Portraits

Leonhard Euler
Daniel Bernoulli
Johann-Heinrich Lambert

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Contents

Foreword

Along with the main eulogies and biographies [i, ii, iv, v, viii, ix], I have included a recent biography of Daniel Bernoulli [vi], his autobiography [iii], for the first time translated from the Russian translation of the Latin original but regrettably incomplete, and lists of published works by Daniel Bernoulli [vii] and Lambert [x]. The first of these lists is readily available, but there are so many references to the works of these scientists in the main texts, that I had no other reasonable alternative. A very short Supplement [xi] provides notice of instructions on geophysical observations compiled by Daniel Bernoulli.

The older eulogies and biographies are certainly dated and sometimes contradict each other; in such cases, however, it is easy to discover the truth and in any case they provide valuable information about the life of their heroes and the attitude of the contemporaries to them.

I have separated each contribution into sections which at the very least ensures the possibility of referring to their texts more definitely. The references such as [1736/15] show the year of publication and the number of the book or memoir in the lists of publications of Daniel Bernoulli or Lambert. For Euler, the notation is similar, but the number of publication is that given by Eneström (1910/1913) as reprinted in Euler (1962, pp. 352 – 385), see Joint Bibliography to [i] and [ii].

The Notes to each contribution are initialled by the appropriate author or by me. The initial F. R. in the Notes to [i] stand for Ferdinand Rudio, the Editor of the appropriate volume of Euler's *Opera omnia*. The references to sources mentioned below in my Foreword are included in the Bibliographies to the appropriate contributions.

A year ago I have published a collection of almost the same contributions translated into Russian. Now I see that it contains some mistakes, and the only explanation (not an excuse) seems to be that I have somehow failed to check my first draft.

General Comments on Separate Contributions

Comments on [i]

The original French text of the Eulogy was published separately, then reprinted in the *Nova Acta Acad. Scient. Imp. Petropolitana*, 1787, pp. 159 – 212. Its German translation by the author himself was subdivided into sections separated by intervals; instead, I numbered them. Professor Gautschi had kindly sent me a draft of his own translation of the German text but avoided further contacts. Consequently, I have only made use of his work for checking here and there my own work and I also inserted his own translation of a Latin passage in § 29.

In his Introduction, Fuss mentions von Michel, who allowed "Euler's life to appear in a native guise by means of his art". Rudio (p. XL of Euler's *Opera omnia*, ser. 1, t. 1) explains that the German translation of the Eulogy was published in Basel at the expense of the state (of Switzerland) and "adorned" by Euler's portrait copying an engraving by Christian von Mechel (reproduced after the title page in that volume).

Nikolai Ivanovich Fuss (1755 – 1826) was Euler's disciple and became a member of the Petersburg Academy. He is known by his work in geometry, but mainly as Euler's "small satellite" (Youshkevich 1968, p. 196). For a description of his life and work see Lysenko (1975).

Two shortcomings of his Eulogy are, first, that he referred to the memoirs of his teacher not definitely enough; and, second, that he obviously prettified Friedrich II. Here is what Youshkevich (1968, p. 108) had to say about that monarch: Euler and Friedrich II "much disagreed about everything", mathematics in particular. The monarch "did not appreciate any abstract investigations" and "all the time interfered" in the management of the Berlin Academy. Finally, with the best intentions Fuss invariably called Euler a genius and a great man, but, as far as style goes, he had thus overdone his admiration.

Literature about Euler is of course immense. Among the newest sources I mention, Du Pasquier (1927), Spieß (1929), Michajlov (1985), Fellmann (2007) and of course Truesdell (in particular, his appropriate essays in vols. 11 and 12 of Euler's *Opera omnia*, ser. 2). Many more worthy publications about Euler are appearing/will yet appear in connection with his jubilee.

Comments on [ii]

Condorcet provides some interesting details about Euler, but, taken as a whole, his *Eloge* is simply inadequate and shows disrespect to its readers. Repetitions abound, the description of Euler's life and work is superficial, in places difficult to understand, sometimes illogical, in other places difficult if at all possible to understand, and ends (§ 38) with an astonishing statement to the effect that Euler's life was almost cloudless. I have substantiated all this in my Notes which follow Condorcet's text. Then, a few alleged facts contradict Fuss who undoubtedly knew everything relevant incomparably better.

I have translated this *Eloge* and thus hopefully done away with Condorcet as an authority on Euler but I ought to add that France, one of the most enlightened European nation, as Condorcet reasonably believed, very soon found itself in the turmoil of a bloody revolution and he himself committed suicide while in detention.

Comments on [iii]

Here is what is known about Daniel Bernoulli's Autobiography (Smirnov 1959, p. 501): it is

A translation [into Russian] *of a Latin Autobiography of Daniel Bernoulli kept at the Archive of the Academy of Sciences of the USSR. The Petersburg Academy of Sciences received it on 21 July 1776. Its ending is apparently lost.*

Smirnov did not name the translator, likely Gokhman, who translated the entire *Hydrodynamica*, and in any case, according to its style the Russian text could not have been written earlier than in the 1920s or 1930s.

The translator had inserted in brackets a few Latin words from the original text. Bernoulli had sent his Autobiography to Petersburg and likely therefore somewhat prettified his relations with the Imperial Academy.

Comments on [iv] and [v]

Daniel Bernoulli (1700 – 1782) was a most prominent scientist mainly known for his pioneer work in mechanics and physics; see Straub (1970) for a modern description of his life and work.

The first Eulogy [iv] is superficial (§§ 11 – 13), difficult to understand (§ 5 and description of works in mechanics), partly wrong (Daniel Bernoulli had not been happy and healthy all his life, see Wolf [v]) and includes long passages not directly bearing on his subject whereas his § 17 is as good as incoherent twaddle, cf. Note 19.

I have checked his text against its German translation by Daniel II Bernoulli and in many places followed him rather than Condorcet but he obviously did not know as much as Wolf [v] about the life of his uncle. I note that he called D. B. *our Daniel* and *our Bernoulli* and manifested excessive respect for Condorcet.

I am grateful to Dr. Fritz Nagel Basel) who sent me a photostat copy of that translation.

Wolf (1858 – 1862) fulfilled a great work on the history of science in Switzerland and in particular provided much information on the Bernoulli family and on scientists more or less connected with Daniel Bernoulli including many passages from their correspondence with each other and him.

Regrettably, he [v] documented his sources quite insufficiently and I was often unable to improve the situation. And he also felt himself at liberty to leave his sentences poorly connected with each other which sometimes hinders understanding (and translation). I left out many passages concerning other scientists. Some of his phrases are italicized or spaced out but it remains unclear whether by Wolf himself or by the authors whom he quotes.

Neither Condorcet, nor Wolf (nor Daniel II Bernoulli) were able to describe satisfactorily Daniel Bernoulli's work in statistics. On this subject see Todhunter (1865), Sheynin (1972) and Hald (1998).

Comments on [viii] and [ix]

Lambert is known less than Euler or Daniel Bernoulli; his modern biographer is Scriba (1973). Concerning the eulogies on him, I note that Formey [viii] compiled it as a philosopher or historian and Wolf [ix], as an astronomer, but did not notice Lambert's pioneer work on the theory of errors (Sheynin 1971). In addition, Wolf mentioned hardly known Swiss place-names which I was unable to identify.

I

Nicolaus Fuss

Eulogy on Leonhard Euler

Translated from French
and extended by various additions by the author himself. Basel, 1786

Euler L. (1911), *Opera omnia*, ser. 1, t. 1.
Leipzig – Berlin, pp. XLV – XCV

To My Fatherland

When the lustre spread by a great man over his epoch is also transmitted to his place of birth; when a city may be proud of the merits of extraordinary geniuses who came from its walls to benefit the world by their superb talent, – so whom could have I more rightfully dedicated this eulogy than to You, dear unforgettable Basel, to You, the cradle of the Bernoullis, Hermanns and Eulers whom Europe mentions with deep respect and whose memory is sacred for every admirer of sciences!

Accept benevolently this donation that one of Your sons presents You from the banks of the Neva river out of gratitude and patriotism as a token of his invariable favour and loyalty.

Illustious Fathers of the state, fellow citizen, friends! For you am I laying out this document, holy for my fatherland, intended as an unforgettable recollection of one of the greatest men raised by Basel to be preserved by You and in every place where he worked indirectly or directly.

St. Petersburg, 28 April 1785

Introduction

1. The undeserved approval with which my sketch of Euler's work has met everywhere, although not unexpected, was very flattering. The ten-year daily contact with the great man had given me an opportunity to find out much about the because the circumstances of his life not generally known in spite of the contemporary taste for authoritative funny stories. And the study of his writings for such a long time under his guidance had acquainted me not only with their contents, but with the motives for most of them. However, the history of his works is almost the complete history of his life devoted to science, and, notwithstanding my rather mediocre talent granted me by nature for compiling an eulogy, I was therefore sure that no admirer of Euler will read it without sympathy.

I am translating my eulogy into German both because of the slow dissemination of our academic editions [over Europe] by the book trade and taking into consideration that many of my foreign friends had encouraged me to do so. And I have gladly made use of the leisure presented me by the passed Easter holidays for this task as well as of the offer of my generous friend von Mechel to allow Euler's life appear in a native guise by means of his art.

Whether I have not disappointed the expectation of my friends; whether the unadorned expression of my feelings in German was not once more displeasing; and whether some strain in the structure of my phrases, etc will not betray here and there that my work was first done in French, – all that I ought to leave to the readers' judgement.

That I was only able to devote a short time to this task may excuse my mistakes, just as the imperfection of the original text was excused previously. I have enjoyed the rights of a translator of one's own work by shortening or expanding, deleting or adding material as clarity, coherence and other circumstances apparently demanded.

The additions concern points to which readers, and especially mathematicians will not be quite indifferent. Had I intended to say everything remarkable presented me by such a fruitful subject, I could have easily multiplied their number, but the requirements of the original text had determined the boundaries which I did not want to overstep too much even in the translation.

