s)(t)}\frac{1}{r^2} + \frac{(t+3)(s+3)}{(s)(t)}\frac{1}{r^3} \text{ \&c.} = Q,$$

one has

$$\begin{aligned}
&(t+1)(s+1) + (t+2)(s+2)\frac{1}{r} + (t+3)(s+3)\frac{1}{r^2} \text{ \&c.} = Q(s)(t)r,\\
&\frac{(t+2)(s+2)}{(t+1)(s+1)}\frac{1}{r} + \frac{(t+3)(s+3)}{(t+1)(s+1)}\frac{1}{r^2} \text{ \&c.} = \frac{Q(s)(t)r}{(t+1)(s+1)} - 1 = Q'.
\end{aligned}$$

Likewise

$$\begin{aligned}
&\frac{(t+3)(s+3)}{(t+2)(s+2)}\frac{1}{r} + \frac{(t+4)(s+4)}{(t+2)(s+2)}\frac{1}{r^2} + \text{ \&c.} = Q'\frac{(t+1)(s+1)r}{(t+2)(s+2)} - 1 = Q'',\\
&\frac{(t+4)(s+4)}{(t+3)(s+3)}\frac{1}{r} + \text{ \&c.} = Q'' \qquad \frac{(t+2)(s+2)r}{(t+3)(s+3)} - 1 = Q'''.
\end{aligned}$$

Now Q being an annuity on the linked heads of two persons, one of s years, the other of t years, Q' is an annuity on the linked heads of two persons, one of $s+1$ years, the other of $t+1$, Q'' is an annuity on the linked heads of two persons, the one of $s+2$

years, the other of $t+2$, & thus in sequence. One has therefore

$$Q' = \frac{Q(s)(t)r}{(t+1)(s+1)} - 1,$$

$$Q'' = Q'\frac{(t+1)(s+1)r}{(t+2)(s+2)} - 1 = \frac{Q(s)(t)r^2}{(t+2)(s+2)} - \frac{(t+1)(s+1)}{(t+2)(s+2)}r - 1,$$

$$Q''' = Q''\frac{(t+2)(s+2)r}{(t+3)(s+3)} - 1 = \frac{Q(s)(t)r^3}{(t+3)(s+3)} - \frac{(t+1)(s+1)r^2}{(t+3)(s+3)} - \frac{(t+2)(s+2)r}{(t+3)(s+3)} - 1,$$

$$Q^{\text{iv}} = \frac{Q'''(t+3)(s+3)r}{(t+4)(s+4)} - 1 = \frac{Q(s)(t)r^4}{(t+4)(s+4)} - \frac{(t+1)(s+1)r^3}{(t+4)(s+4)} - \frac{(t+2)(s+2)}{(t+4)(s+4)}r^2 - \frac{(t+3)(s+3)}{(t+4)(s+4)}r - 1$$

& in general

$$Q^{\mu} = \frac{Q(t)(s)r^{\mu}}{(t+\mu)(s+\mu)} - \frac{(t+1)(s+1)}{(t+\mu)(s+\mu)}r^{\mu-1} - \frac{(t+2)(s+2)}{(t+\mu)(s+\mu)}r^{\mu-2} \cdots - \frac{(t+\mu-1)(s+\mu-1)}{(t+\mu)(s+\mu)}r - 1.$$

One will have therefore

$$P' - Q' = \frac{P(t)r}{(t+1)} - \frac{Q(s)(t)r}{(t+1)(s+1)} = \frac{(t)r}{(t+1)}\left(P - \frac{Q(s)}{(s+1)}\right) = \frac{(t)r}{(t+1)}\left(\frac{P(s+1) - Q(s)}{(s+1)}\right).$$

§ 19. Let now $(s+1) = (s) - m'$, $(s+2) = (s+1) - m''$, $\ldots$ $(s+\mu) = (s+\mu-1) - m^{(\mu)}$, m', m'' $\ldots m^{(\mu)}$ being the number of dead men in the corresponding years, one will have

$$P' - Q' = \frac{(t)(s)r}{(t+1)(s+1)}(P - Q) - \frac{(t)rm'P}{(t+1)(s+1)},$$

$$P'' - Q'' = \frac{(t+1)(s+1)r}{(t+2)(s+2)}(P' - Q') - \frac{(t+1)rm''P'}{(t+2)(s+2)} \text{ \&c.}$$

& in general,

$$P^{(\mu)} - Q^{(\mu)} = \frac{(t+\mu-1)(s+\mu-1)r}{(t+\mu)(s+\mu)}(P^{(\mu-1)} - Q^{(\mu-1)}) - \frac{(t+\mu-1)rm^{(\mu)}P^{(\mu-1)}}{(t+\mu)(s+\mu)}.$$

Now one has some tables of values P, P', P'' &c. $P^{(\mu-1)}$; thus have calculated in detail $Q-P$, one will have successively $P'-Q'$, $P''-Q''$ $\ldots P^{(\mu)} - Q^{(\mu)}$.

§ 20. When the difference between $P-Q$ & $P'-Q'$ is very small, one will be able to have immediately the value of $P-Q$ by an approximate manner, make $P'-Q' = P-Q$, this will give

$$P - Q = \frac{(t)rm'P}{(t)(s)r - (t+1)(s+1)} = \frac{(t)(s)rP}{(t)(s)r - (t+1)(s+1)} - \frac{(t)(s+1)rP}{(t)(s)r - (t+1)(s+1)} = \frac{\frac{(t)(s)rP}{(t+1)(s+1)}}{\frac{(t)(s)r}{(t+1)(s+1)} - 1} - \frac{\frac{(t)}{(t+1)}rP}{\frac{(t)(s)r}{(t+1)(s+1)} - 1}$$

this which can be useful in certain cases.