2. A biographer describing the life of a great man who had honoured his century by considerably enlightening it, invariably praises the human mind. However, no one ought to paint such an interesting picture if he does not combine his most perfect knowledge of the science, whose advance must be noted, with all the conveniences of style needed for him but thought to get along rarely with studies of abstract sciences.

Even if the biographer is spared from casually decorating his subject, great as it is by itself, and only keeps to the facts, he is still compelled to arrange them clearly and tastefully and describe them in a dignified manner. He ought to show the means by which nature brings forth great men; should track down the circumstances that benefited the development of their superb talent; and must indicate what did his hero do for the sciences by sufficiently referring to their scientific works. Finally, he ought not forget to show the state of those sciences before his appearance and thus establish his point of departure.

3. Already when, at an assembly of the Academy, I had offered to describe the life of the immortal Euler, I had known all these demands and felt how difficult it will be for me to fulfill all of them, and imagined it all the more since the painful loss of my unforgettable teacher had increased my awareness that the narrow confines of an academic report will not allow me to achieve sufficiently all the duties of a biographer.

So now I am offering what the circumstances permit me to report: an attempt to describe the life of that great man, and I am satisfied that I have thereby scattered some flowers on the grave of my dear teacher. and provided the necessary sources for anyone feeling himself strong enough to compile his worthy eulogy.

[The Main text]

1. Leonhard Euler, Professor of mathematics, Member of the Petersburg Imperial Academy of Sciences, formerly Director at the Berlin Royal Academy of Sciences, member of the Paris Academy of Sciences and the London Royal Society, member of other learned societies, was born in Basel on 4/15 April 1707. His father was Paul Euler, then a designated minister in Riehen, and his mother, Margaretha Bruckner, belonged to a family commendably known to the world owing to many scientists of that name.

2. Euler spent the first years of his childhood in Riehen. To that rural life, in a country where in general moral standards had been dropping slower than elsewhere, and to the example of his parents he probably ought to be indebted for his simple character and that natural morality which distinguished him and only due to which he was presumably able to be living his long and brilliant life that made his name immortal.

3. He received his first lessons from his father, who, being a lover of mathematical sciences and a pupil of the celebrated Jakob Bernoulli, did not fail to teach his son mathematics as soon as Leonhard's age allowed it. He did not imagine that those studies, which should have only been an educational pastime for the son destined for theology, will soon become the subject of most earnest and persistent efforts.

But the seed was planted in the soul of the young geometer and soon became ineradicably rooted. However, Euler was too well organized for showing his exclusive talent for mathematics although feeling that it was his own vocation and remaining faithful to it.

4. Happily enough, his father for a long time did not think to remove him from the studies, to forbid them to him in earnest. He himself loved them too much and understood too well their influence on the development of mental power as well as their usefulness in all branches of human knowledge.

Therefore, the talent of the young Euler had all the time for developing, and, for that matter, with such rapidity that always foreshadowed an extraordinary talent and heralded his future greatness.

5. After those lessons had prepared him for academic studies he was sent to Basel where he regularly attended the lectures of the professors. His extraordinary memory allowed him to understand rapidly everything not belonging to geometry and to be able to devote all the time left over for that favourable science of his. Having such a strongly pronounced inclination to mathematics and a mind ever more inspired by considerable success, he had been inevitably noticed by the then greatest living mathematician, Joh. Bernoulli.

The latter had soon distinguished him from his other listeners and, although not agreeing to tutor the young mathematician privately, as Euler had asked him, nevertheless offered to remove on Saturdays all his doubts that could have arisen during the week when reading most difficult writings or on other occasions. A marvellous method! However, it could have only succeeded with such a passionate genius combined with such a tireless diligence as possessed by Euler. Already then, as it seems, he was destined to overcome his teacher even if Bernoulli had marked an epoch in the history of mathematics[1].

6. In 1723[2] Euler earned the degree of master and on that occasion read a report in Latin comparing the philosophies of Newton and Descartes. After that, complying with his father's wish, he began studying theology and Eastern languages under the guidance of the celebrated Frey, and not barely successful at that, although these studies so little corresponded to his inclinations.

However, he soon obtained from his father the desired permission to devote himself completely to mathematics from which nothing could have separated him. He made use of that permission with a redoubled diligence, resumed asking advice from the venerable Joh. Bernoulli and became closely acquainted with both his sons, Nikolaus and Daniel, to whom the [Petersburg] Academy is grateful for enjoying the benefit of enlisting Euler.

7. Ekaterina I brought to conclusion the project of [her late husband] Peter the Great, that is, the establishment of an academy of sciences in Petersburg. Both the young Bernoullis were invited to Petersburg in 1725 under very advantageous conditions; when departing from Basel, they promised the young Euler, who passionately wished to follow them, to do everything possible for securing him a decent position. Next year they wrote him that they had achieved that goal and advised him to direct his mathematical knowledge to physiology.

8. A superb talent is always successful. To become a physiologist Euler only needed wishing it. He at once registered at the medical faculty and started attending the lectures of the most excellent Basel physicians with all the zeal that the perspective of a brilliant career can instil in a courageous genius.

9. Meanwhile, these studies were not sufficient for completely occupying his so active and all-embracing mind. During that period he prepared a memoir [1728/4] on the nature and transmission of sound and an answer to a prize question of the Paris Academy about the best number, height and arrangement of masts on a ship. In 1727 the Academy conferred an *accessit* [honourable reference] on his answer.

This writing as well as one of the theses that he defended on the occasion of [competing for] the vacant chair of physics in Basel prove that Euler had very early begun thinking about the improvement of seafaring, which he later furthered with so many discoveries and developments.

10. Happily for our Academy the lot that decides in Basel the filling of administrative and scientific positions was against Euler[3] who then, a few days later, left his fatherland for Petersburg[4]. There, he found a proper arena for the part that he had to play later in the scientific world; there, in Petersburg, he soon showed himself in a way wholly justifying the expectations excited at the Academy by his friends and fellow countrymen, Hermann and Daniel Bernoulli.

Nikolaus had meantime died, too early considering his increasing fame, his worthy family and the Academy.

11. Euler was appointed adjunct of the mathematical class with physiology never mentioned, and completely devoted himself according to his calling to studies to abandon which he refused in spite of his father's wish or owing to considerations of the slim chance of happiness that can usually be expected from them. He enriched the first volumes of the Academy's yearbook with many memoirs which became the main cause for arousing a noble competition between him and D. Bernoulli that lasted all their lives in a manner certainly befitted noble minds, without ever degenerating into envious jealousy. It merits to be cited as a model but regrettably rarely occurs in science.

12. The mathematical career when Euler started it was not at all encouraging. A mediocre mind could not have hoped to distinguish himself there. The memory for the great men who imparted lustre to the end of the previous, and the beginning of this century was still too fresh. Scarcely had the creators of the new mathematics, Leibniz and Newton, died, and in addition the important discoveries made by Huygens, the Bernoullis, De Moivre, Tschirnhausen, Taylor, Fermat and so many other mathematicians were still well remembered. What remained for Euler after such a brilliant period? Could he have hoped that, after their superb talents, nature, so sparing as it is, will work wonders for him after having at once created so many mathematical minds?

He began his career with a noble self-confidence, with a feeling of his own decided worth without which no great man can originate, and he soon found out that his predecessors had not exhausted all the treasures of geometry and analysis, so that for a mind similar to his there still remained enough work.

13. Actually, it could not have been otherwise. The calculus of infinitesimals was still too near to the epoch of its discovery and therefore could not have arrived at a considerable degree of perfection. Mechanics, dynamics[5], and especially hydrodynamics and physical astronomy had been still feeling the imperfection of that new method of calculation. True, the application of the differential calculus did not meet with any difficulties, but the art of integration, that is, of returning from the elements to the magnitudes themselves, felt them all the more.

Fermat discovered proofs of many properties and of the nature of numbers, but they died together with him. Artillery and navigation based themselves on a pile of unsuitable and often self-contradicting experience rather than on a sound scientific structure. The irregularities in the motion of heavenly bodies and especially the involved forces influencing the Moon's motion often occurred to be the subject of fruitless efforts of even the greatest mathematicians. Practical or observational astronomy still struggled with the imperfection of instruments, especially telescopes, for whose manufacturing there still did not exist any reliable rules. Time and time again Euler turned his attention to all these various subjects. He expanded the boundaries of the so imperfect integral calculus, invented the calculus of trigonometric magnitudes, re-established many of Fermat's proofs, simplified an indescribable number of analytical operations. And these powerful aids coupled with the amazing ease with which he was able to handle the most involved expressions made it possible for him to throw new light across all branches of mathematics.

14. Meanwhile Euler had not been long at the Academy when a coincidence of various circumstances threatened to put him out forever from his path which he was following according to his own inclination. The demise of the Empress Ekaterina I threatened the existence of the Academy as an institution costing a lot of money to the state but providing no noticeable benefit. As it often happens, the proper attitude towards similar learned societies with respect to their usefulness and influence is overlooked or, rather, is not known at all.

So the academicians were compelled to take steps for preventing them from being caught unawares by the abolition of their institution, and Euler decided to enlist into the fleet. Admiral Sievers, who understood Euler's worth and perceived him as a godsend for the sprouting Russian Navy, offered him a position of lieutenant with a promise of a speedy promotion. Happily, however, the circumstances changed to the benefit of the Academy which consolidated its position anew under the Empress Anna Ivanovna. So, when in 1730 Hermann and Bülffinger had returned to their fatherland[6], Euler received the professorship of natural sciences and held it until in 1733 his friend Daniel Bernoulli left the Academy and Euler became his successor.

15. The extraordinary number of memoirs, which Euler had read out at the Academy during that initial period of his scientific career, already proved his great fruitfulness, industry and the ease with which he managed to solve the most difficult and involved problems. In 1735 he provided yet another example of his own iron assiduity when a certain calculation[7], which some academicians wished to have several months to accomplish, had to be speedily done. Euler managed to conclude it in three days, but how dearly had he to pay for that strain! It caused high fever bringing him to the brink of death. Although his constitution saved him and he recovered, but he lost his right eye robbed by a boil developed during his illness.

16. For anyone else, the loss of such an important organ would have been a forceful motive for looking after himself and retaining his other eye. For Euler, however, work was a steady habit turned into necessity so that he often forgot even the most important physical needs, food and sleep.

17. His first large work, *Mechanica* [1736/15; 16] comprising two volumes in quarto, appeared only a year after that ill-fated incident. The revolution brought about by the discovery of the differential and integral calculuses to all branches of mathematics had considerably changed the doctrine of motion as well. Newton, Bernoulli [which one?], Hermann et al, and Euler himself had enriched that important branch of applied higher mathematics with many new discoveries. At the same time, except two or three works on mechanics, about whose imperfection Euler could not have been ignorant, there was nothing deserving to be called a textbook.

He noted with displeasure that Newton's principles of natural philosophy [his *Principia*] and Hermann's *Phoronomia* [1716], so excellent in other respects, but being mysteriously and artificially shrouded, were not as helpful as they deserved to be. They almost intentionally concealed the ways leading their authors to such important discoveries. For revealing these ways Euler summoned up all the analytical tricks which he mastered to such an extent and thus succeeded to solve very many problems that no one previously dared tackle. He combined his own discoveries with those of his predecessors, arranged them systematically and in 1736 let all that [i. e., his *Mechanica*, see above] to be published by the Academy.

18. If clarity of notions, definiteness of expressions and methodical ordering are necessary properties of a classical work, then Euler's contribution [1736/15; 16] to a large extent deserves that designation.

How can we suspect vagueness and confusion in a contribution of a man who knew how to throw light on most abstract and deepest investigations? For his *Mechanica*, however, those properties are not at all the most important. It firmly established Euler's reputation and secured him a place among the best living mathematicians, a statement that implies much indeed: Johann Bernoulli was still living. Only an extraordinary mind could have hurried forward so speedily and caught up with a robust old man who, with the approval of his contemporaries and adorned with a reputation of so many victories, who mounted and met so many mathematical challenges and never left the battlefield dishonourably.

19. Above, I have remarked that Euler, from his admission to the Academy onward, had enriched the *Commentarii* with a large number of memoirs each of them bearing the stamp of his extraordinary genius. He essentially improved the theory of curves on which in those times all mathematicians tried out their capability and the advantages of the new calculus of infinitesimals, as well as the integral calculus, the doctrines of the properties of numbers, infinite series, the motion of heavenly bodies and attraction of spheroidal bodies. He also carried out many other investigations a hundredth part of which would have been sufficient for making anyone else famous.

What, however, completed his reputation and established beyond any doubt his superiority as an analyst was the solution of the isoperimetric problem that became so famous owing to the quarrel between the brothers Jakob and Johann Bernoullis. Each of these great mathematicians wished to resolve it, but neither had been able to accomplish that aim in full.

The number and significance of all of Euler's memoirs published during that period ought to wonder anyone who only had to glance at their list, and it is barely possible to understand how so much work was up to one single scientist.

20. Certainly so extraordinarily hard-working man had not participated in any diversions although the connections caused by a great reputation could have so easily dragged the generally admired scientist into their whirlpools, and he would have been readily excused owing to his youth and cheerful character created to delight society.

Euler devoted his rest hours to music and he also applied his geometrical mind to the piano. Abandoning himself to the pleasant feelings of harmony, he absorbed himself in investigations of the causes of their effects and calculated musical proportions in their accords. It can be quite really said that his attempt to introduce a new theory of music [1739/33] was the fruit of his rest hours.

21. This deeply thought out theory filled up with ideas either new or shown from a new point of view did not however cause special sensation. The only reason perhaps was that it contained too much mathematics for musicians and too much music for mathematicians. Nevertheless, we find there a theory partly based on Pythagorean principles as well as many important hints for manufacturers of musical instruments and composers, and the doctrine of keys is provided with the same clarity and definiteness that mark all his works.

22. As far as the theory itself is concerned, its physical part is not called into question. Euler issued from the principle that the presentation of any perfection causes pleasure; that order is a perfection that excites pleasurable sensations in our soul and that, consequently, the pleasure that we feel from nice music is based on hearing the proportions in the system of notes with respect both to their duration and the number of air vibrations which generate them. This psychological principle applied to all aspects of music served as the basis of Euler's theory.

23. That explanation was judged unsatisfactory and since a mathematician cannot calculate feelings[8] it is difficult to justify that principle. If, nevertheless, it is accepted, we will have to admit that its application to the entire theory of music could not have been more fortunate.

24. Even before the appearance of that work Euler had published a treatise on calculation [1738/17]. Answering the wish of the President, several academicians took upon themselves the preparation of handbooks for educating young men, and the greatest analyst did not think that such a task, although much below his capabilities, will lower him since its aim ennobled it. The willingness and zeal with which he normally undertook and carried out unusual assignments incurred many similar tasks, and, among others, the supervision of the Geographical Department as commissioned in 1740 by the Governing Senate.

25. In 1740, the Paris Royal Academy of Sciences, that had already, in 1738, awarded its prize to Euler for his memoir on the nature and properties of fire [1739/34], proposed an important problem about sea tides. Euler thus had a new occasion for exerting himself. His memoir [1741/57] on this difficult problem demanding most involved calculations is a masterpiece of geometry and analysis, but, nevertheless, he only shared the prize with two other worthy rivals, D. Bernoulli and MacLaurin. The Academy had barely indeed seen such a brilliant competition and I would really state that none of their problems had until now answered by three memoirs of such unquestionable worth.

26. Euler's contribution especially commended itself by clarity of explanations about the forces of the Sun and the Moon exclusively [ausschließlich; separately one from another?] exerted on the sea; by an excellent determination of the figure of the Earth as changed under the influence of both those forces; by the skill with which he allowed for the necessarily neglected inertia of water thus correcting his initial findings; by many lucky integrations demanded by the investigation of the fluctuations of the sea; and, finally, by the extraordinary acuteness with which he was able to explain the main manifestations of the tides by his theory.

27. Nothing is more capable to increase the trust that Euler's sublime investigations of that great phenomenon corresponding with observations deserve to such an extent than their coincidence with Bernoulli's. Although different were the principles from which these great mathematicians issued, they closely agreed on many aspects, like for example on the determination of the tides in the cold zones of the Earth. Thus, truth sometimes apparently duplicates itself for revealing itself to its veritable confidants even when they are seeking it in different ways.

28. In general, as I noted above, Euler and Bernoulli, between whom there always existed a noble competition, often encountered each other in physico-mathematical investigations. The latter sometimes gained an advantage over Euler by his greater certainty about physical principles. He exerted all efforts to rectify the assumptions demanded by his calculations by many very skilful and thought out experiments.

Euler, whose fiery mind spurred him to complete the task (zur Vollendung), only rarely made experiments. Entirely confident of his natural feeling for distinguishing truth and falsity and of his skill in appraising combinations and similarities, he introduced hypotheses often too bold, but his superiority in analysis mostly (mehrentheils) corrected everything. And concerning the simplification of analytical formulas, the art of applying them to experiments and deriving thereby reliable results he had left Bernoulli and every other mathematician of his time far behind.

29. A rich correspondence is not always a most reliable measure of a scientist's reputation, the less so since some of them ought to be grateful for reputation only to it. It is therefore not so important to note that Euler's merits already early connected him with the greatest mathematicians. It is more remarkable that such a correspondence with the great Johann Bernoulli began already in 1727 and continued without interruption until the death of the latter in 1748. The Nestor of geometers did not see it beneath himself to ask advice from his former student and often to subject his works to Euler's verdict[9].

30. We arrive now at one of the remarkable periods of Euler's life. The variability and the brilliant success of his works made his name known over all Europe, and he had already received various advantageous offers [invitations] which he, however, invariably turned down. Then, in 1741, the Prussian envoy, Count von Mardefeld, offered him to enter the service of his King.

The old Royal [Scientific] Society established by Leibniz was strengthened by the attention of Friedrich II since his enthroning. He had worthily decided to recast it as an Academy of Sciences which was the reason for inviting Euler. The shaky state of our Academy under regency[10] still more increased the weight of the advantageous by themselves conditions.

Euler therefore accepted the King's invitation and in June 1741 left Petersburg with his family to add lustre to the Academy developing under the patronage of a crowned wise man and to gain glory in that body.

31. As soon as Euler had come to Berlin, the King showed him a flattering sign of attention by writing him from his camp in Reichenbach in the midst of his military pursuits. Against that, Euler found the Royal Society of Sciences almost in its last breath. War, always harmful to science, had thwarted or postponed the monarch's generous intention.

Meantime, a new learned society was emerging consisting partly of the members of the previous society and partly of other scientists, including Euler who then enriched the last issue of the *Miscellanea Berolinensia* by five memoirs [1743/58 – 62], unquestionably the best ones in that collection. Inconceivably rapidly there followed a large number of the most important investigations scattered among those included in the volumes of the memoirs that the Academy had been regularly yearly publishing since it origin [1746].

32. This extraordinary number of contributions on everything important, difficult and great contained in the mathematical science where new ideas were always, sublime truths often, and most important discoveries sometimes present. This is all the more amazing since Euler did not stop from regularly sending memoirs to our Academy that beginning with 1742 granted him a pension. A half of the *Commentarii* consists of the fruits of his admirable industry. He, who glances at his speedily increasing works, will barely hold back thinking that the most elevated meditations, the most involved calculations had only cost him to write them down. And posterity will be hard put to believe that the life of one man was sufficient for producing them[11].

33. Among the extraordinary or special works of that period there is a contribution on the general method of finding curves possessing some property of maximum or minimum. Already when Euler studied the isoperimetric problem he discerned the great usefulness of that investigation both for pure analysis and treatment of physical matters.

He noted that all curves presented by that kind of problems have a maximum or minimum[12] which in many cases can be found by the method of isoperimetry. He even stated that all natural phenomena could be just as well explained by the doctrine of the greatest and the least when issuing from final causes as by the effective causes if only always knowing how to distinguish the kind of maxima and minima applied by nature[13].

Daniel Bernoulli also applied the same method for determining the form of a curved elastic band, but arrived at a differential equation of the fourth order and was unable to find the general equation of an elastic curve. He informed Euler about that and conjectured that the paths described about one or more centres of forces[14] can be determined by the same method.