§ 21. Let there be, for example, the case that Mr. Karstens treats p. 74 for a woman of 20 years & a man of 35, this which gives $(t) = 516$, $(t+1) = 512$, $(s) = 422$, $(s+1) = 414$, $r = \frac{104}{100}$, $P = 16.408$, one will have

$$P - Q = \frac{1.0686 - 1.0486}{0.068} = \frac{0.02P}{0.068} = \frac{0.02(16.408)}{0.068} = \frac{0.32816}{6800} = 4.826,$$

therefore $Q = P - 4.826 = 11.582$. In order to have the annual annuity, it is necessary following the process of Mr. Karstens, to divide $P - Q$ by $Q + 1$, this which gives $\frac{4.826}{12.582} = 0.383$. Thus a husband aged 35 years who wishes to buy an annuity of widower of 100 écus for his wife aged 20 years, must pay annually 38.3 écus. Mr. Karstens finds by a rigorous calculation 37.6 this which does not differ much.

§ 22. We suppose a woman aged 30 years, & a man aged 40, we will have $(t) = 459$, $(t+1) = 452$, $(s) = 382$, $(s+1) = 374$, $P = 14.166$,

$$P - Q = \frac{1.079P - 1.056P}{0.079} = \frac{0.023P}{0.079} = 4.269,$$

therefore $Q = P - 4.269 = 10.367$. Dividing 4.269 by 10.367, one has 0.374. The annual annuity will be therefore 37.4 écus. Mr. Karstens finds 37.2. I will treat in another Memoir some limits of these approximations & of some other analogous approximations.

§ 23. I will terminate here this Memoir, in which I have had no other end but to show that it was inappropriate that one had taken pleasure to represent the calculations of the geometers, & in particular those of Mr. Karstens as destitute of foundation, & as not being able to serve the investigation of the establishments which subsist. The late Mr. Michelsen[5], Member of this Academy, had undertaken a long work in order to prove it, & I awaited until he had finished it, in order to try to make it return some prejudices that I had concluded on this subject. I flattered myself so much more to have successful results that having read to the Academy a little essay on the mortality of smallpox based on some principles absolutely similar to those which he assailed, this scholarly Academician asked of me this Memoir, gave his approval to it, & promised to procure for me some detailed observations of which I have need in order to continue. But death has prevented Mr. Michelsen to finish his enterprise, & the two Memoirs that one has published in the last German volume, contain only some generalities which require not at all particular refutation. Mr. Michelsen ascended so high, that he had never been able to descend to the particular subject which had been the occasion of his researches. If the government demanded to the Academy his opinion on a proposed machine, I believe

[5] *Translator's note*: Johann Andreas Christian Michelsen, 1749–1797. Professor of Mathematics at Berlin. He read to the Academy at Berlin several papers on this topic of which we mention only four: "Preuve que cette caisse ne viendra pas, comme on l'affirme, à manquer dans trois ans" (1793), "Sur les dispositions les plus convenables pour subvenir aux veûves, Section premiere" (1796), "Comparison des expériences faites jusqu'ici relativement à la caisse des veuves avec les résultats théoretiques du calcul" (1796), "Sur la théorie des objets mathématiques qui tiennent à la vie civile" delivered on 31 March and 11 May 1797. He died August 8. His papers are listed at the end of his Eloge printed in the *Histoire de l'Académie Royale . . . Berlin* 1796.

that the response of the Academy would be poorly received if it held that Berkley having denied the existence of the body & having never been refuted demonstrably, it was necessary first of all to know if the body exists or not. All the calculations of political arithmetic are based on the tables of mortality, & on this prejudice that the course of nature will be in the future such as it has been in the past. This put, the calculations proceed simply & regularly. Their justice will depend always on the exactitude of the tables of mortality, which will be always more or less imperfect; it is to perfect them, & especially to choose those have the most analogy with the end as one proposes it, that one must work with care. Next one of the most interesting objects is to calculate the risk that any establishment incurs, that is to say the probability that the real result will deviate from the mean result by a given quantity, a probability which must determine the sum that one must destine to cover this risk in order to give to the establishment all the solidity that one can desire. This calculation is long & delicate. Mr. Tetens[6] has given an essay on that in his scholarly work; the methods of Mr. de la Place on the probability of the causes deduced from events can be applied with success, as I will demonstrate it elsewhere, but these calculations will rest always on the same foundations as the first, & there will never be two methods to proceed. I have believed it necessary to add these remarks, because it is good that the public know that the Academy has held no other judgment neither expressed nor tacit in favor of the validity of the doubts raised by Mr. Michelsen, & that this scholar has nothing to advance which could destroy the received principles & to invalidate the conclusions contained in the good work of Mr. Karstens, where this able mathematician has united under the most simple & most elementary form a multitude of things useful in all places & in all times.

[6]*Translator's note*: Johann Nicolas Tetens (1736–1807) Professor of Philosophy and Mathematics at Kiel. The reference must be to *Einleitung zur Berechnung der Leibrenten und Anwartschaften*, 1785–86.