Euler dwelt on that important subject once more and published a complete contribution [1744/65] on the isoperimetic problem. It can be maintained that he had expended there the whole treasure of most elevated analysis. It also contained the first ideas about variational calculus [calculus of variations] later elaborated by him and the famous Lagrange.

34. The same year, 1744, when the Academy was established anew and Euler appointed Director of the mathematical class, there appeared his general theory of motion of comets and planets [1744/66] and the Paris Academy crowned his memoir on magnets [1748/109][15].

35. The doctrine of the causes of magnetic phenomena that he reported is generally known so that I do not have to dwell much on it. However, since that subject is more readily comprehensible to each reader than any other described here, I cannot pass it over in silence. Euler issued from the Cartesian principle that the circulation of infinitely fine elastic matter through imperceptible canals of a magnetic body is the cause of the visible peculiar phenomena.

He imagined the pores of the magnet as so much openings of narrow parallel tubes joined together and supplied from within by valves similar to those of the veins and lymphatic vessels of an animal body. These narrow tubes, as Euler presumed, only let through the fine matter contained in the ether[16] and pushed forward by their resiliency whereas their backward movement was hindered by the valves.

When flowing out, that matter turns to both sides of the magnetic body because of the ether's resistance, returns back from the outside to the openings and is squeezed into them anew by the ether and in this manner generates the magnetic whirlpool visible by the formation of rays on a paper with scattered iron fillings placed on the magnet.

Thus by a very perceptively developed idea Euler explained all the properties of magnets and the coincidence of the phenomena with hypotheses so nicely corresponding to the general laws of natural science had won over many followers.

36. During the same work-filled year, 1744, Euler [1745/77] translated Robins' principles of gunnery [1742]. The King asked his opinion about the writing most suitable in that field. A few years ago, Robins who did not understand Euler's *Mechanica*, rudely attacked it. Euler [however] praised his book to the King and at the same time volunteered to translate it and accompany the text by additions and explanations. These explanations contain a complete theory of motion of shells, and during the next 38 years nothing has appeared that could have thrown away anything done by Euler in that difficult branch of mechanics.

The worth of his excellent work has been generally recognized. An enlightened statesman, the French marine and finance minister Turgot let it be translated into French and introduced it into French artillery schools[17]. Almost at the same time appeared its English translation published in the excellent typographic design only possible in England.

In his translation, Euler wherever possible justly treated Robins and with a rare modesty corrected his mistakes against theory. Euler's only revenge for the previous injustice on his enemy consisted in making Robins' work so famous as it would have never been otherwise. I abstain from any remarks about such a dignified behaviour of a great man. Who will not approve, not wonder?

37. Various physical investigations followed after that work and one of the most remarkable of them was a new theory of light and colour [1746/88]. Euler found the cause of fire, gravity, electricity and magnetism in the ether, and he even calculated the weak resistance experienced by the heavenly bodies moving through that fine matter. It is easy to understand that for him the Newtonian theory of emission of light could not have been sufficient for explaining the phenomena of light.

When justifying such a [his own] theory that served as an introduction to his theory of light and colours, Euler showed how strongly does the assumption of an empty space contradict the material outflow from the Sun and fixed stars whose intersecting rays necessarily fill all the space and will much more strongly than the ether resist [the movement of] the heavenly bodies. Newton denied the existence of ether exactly for this reason.

Euler showed how impossible it was for the material particles to move with such an inconceivable velocity without hindering each other. He calculated the loss of matter that the Sun would have to experience from such an outflow and showed that its entire enormous mass will be then exhausted in a few seconds. And, finally, he put forward another equally serious objection by noting that for transparent bodies to allow material rays of light a free passage from all directions they themselves should be deprived of all matter, or, what is the same, should not be bodies anymore.

38. Even Descartes suspected that light propagates the same way as sound. Actually, it is impossible to underestimate the striking similarity existing between the impressions of vision and hearing. Sound and light are transmitted to us from distances inaccessible to other senses, both propagate along straight lines, both can be reflected.

Euler [1750/121?] took into consideration these similarities and, when comparing [sound and light] later [1750/151], showed that light originated from the vibrating motion of the ether, and that the cause of sound is a similar air flutter; that the difference between colours as also between tones depends on the velocity of vibrations;

and that sound when passing through suitable bodies changes its direction just as the rays of light do, and refracts in a certain way.

This main proposition, proven as rigorously as possible in physical reasoning, enabled Euler to explain easily and naturally all the phenomena of light. The uneven refraction of rays of light never explained by Newton followed so naturally from Euler's theory that it could have been thus discovered *a priori* had it not been known for a long time from experience.

39. Exactly at the time when Euler was busy with refuting the Newtonian theory of light, the philosophy of Wolff attained in Berlin its greatest glory. Everyone spoke only about monads and sufficient causes. The scope that Wolff and his followers attached to that [Newtonian] principle was for Euler only a topic for friendly jokes, but the doctrine of monads, as he saw it, was too mistaken for him to abstain from publicly discussing it.

He had done just that in his thoughts about the elements [particles] of bodies [1746/90; 91; 81] where he showed that simple things [monads] cannot be however small without becoming infinitely small, i. e., disappearing; that the force of friction is as an important property of bodies as their expanse or impenetrability; that that force contradicted the attributed property of simple things to change incessantly their position; that those simple things can therefore exist not more than the epicurean atoms[18] and that everything following from the principle of the indistinguishable falls down.

After refuting that systematic doctrine, whose future fate was the same as of those many other false although great systems, Euler became able to replace the properties attributed to the monads by Leibniz and Wolff by the force of friction or resistance which was one of the properties of material acknowledged already by Leibniz and to regard it as the cause of all the changes discovered in nature. He later applied that same principle for explaining the action of pressure and shock and proving that material cannot think.

This attack against the doctrine of monads so popular at the time encountered many opponents whose writings are now forgotten together with that doctrine that they attempted to defend. They are only remembered as a vivid example of delusions to which the human mind is sometimes liable.

40. Concerning the principle that according to Euler the force of inertia is the cause of all forces and all the laws of motion, it is of an extensive scope and corresponds to the simplicity shown by nature in all of its laws. Although its cognition is only metaphysical, its action can be calculated. And all that we can demand of a hypothesis is that it is sufficient for explaining phenomena.

41. It would have been quite proper to recall a number of other philosophical investigations published at that time in the academic yearbook. There, with as much pleasure as wonder, we can see most sensible physics coupled with the most elevated geometry. To those studies belong Euler's investigations of the comet tails [1748/103]; northern lights and the zodiacal light [Ibidem]; propagation of sound and light [1748/104]; space and time [1750/149]; origin of forces [1752/181], etc.

The boundaries of this academic speech do not however allow to indicate all the remarkable contained in the large number of memoirs published in the collections of the Petersburg and Berlin academies. Happy and fruitful was Euler in discovering important mathematical truths, and to the same extent was he penetrating in explaining physical phenomena. Although bold to introduce assumptions which could have been justified by calculus, he was cautious about hypotheses not subjected to them. And he was the originator of brilliant and lofty systems; the world has recognized the worth some of them, and the posterity will decide about the others. The biographer had attempted to simplify the future verdict without prejudging it.

42. We return from the philosopher to the mathematician. Among all useful knowledge that the united forces of geometry and analysis can lead to an essential degree of perfection only the ship handling did not benefit from the general advance of the physical and mathematical sciences. Except for the hydrographical part and the art of navigation nothing yet had been tackled by professional mathematicians; the imperfect attempts by Huygens and the Cavalier De Renau [1689] about directing ships and their velocities could have hardly been taken into account. Euler was the first bold enough to elevate ship handling to a perfect science[19].

A writing about the motion of swimming bodies sent by its author, La Croix [1735], to the Petersburg Academy, suggested him his first pertinent ideas. After a few fortunate investigations about the equilibrium of ships, he was able to determine to a certain extent their stability. The success of these first attempts had encouraged him to deal with navigation in full, and thus appeared his great work [1749/110]. Its first part systematically dealt with everything difficult and elevated in the theory of equilibrium and motion of swimming bodies and the doctrine of the resistance of fluids.

43. These general principles were not yet however sufficient. Navigation has to do with swimming bodies of a certain form, and involved are not only resistance and force. A ship ought to ensure that the former is weakened and the latter increased as much as possible. It must properly resist the attempt of water to bend and rock it, ought to possess all the properties demanded and made possible by its purpose.

Therefore, the theory should give us general knowledge about the construction and handling of ships and indicate means for combining all the properties of a good vessel some of which can only be ensured by sacrificing others. For example, greatest stability and greatest speed are incompatible. It is therefore most important to know how much ought to be sacrificed against any benefit. This is the subject of the second part of Euler's work where all that the shipbuilders and navigators can expect from the new theory is recapitulated. Later Euler had enriched that important branch of applied mathematics by many new and useful views, in particular with two memoirs on the best means for replacing the lacking wind in case of big ships [1771/413] and about the action of pitching and rolling [1771/415]. In 1759 the latter earned the prize of the Paris Academy.

44. Owing to those various contributions the shipbuilding, previously for a long time lacking reliable principles and keeping to collected practical rules. Euler provided it with a theory which other sciences only obtained step by various step by trial and error.

45. That theory was not, however, written in the language typical for skilled workers; it assumed such mathematical knowledge as could not have been expected from master shipbuilders or helmsmen. To actually benefit from Euler's important discoveries it was necessary to omit too difficult calculations and too complicated and deep reasoning, and Euler himself felt it.

This consideration and many conversations with Admiral Knowles, that took place after Euler's return to Petersburg, convinced him to delete from his theory everything not really necessary for seamen or less understood by them. Thus emerged a complete theory of shipbuilding and handling of ships comprehensible to all seamen [1773/426].

46. Never had a contribution of a geometer met with such brilliant success. A new edition was at once published in Paris. It was introduced in the Royal marine schools and the King presented Euler 6000 livres[20] for his numerous discoveries benefiting the French nation as also all enlightened nations as honourably stated by the Editor of the French edition. Almost at the same time there appeared translations into Italian, English and Russian and on the occasion of the last-mentioned translation our great Empress presented Euler 2000 roubles.

47. I have intentionally summarized here Euler's most distinguished writings on the indicated subject although they had been published over a very long time; indeed, it is agreeable and interesting to survey at once how much two of the most useful branches of knowledge, shipbuilding and ship handling have to be grateful to the great Euler.

48. Now, however, we must go twenty years back for recalling Euler's previous works. There, I find first of all many assignments directly from the King concerning the levelling of the Finow channel between Havel and Oder; the salt-works of Schönebeck; the waterworks of Sanssouci; the Calzapighi plan of a lottery[21] and other financial projects. These commissions gave Euler an opportunity for directly applying his views for the state's benefit [in particular] by preventing the accomplishment of many harmful projects and many unnecessary or burdensome expenditures. A collection of 54 handwritten letters from the King to Euler, some of them written by him personally, prove the great trust with which the monarch honoured the views and truthfulness of the worthy man whose advice about the Academy and the Halle University he had also on many occasions asked[22].

49. For arranging in perfect order all of Euler's important findings in, and expansions to the differential and integral calculuses made during almost 30 years and scattered in the academic collections, as was his own early intention, he had to compile a preparatory contribution containing the necessary preliminary introduction to the analysis of infinitesimals [1748/102]. There we find a discussion of the entire doctrine of algebraic and transcendental functions, their transformation, resolution [Auflösung ?] and expansion [into series]. It includes all necessary and worthy of being known about the properties and summing of infinite series and indicates a new and remarkable manner for dealing with exponential magnitudes. It ensures a clear and fruitful notion of logarithms and their application and explains Euler's newly discovered algorithm [rule of calculation] of circular functions.

The second part of the *Introduction* provides the general doctrine of curves and their subdivision, an addition about the theory of solids and their surfaces with the emerging equations in three variables. In conclusion, that important writing contains the notion of doubly curved lines occurring at the intersection of two curved surfaces.

50. After that *Introduction* there followed a treatise on differential [1755/212] and integral [1768/342; 1769/366; 1770/385] calculuses published at the Academy's expense. The main merit of the first of these contributions, whose subject its discoverers had already brought to a considerable extent of perfection[23], consists in the point of view from which Euler dealt with its principles and their systematic arrangement; in the methodical presentation of all that; in the clarity shown in applying that calculus concerning the theory of infinite series and the doctrine of *maxima* and *minima*.

His own discoveries are mixed with those of the originators but the traces of his genius are ineradicable. Even when the great mind was unable to find anything new, it brought to maturity the discoveries of others, simplified their principles, made them more evident and derived from them new corollaries. Who would deny those signs of a genius in Euler's works? Each page contains something belonging only to him, but showing there all the novelties would have been too verbose for this eulogy.

51. The integral calculus whose beginnings are lost in the origin of the analysis of infinitesimals is still far from perfection of the differential calculus. And for calculating the magnitudes from their elements[24] no general rules have yet been discovered which is unlike the matter with the opposite operation. Should this happen, the just posterity will admit that Euler, by his innumerable fortunate integrations which only he was able to perform, had prepared that advance.

He triumphed by greatly widening the boundaries of that lofty science as compared with the expectations of its discoverers, and even Newton, had he been able to return, would have been surprised by the immense difficulties that Euler was able to overcome.

52. The third part of the integral calculus contained a new branch, the calculus of variations with which he enriched analysis. Above, I have said that the isoperimetric problem had suggested Euler his first ideas about it when he considered curves deviating however little from a given curve. Lagrange, his worthy follower, had grasped that idea, transformed it into an analytical problem divested of all geometrical considerations and solved it by a new kind of calculus. Euler later made that so productive of new truths and named it *calculus of variations*.

53. We already had occasion to remark that Euler's all-embracing and active mind did not always remain in the realm of mathematics, though vast indeed it is. Everything having even a tiny relation to it, became an object of his thoughts; he calculated all that was possible. We will see how much optics, natural sciences and sidereal astronomy have to be grateful for his single theory of light and colours. The examination of the Newtonian theory had already given him the opportunity to investigate the differing refrangibility of the rays of light and the harmful action of the dispersion of colours [chromatic aberration] in refractive telescopes which were completely abandoned for that reason since reflecting telescopes proved to be better.

Thoughts about the wonderful structure of the eye made him believe in some combination of various transparent bodies and Euler proposed an objective from two combined glasses with the space between them that could have been filled with water; he assumed that the shortcoming of the refracting telescope will thereby be overcome.

The celebrated English optician Dollond attacked his opinion since it opposed Newton's authority but Euler was quick to indicate that Dollond's statement was insignificant. Many experiments with menisci [lenses, convex on one side, concave on the other] with the space between them that can be filled by various liquids strengthened his assumption whereas Dollond had meantime found a [fortunate combination of] lenses with differing refrangibility that could have served for testing Euler's opinion and crowned it in 1757 by discovering the so-called achromatic telescope that marked an epoch in astronomy and dioptrics.

54. Dollond's happy success in applying a discovery, that he initially regarded as contradicting experience and Newtonian principles[25], prompted Euler to go ahead and investigate the improvement of dioptrics instruments. First of all he attempted to rectify the shortcoming occurring because of the aberration of rays of light and the spherical form of the lenses. He finally provided general rules for manufacturing telescopes and microscopes having experimentally convinced himself in their thoroughness, and such instruments were indeed made according to his indications[26].

55. One of the greatest discoveries of this century should be thus grateful to that dispute with Dollond. It greatly benefited astronomers by simplifying observations and made them capable of discovering new phenomena.

56. The controversy between Euler, D'Alembert and Daniel Bernoulli over the vibrations of strings[27] is only important for mathematicians but it prompted many superb discourses and thus deserves to be mentioned. Daniel Bernoulli, who was the first to develop the physical aspect of those investigations and explained the origin of keys by the vibration of musical strings, claimed that Taylor's solution of that problem was sufficient.

Euler and D'Alembert applied all the power of geometry and analysis to that difficult object of physical mathematics and showed that Bernoulli's solution issuing as it did from the Taylor's trochoid could not be general. That dispute lasted for a long time with mutual respect shown throughout as befitting scientists of such unquestionable merit and actually only ended with their death.

Such discussions about subjects belonging to applied (vermischten) mathematics are perhaps needed sometimes for mathematicians who are accustomed too much to the certainty and evidence of geometrical truths so as to excite in them healing doubts about other investigations incapable of such high obviousness[28].

57. Another controversy that did not last as long but had been conducted with more bitterness on both sides involved Euler in 1751. Professor König had attacked the general mechanical law of least action[29] and denied [the Academy's] president Maupertuis the honour of being its first discoverer.

Euler, however, had only entered that dispute because of his friendly relations with Maupertuis and of the Academy's honour. I am only mentioning all that since it inclined Euler to compile various excellent memoirs and also because it offers me a desired opportunity to remark that Euler, with his rare modesty, defended Maupertuis' claims on a discovery that he himself could have partly appropriated had he more self-respect or been less decent[30].

58. The solution of the important problems of the precession of the equinox and the oscillation of the Earth's axis first discovered by D'Alembert[31] prompted Euler to offer his own investigation [1751/171]. Also included there was a fortunate solution of the dispute between Leibniz and Johann Bernoulli about the logarithms of negative and imaginary numbers.

The problem about the precession of the equinox compelled Euler to investigate the rotary motion of bodies having a travelling axis. The known by then laws of motion were not adequate here and he had to go back to the initial principles of mechanics and derive from them general rules for determining the rotary motion. This investigation brought him to a new mechanical law and made him capable of solving that problem of the motion of solids in all generality[32].

59. Taken together, these investigations throwing new light on mechanics deserve to be reported. In his previous extensive work on that science Euler [1736/15; 16] only dealt with the motion of infinitely small and reserved for himself the right to investigate finite bodies, either elastic or not. And so, he [1765/289] published a theory of [motion of] finite non-elastic bodies. Since its Introduction contained an excellent and easily understood discussion of all the laws of motion of infinitely small bodies, it can be considered as a complete mechanics. He summarized there all his previous scattered investigations of the motion of solids and was able to render such an important service to physical astronomy[33].

60. Those were the most excellent works which so much distinguished Euler's stay in Berlin. At the same time, during that long period he never quit to render very important service to the Petersburg Academy: we have already noted that he had been sending there a very considerable part of his scientific writings. And he took great care about our Academy and had even been eagerly concerned with the education of many of our disciples sent to Berlin[34]. Therefore, he never in any sense left it and it seems that that was acknowledged by our Court and our army. During the stay of Russian troops in Berlin[35] they guarded him and indemnified him for the damage inflicted him on his estate.

61. Having such a marked predilection for the country where he had spent his younger days and for the [Academy's] corps, the cradle of his glory, Euler must have cherished hope to return there. And soon a new cause for the move had occurred. The enthroning of Ekaterina II [in 1762], the lustre of her government both mild and wise, both just and charitable, astonished the world. The protection that she provided sciences and men of science strengthened the Academy anew which reinforced Euler's decision to end his days at the service of a monarch embodying her peoples' happiness and the world's pride.

62. May 1766 brought nearer the realization of his desire. The Russian envoy in Berlin, Prince Vladimir Sergeevich Dolgorukiy on behalf of the Empress approved all the conditions that Euler raised for himself and his family[36]. After many impediments the King allowed him and two of his elder sons to leave but flatly refused to permit the youngest son, an army lieutenant with the artillery, to accompany them.

63. In June 1766 Euler left Berlin where he had been living for 25 years and enjoying respect befitting his great merits. Princes of the Royal family, and especially the governing Margrave of Brandenburg-Schwedt, reluctantly saw him leaving and in a most flattering way expressed their regret about losing him[37].

64. He was just about leaving when the King of Poland invited him through Prince Adam Czartorinsky [Czartoryscy] to visit Warsaw on his way. Euler [indeed] spent there ten days in all comforts that such a benevolent and amiable prince could have rendered a wise man who was able to value their pleasure without, however, attempting to ensure them[38].

On 17 July he finally came to Petersburg after being absent for a long time, was at once presented to the monarch and invited to her table. And owing to the mediation of the mighty Empress his youngest son was set free to follow his father and join the Russian [army].

65. Almost as soon as Euler settled in his house for whose purchase the monarch presented him 8000 roubles, he was taken seriously ill and recovered with a complete loss of his eyesight. A cataract formed in his left eye deprived him of his last organ of sight weakened by excessive work.

66. What a terrible accident for a person for whom a habit of working became a need, whose incessantly busy mind suddenly found itself impossible to finish an important work already sketched in his soul; and whose new deep connection with our Academy should have been marked by a still extraordinary degree of fruitfulness. Complete inactivity would have been the lot of any lesser mind, but Euler's astonishing memory and power of imagination increased because of the total removal of all the scattered external impressions soon compensated that loss which threatened to wind up the scientific career of the great man.

67. The first that he undertook was the compilation of an algebra textbook. A young man whom Euler had taken from Berlin as a servant and who, except for some skill in [arithmetical] calculations, had not a slightest idea about mathematics, became the means that he applied for that purpose. And thus appeared the delightful *Introduction to algebra* [1770/387; 388], so well-known and so admirable owing to the circumstances accompanying its preparation and also because of the extraordinary clarity of exposition. Even there, in a textbook only intended for beginners, the inventive mind of its author manifested itself by new methods. And, to my knowledge, it is the only one where the so-called Diophantine problems [leading to his equations][39] was connectedly discussed. A Russian translation of that textbook was published two years in advance and soon followed by a French translation.

68. The arrival of Krafft soon made it possible for the blind old man to undertake a larger work whose sketch he had prepared long ago, that is, a summary in a special contribution of everything done during 30 years for improving optical instruments and their theory. As was characteristic of him, he got down animatedly to processing that outline and published his *Dioptrics* in three large volumes in quarto [1769/367; 1770/386; 1771/404].

69. The first part of the important contribution mentioned contained the general theory of that *new* science. I will be allowed to insert that adjective after recalling that the present state of dioptrics is only due to Euler before whom it barely deserved to be called a science. The excessive length of telescopes which was necessary before the invention of the component objective for ensuring a considerable magnification and the entanglement of images due to the rainbowed colours compelled astronomers to abandon completely the refractive telescopes.

The calculation of the most advantageous composition of these instruments as also of the reflective telescopes replacing them was absolutely chaotic. Although that problem squarely belonged to elementary geometry and only demanded a little knowledge of the infinitesimal calculus, its solution had been lagging so much behind that the advance of the theory can only be measured from Euler onward.

70. The second and the third parts of that *Dioptrics* contain an entire set of rules for the best composition of telescopes [spyglasses?], reflective telescopes and microscopes. The calculation of the aberration of the rays of light due to the spherical form of lenses is a masterpiece of most subtle analysis and we are rightfully surprised by the extraordinary tricks applied for combining in these instruments all the possible advantages of clarity of images; wide-angle fields of vision; and shortness of instruments, for every magnification and number of eyepieces as well as by the simplification of dioptrics calculations formerly so boring because of the number and complexity of the [optical] elements, – all that deserves the gratitude and approval of the whole world.

71. At the same time as the academic printing house processed those works they were also busy with putting out the *Letters to a German Princess* [1768/343; 344;1772/417], the *Integral Calculus* [1768/342; 1769/366; 1770/385], the *Introduction to Algebra* [1770/387; 388], the investigation of the comet of 1769 [1770/389], the calculation of the solar eclipse and the transition of Venus [over the solar disc] [1770/397], the new lunar theory [1770/399], the lunar tables [1772/418A] and a contribution on ship building and steering of ships [1773/426], all that apart from a large number of memoirs included in the volumes of the *Commentarii* published during that period.

72. Just as the first volume of *Dioptrics* had been published, as its Russian translation and also a new French edition[40] appeared and made it one of the most popular and favourite physical textbooks which essentially contributed to acquaint, among others, the fair sex and those unable to judge Euler's merit in enlightenment by his important writings, with the name of its celebrated author.

73. We go back now to the year 1769 such remarkable in the history of science and especially of astronomy. The general desire of the mightiest European princes to support the highly attentive astronomers in observing the transit of Venus over the solar disc had then manifested itself so marvellously. That year and partly the previous year the Empress of Russia, like the kings of France, England and Spain had sent a large number of astronomers equipped by everything possible by supporting their plans to all parts of the world for observing that phenomenon so important for determining the linear scale of the solar system.

Ten astronomers, inspired by the honour of participating in that event and generously assisted by our incomparable monarch, scattered themselves across the territory held by Russia's mighty sceptre. Euler had then been thinking about a method of applying their observations for the determination of the solar parallax. He discovered a new method to treat the observations of the transit and in addition of the preceding solar eclipse[41] and to simplify the determination of the geographical position of the points of observation. He is therefore also partly responsible for the improvement of astronomy which followed that determination of the parallax.

74. We come finally to one of his most important contribution concerning the lunar theory with which he had so often and so successfully dealt. He [1746/87] previously published lunar tables, then [1753/187] a theory of lunar motion. Tobias Mayer had applied that theory for calculating his tables, and the British Commissioners of the Longitude later rewarded it. On that occasion the British parliament had also presented Euler 300 pounds sterling as a reward for the investigations that paved the way for Mayer to make such a considerable step in the solution of one of the most important problems [of navigation][42].

75. Meantime, the Paris Academy that since electing him to foreign membership [in 1755][43] crowned three of his contributions on the inequalities of the motion of heavenly bodies. For 1770 and 1772 the Academy had formulated as its prize problems the perfection of the lunar theory and Euler, together with his eldest son who had already in 1761 shared a prize with the Abbot Bossut for the best method of loading ships, earned both prizes [1777/485; 486][44].

76. In the latter of these contributions [but not that of 1761] Euler discovered a method of reporting about those inequalities in the lunar motion which he was unable to determine in his first lunar theory owing to the entangled calculations. That encouraged him to study its complete theory anew and to compile new tables. With the assistance of his eldest son, Lexell and Krafft, they were published together with his great work [1772/418].

Instead of issuing from a fruitless integration of the three differential equations following from mechanical principles, Euler now wrote down these equations [separately] with respect to the coordinates determining the position of the Moon[45] and subdivided all the inequalities into classes as far as they depended on the mean elongation of the Sun, the Moon, the eccentricity, parallax or inclination of the lunar orbit[46].

Application of that method with deep perception and most subtle tricks only available to analysts of the first rank, led to success surpassing all expectations. The frightening calculations demanded by these studies and the means for shortening them and making them applicable for determining the lunar motion astonishes us.

77. And still more are we astonished by the patience and peace of mind demanded by that immeasurable work after considering the time and circumstances of its preparation. Being deprived of eyesight; compelled to apply all the power of memory and imagination to help him with those horrifying calculations; hurled from his house by the fire that destroyed it along with the larger part of his and his family's belongings[47], the fire that banished him from the place where for that reason he had been acquainted with every nook and cranny and where habit could have replaced eyesight; confused by the muddle that must have been occasioned by such a sudden and sorrowful change and the establishment in the new house, – Euler was still able to complete a contribution that would have all by itself immortalized his name even had it been done in a calmest and happiest state of mind. Who can abstain from wondering about that willpower, that composure bordering upon heroism of the wise man which provided him with the peace of mind necessary for such work and allowed him to hold his ground even under redoubled strokes of evil fate?

78. A few months after that sad occurrence alleviated by the monarch's generous gift of 6000 roubles, the well known oculist, Baron von Wenzel removed Euler's cataract. That operation restored his eyesight and indescribably gladdened him and his nearest and dearest. But how fleeting was that happiness! Perhaps he had been inattentive during the following treatment or too passionately made use of his restored vision and did not sufficiently spare the eye. He again lost the eyesight and had to experience dreadful pain.

79. And thus was the unhappy old man once more compelled to be assisted in his work. His sons, the learned man and the lieutenant colonel, and Krafft and Lexell had alternately been at his side helping him with the preparation of lengthier works and memoirs to be found in the last volumes of the *Novi Commentarii*[48] which I do not mention so as not to enlarge too much on my description.

80. Nevertheless, for the sake of comprehensiveness I ought to dwell a bit on some of the most important of them since they contain either improvements or expansions of his previous lengthier works. To those contributions belong writings on the equilibrium and motion of fluids [1776/481] and perfection of achromatic telescopes [1774/460?].

81. The improvements achieved by him that concerned all parts of the analysis since the appearance [in 1738] of the *Hydrodynamica* of the famous Daniel Bernoulli naturally led him to think of studying anew that part of mechanics and he had already expressed his hope about this in the Introduction to his previous *Mechanica* [1736/15; 16] and published some preliminary work in the *Berliner Memoiren*[49]. He finally fulfilled that excited expectation in four lengthy memoirs [1754/206; 1757/225 – 227] exhausting the entire theory of hydrostatics and hydrodynamics.

82. That theory is infinitely fruitful both in successful applications of general laws and in sufficient explanation of the most important phenomena. For example, Euler derived the general cause of winds and especially of the trade winds or monsoons of the Indian Ocean[50] by examining the disturbance of the air equilibrium owing to the differences of densities and heat [temperatures].

And when studying the state of equilibrium of fluids attracted by one or more centres of forces [see Note 14], he determined the figure of the Earth and the state of the equilibrium of its surrounding waters which led him to the explanation of the phenomena of tides [cf. [1741/57]]. After discovering the means for reducing the theory of motion of fluids to two differential equations of the second order [1761/258 – 260?], he applied the general principles to the motion of water in vessels, pumps, pipes, etc. Investigations of the movement of air led him finally to the theory of propagation of sound, the formation of the tones of the flute and to other acoustic matters.

Those are the various and important subjects that Euler explained in his new theory of hydrodynamics. It was so little known about this difficult branch of general natural sciences[51] so that what Euler had written about it was so much more elevated that a special reprint of those four contributions [1754/206; 1757/225 – 227] would be an important present for each mathematician unable to acquire the expensive publications of the Academy.

83. When writing his *Dioptrics* [1769/367; 1770/386; 1771/404] Euler did not consider in his theory of perfect objectives either the distance between the lenses or their thickness although there were cases in which these magnitudes were not so insignificant for being neglected without strengthening the effect of the entanglement of the rays of light in those objectives. The contributions about compound objectives and their application in many types of telescopes [1774/459; 460] are intended to remedy that imperfection. There we find an intelligible discussion of the various means for manufacturing shorter optical instruments with a wider field of vision. Following these indications, I have compiled instructions for manufacturing telescopes as perfectly as possible which was published in 1774 by our Academy in French and then translated into German by the meritorious Professor Klügel (1777 – 1778) in Helmstedt in his contribution.

84. At that time, the general defect of almost all German widow funds and funeral brotherhoods and most financial operations involving mortality such as annuities, tontines, were accused of being too advantageous for their entrepreneurs[52]. This prompted Euler to think about basing such enterprises more solidly to the extent possible by the known imperfection of most tables of mortality. His investigations [1776/473] explained the essence of widow funds and funeral brotherhoods and offered me my first incentive for an outline [1776] of a general loan-office.

85. Euler had several times pledged Count Vladimir Grigorievich Orlov to leave so many memoirs to the Academy that they will last for twenty years after his death, and had kept his promise. His previous enthusiastic attitude to work was completely preserved in spite of the loss of eyesight and the weakening of the power of mind due to old age and his innumerable discoveries did not yet exhaust his fruitful intellect[53].

A convincing proof of that is presented by the 70 memoirs he had dictated to Golovin during seven years and 250 others for which I myself made the necessary calculations and read them out at the Academy[54].

86. None among them lacks a new discovery or a penetrating idea whose development can lead others to new discoveries. We find there most successful integrations; many tricks and improvements belonging to the most elevated analysis; deepest investigations of the nature and properties of numbers; most witty proofs of many Fermat's theorems; solution of many very difficult problems about the equilibrium and motion of flexible and elastic solids; explanations of many apparent paradoxes.

Everything abstract and difficult contained in the doctrine of motion of heavenly bodies, their mutual perturbations and irregularities is brought there to perfection only made possible by the improvement of analysis at the hands of the greatest geometer. Each branch of mathematical knowledge has something to be grateful to him for.

87. These are Euler's merits of enlightening his time and his works deserving immortality. As long as it remembers science, the posterity will retain his name alongside Galileo, Descartes, Leibniz, Newton and very many other great men who honoured mankind by their genius. And that memory will still be preserved when so many others transiently glorified by the frivolity of our century will be long and forever forgotten.

88. Only a few scientists have written as much as Euler did, no geometer had ever covered so many subjects or threw so much light on all branches of mathematics.

89. He, who understands the mighty influence exerted by men of extraordinary genius on the widening of human knowledge, who weighs the extreme rarity of excellent talents whom nature apparently privileged by granting them the right to enlighten, while noting that they are withdrawing from their brilliant career, – he can hardly restrain himself from wishing that they should have been spared the general fate of mortality or that at least their demise should have been delayed beyond the usual boundaries of human life.

And Euler's life was long and active. Apart from losing eyesight he did not suffer from so usual harmful consequences of overexertion and retained up to his last day the same strength of mind as he had been enjoying all his life and its traces are not missed in his last works.

90. A few fits of giddiness about which he complained in the first days of September 1783 did not hinder him from calculating the motion of balloons [1784/579] that began then to attract general attention, and he managed to achieve a difficult integration to which that investigation had led him.

That giddiness was, however, a harbinger of his death which occurred on 7 September. Even at dinnertime, retaining his invariable intellect, he had yet connectedly conversed with the deceased since then Lexell and me about the new planet[55] and other topics, then left for his usual after-dinner rest. At tea he still joked with one of his grandsons, but then suddenly experienced a stroke. He lost conscience saying *Ich sterbe* (I am dying) and died a few hours later thus ending his glorious career at the age of 76 years, 5 months and 3 days.

91. Thus died our oldest academician after being for 56 years the pride and ornament of our academy, a witness and participator of its origin and development. His influence on our contributions was so essential that, even after leaving out all that he had accomplished in Berlin, the *Commentarii* [including the *Novi Commentarii*] show clearest marks of his departure and return as though his existence all by itself were sufficient for spreading on everything life and activity[56].

Before dying, he had experienced consolation of seeing the dawn of a clear day that the wise direction of Her Highness Princess Dashkova had broken over the Academy and his appropriate happiness was just as great as his devotion that he always felt to our corps.

92. Euler had a healthy and robust constitution without which he would have hardly endured so many shocks with which the acute and numerous diseases racked his body[57].

93. His last days were quiet and joyful. Apart from some weaknesses unavoidably connected with old age he enjoyed the health that allowed him to devote his time to study although age usually compels to pass it restfully. He thus continued to be active to the end of his life wholly sacrificed to science and combined the pleasure of his glory and public respect with the fruits of his mind and virtue, – the much purer pleasure of remaining true to his inner conscience and duties until his last hours. He always found relaxation within the family circle, in the delightful domestic bliss it is able to spread over the life of a paterfamilias.

94. He largely possessed what is usually called erudition; he read the best writers of ancient Rome; sufficiently knew the old mathematical literature; was intimate with the history of all ages and nations. He even knew more about medicine, herbal remedies and chemistry than expected from a scientist not specially studying those sciences.

95. Great glory and public respect still more essentially based on his virtue which does not always go with scientific merit often attracted travellers to him. I saw many of them leaving him full of astonishment and admiration. They were unable to understand how could have a man who had apparently only been busy for half a century with discoveries in mathematics and natural sciences retained so much unnecessary knowledge alien to the subjects of his studies.

That, however, was the effect of his fortunate memory holding ready at hand everything once imprinted by extensive reading. Who, like Euler, knew *Aeneid*[58] by heart from beginning to end and was able to tell you its first and last verse of every page in his own copy, would have really held [everything] read in his old age when the impressions are the most vivid[59].

96. Perhaps owing to his memory Euler lacked that adaptability because of which we are usually imperceptibly acquiring the pronunciation of those surrounding us. Euler, however, invariably kept to the Basel pronunciation and to all the peculiar idioms of his home town. He often enjoyed himself by reminding me certain provincialisms and inversions or by admixing his talk with Basel expressions whose use and meaning I had forgotten long ago.

97. Nothing squares with the inconceivable ease with which he without a trace of annoyance abandoned his calculations to lower himself by participating in a shallow small talk and was later able to return to his calculus. The capability to discard the scientific atmosphere of a study, conceal superiority and tune in to the aptitude of common people is too rare not to be reckoned as one of Euler's merits. An invariable mood, a mild and innate cheerfulness, a certain good-natural banter (Kausticität), a very naïve and amusing manner of narrating made his conversation equally pleasant and welcomed.

98. His great vivacity, without which his mind could not have been astonishingly active, sometimes carried him away, he often flared up. However, his kindness, the main trait of character, extinguished his anger as rapidly as his sensitivity inflamed it. He was unable to detest anyone for a long time.

99. He was most sincere and unfailingly honest, the accepted national virtues of a Swiss. As a sworn enemy of all injustices, he often ventured to criticize them bluntly or under certain circumstances even to attack publicly those who wished to act unfairly. And we all still remember how happy he sometimes was after having been able to console the oppressed, to do away with abuse.

100. Not as every great man is, he justly appraised the merits of others, and even of his opponents. How often did I hear him expressing unreserved satisfaction and most sincerely praise the merits of a Daniel Bernoulli, a D'Alembert[60], a Lagrange et al. Each new discovery gladdened him as much as though it was his own, and that proves that the spread of the realm of knowledge meant more for him than the world's approval.

101. For him, religion was sacred and venerable. His piety was sincere, his worship arduous and heartfelt. He exercised all the duties of Christianity very attentively and without bigotry or pomposity, was benevolent and tolerant to a great extent, although not towards enemies of religion and especially professed apostles of freethinking, against whom he [1747/92] publicly defended revelation.

102. As husband, father, friend and citizen he was a model of conscientious discharge of duties arising from those relations, and all this combines to justify our sincere pain due to his death and to show the world what we lost with him[61].

103. Euler married twice, the first time in 1733. His first wife was Catharina Gsell, the daughter of an artist from St. Gallen, whom Peter the Great while in Holland had taken into his service, and a sister of the famous von Loen. After losing her in 1776, his housekeeping obliged him to marry once more, this time Salome Abigael Gsell, a half-sister of the deceased, daughter of Maria Graff and granddaughter of Sibylle Merian, both famous for their drawings of the insects of Surinam.

104. From the thirteen children whom Euler had fathered in his first marriage eight died at an early age; from the three sons and two daughters who accompanied him back to Petersburg the daughters predeceased him. The eldest son, who already for a long time had been following in his father's footsteps is well known by his own contributions, many of them earning prizes of the Petersburg, Paris, Munich and Göttingen academies, and by participating in the last works of the deceased.

The second son, a physician at the Court and collegiate councillor, enjoys deserved glory for his knowledge and zeal shown in the execution of his duties. The youngest son is the chief supervisor at the Sestroretsk arms mill and lieutenant colonel on active service with the artillery[62]. He is also known in the scientific world since he was one of the astronomers sent in 1769 by the Academy to observe the transit of Venus (in Orsk).

The elder daughter who died in 1781was married to the chief quartermaster and premier major[63] von Bell; the younger daughter married Baron von Dehlen and died in 1780 on his estate in Jülich. Those five children gave the deceased Euler 38 grandchildren of whom 26 are still living.

105. For a long tome the image of the venerable old man will still float in my mind as sitting like a patriarch in a cheerful circle of his numerous grandchildren who were attempting to gladden him and sweeten his last days by all possible attention and tender worry. I will never again see such a touching spectacle as I did almost daily in those times.

106. It will be a vain attempt, Illustrious Assembly, to describe these moving and thrilling scenes of domestic happiness which were the triumph of nature and the best possible reward for faithfully exercising family duties. Many among you were yourself eye witnesses of this and especially those of you who are proud of having had him as your teacher.

There are five of his disciples here[64]. Who among scientists can boast of having so many of them in one and the same corps? Oh, had we been able to attest our affectionate and indelible gratitude; to show it in front of the eyes of all the world as vividly as we feel that our immortal teacher was equally admirable for his rare virtue and the power of his genius!

Dear friends, academicians! Mourn for him with the sciences that have so much for being grateful to him; with the Academy that had never lost so much; with his family whose pride and support he had been. My tears are mixing with yours and the remembrance of what I myself owe him will only disappear with my last breadth.

Notes

1. Euler often told me that he strained every nerve to doubt as little as possible and attempted, partly because of impatience, and partly owing to ambition, to resolve the encountered difficulties himself, and the happy outcome of his efforts was an inexhaustible source of purest pleasure of mind of the young geometer thirsting for truth. N. F.

2. That indication is not altogether correct. F. Burckhardt (1884) reported:

According to his father's wish, Euler registered on 29 October 1723 at the theological faculty and at the age of seventeen earned the degree of master on 8 June 1724, at the same time as the three years younger Johann II Bernoulli. F. R.

3. That indication is wrong: Euler was not at all admitted to draw lots. A. Burckhardt (1910) stated that the drawing was introduced in 1718 under the condition that "lots will be drawn after reasonable selection". And, when a professor had to be appointed,

The lot decides between two or three candidates who were found the best among all applicants by examination and ballot. The definitive appointment was the duty of the Council.

It is understandable that in 1727, Euler, a student of theology not yet twenty years old, with other applicants such as Jacob Hermann being alongside, was not at all included among the three main candidates, and therefore did not participate in the drawings of lots.

Neither the University, nor its policy are in any way responsible for Euler leaving his fatherland. Later, when dealing with the appointment of a successor to Johann Bernoulli, the University offered Euler a possibility to return to Basel, see the record of the Regency's proceedings of 26 January 1748 (F. Burckhardt 1884, p. LII). F. R.

4. They left on 5 April 1727 and travelled through Frankfurt, Cassel [Kassel], Hanover, Hamburg, Lübeck, then by sea through Reval [Tallinn] to Kronstadt. F. R.

5. In our time, dynamics is considered as a branch of mechanics. O. S.

6. They left on 25 January 1731 [vi, Note 10]. O. S.

7. A compilation of tables for determining the true midday or the moment of the upper culmination of the Sun's centre by its observation before and after culmination, see Euler [1741/50] and Courvoisier (1964, p. XI). This explanation replaces Fuss' Note on p. LVII.

Bogoluibov et al (1988, p. 380) state, in their comment to the Russian translation of the original French text of this Eulogy, that Euler lost his right eye (see below) in 1738 rather than in 1735. There also, they note that Euler's talks with Admiral Sievers (se § 14 above) are not confirmed by any sources. I have not seen a paper on Euler's eyesight (R. Bernoulli 1983). O. S.

8. Cf., however, the Weber-Fechner law connecting stimuli and sensations. O. S.

9. To justify understandably Bernoulli's early great trust in Euler's knowledge I venture to adduce the following lines from his letter [of 7 March 1739 reprinted by Eneström (1905, pp. 19 – 24). The following quotation contains some departures from the original only one of which deserves to be noted: "exploring the mass" should have been "examining the mass" – F. R.]:

By the way, I was very glad to see that you liked, – nay, even admired, – what I wrote about vertical oscillations because of the simplicity of the expression and the great utility it can have in exploring [examining] *the mass* [distribution] *of ships. However, I should have preferred you to do the calculation yourself, by your own lights, since then I would have seen clearly whether I could have been mistaken in my considerations. Indeed, I admit candidly that I rely on your insights more than on my own. In your further argument on isoperimetric curves I think you have digested everything competently and pondered it nicely on the balance of truth, so there remains hardly anything that has escaped your most acute perspicacity* […].

10. From 9 November 1740 until 25 November 1741 the regent was Anna Leopoldovna, mother of the minor Ivan VI Antonovich. O. S.

So that was what the greatest mathematician of those times wrote to Euler, then barely 30 years old. N. F.

11. Fuss mentioned here the list of Euler's publications appended to the original Basel edition of his eulogy but reasonably omitted from Euler's *Opera omnia.* O. S.

12. Perhaps Fuss meant extreme values with constraints. On the summarizing memoir [1744/65] see Dorofeeva (1972, p. 458). O. S.

13. Euler's statements indirectly corroborated Maupertuis' principle of least action. O. S.

14. The centre of forces "has no mechanical meaning" since it depends on the choice of the origin of the coordinate system, see W. Habicht's Einleitung to Euler's *Opera omnia*, ser. 2, t. 20. Zürich, 1974, pp. x – xi. O. S.

15. The contribution [1748/109] was crowned only in 1746. Anyway, the prize problem was formulated already in 1742, then, being incompletely answered, offered in 1744 and again in 1746. F. R.

16. The supposed all-penetrating medium. Its existence is not admitted anymore. O. S.

17. See Note 20.

18. Epicurus introduced *atoms* and attributed to them arbitrary motion. Why did Euler deny them? O. S.

19. Fuss had overlooked here that after the contribution of Renau had appeared and his strife with Huygens ended Johann Bernoulli (1714) published his famous relevant memoir. F. R.

20. The letter which Turgot wrote Euler on that occasion is too pleasant and honours both of them, each great in his realm, for me to resist the temptation to adduce it in full. Here it is:

Fontainebleu, 15 October 1775

During the time, Sir, that I have been in charge of the Naval Department, I thought that there could not have been anything better for instructing our young men, cadets at Naval and Artillery Schools, than to make it possible for them to study your works which you have written on both these branches of mathematics. And I have therefore proposed to the King to publish by his order your treatises on the construction and handling of ships and a French translation of your commentary [1745/77] *on Robins' principles of gunnery.*

Had I been not so far from you, I would have asked your consent before dealing with your contributions; I believe, however, that you will be well recompensed for that kind of property by a sign of the King's benevolence. His Majesty authorized me to send you a thousand roubles as a reward and He [Elle] *is asking you to receive it as a token of respect which He* [Elle] *feels with regard to your works and which you deserve more than any titles.*

I am happy, Sir, to be the executor, and I pleasurably take this opportunity to express what I had been thinking for a long time about a great man who honours mankind by his genius and the sciences by his principles of behaviour. I am, etc.

21. Sanssoussi, a park and a palace in Potsdam. The mentioned lottery is possibly that *Italian* about which Euler (1749) wrote to Friedrich II. O. S.

22. When Wolff's position at Halle University should have been filled, the King asked Euler's advice. Euler had suggested Daniel Bernoulli who turned down the invitation. Euler then had proposed Segner who was indeed appointed under very favourable conditions. It was also Euler who persuaded the King to acquire for the University the physical apparatus left by Wolff. And it was Euler again whom the King commanded to enter into negotiations with Haller for calling him to work in Prussia. Haller demands were disapproved of and his invitation shattered. N. F.

23. This is hardly correct. The differential calculus was only "perfect" from the beginning in the sense of its application. O. S.

24. That statement is understandable in spite of the dated terminology. Cf. § 13. O. S.

25. Fuss did not mention experience in his § 53. O. S.

26. The King himself, to whom Euler had sent the spyglasses manufactured according to his theory, honoured that work with his approval. I have found the monarch's letter in his own hand and it will undoubtedly please the readers:

I am grateful to you for the spyglasses that I have received since the arrival of your letter of the 14th of this month, and I praise your attempts to make the theory that justifies your work generally useful and your application to science. Since my occupations do not at present allow me to examine them [the spyglasses] *attentively, as everything coming to me from you deserves, I will do it when having more free time. And I am asking God to keep you under His sacred and worthy guard.*
Waldou, 15 September 1759 Féderic

27. See Antropova (1972, pp. 412 – 418). She mentions contributions [1750/140; 1766/317] and Daniel Bernoulli [1755/45 and 46].
28. In 1776 I have informed Bernoulli about Euler's new method of determining the vibration of strings, even more general than all his former ones, allowing for initial bending whose essence cannot be described by equations. The following extract from the answer of that immortal man to me deserves a place here for more than a single reason:

Your outline of Euler's method enjoyed me but did not at all change my ideas about that subject. I was always convinced that my method discusses all possible cases in abstracto. I acknowledge, however that from certain viewpoints Euler's method is much preferable, but, nevertheless, there are other points of view providing an opposite opinion since my method is applicable to any number of finite bodies even if the system [their system] *cannot be expected to return exactly to its previous state or be exactly periodic. As to my claims, I am always ready to lower my flag before my Admiral, etc.*

29. Maupertuis is the author of the principle rather than law of least action. Below, Fuss mentions *laws* (plural). O. S.
30. Euler himself, long before being acquainted with Maupertuis' laws of least action, discovered many minimums in nature, as for example in the motion of heavenly bodies; of all bodies attracted by many centres of forces [see Note 14]; in many curves [?] etc. Above, I had shown in connection with the isoperimetric problem how near he came to those general laws. And since applying them to a large number of mechanical problems, as the discoverer himself [as Maupertuis] publicly acknowledged in one of his contributions, Euler thus acquired a right [of the discoverer] of sorts which he always declined out of generous modesty. N. F.
Speiser (2008, p. 260) mentioned the principle of Maupertuis and Euler. O. S.
31. Rigorously proved (1n 1749). O. S.
32. Rotation of solids had been studied by many subsequent authors as well, suffice it to mention Kovalevskaya. O. S.
33. It was "an important service" for geodesy as well: the oscillation of the Earth's axis meant oscillation of latitudes. O. S.
34. He took many academic disciples to his house and table and instructed them in mathematics. Academicians Kotelnikov and Rumovsky had thus spent some years in Berlin and enjoyed his lessons. N. F.
35. In 1760 – 1761, during the Seven Years' War 1756 – 1763. O. S.
36. They are known to have been very essential. In addition to 3000 roubles yearly and insurance of 1000 roubles as a pension for his [possible] widow his three sons should have been beneficially provided for, and that had indeed happened. N. F.
37. In addition to the loss of friendly and close contacts between the Prince and Euler the regret was intensified by the feeling of gratitude for the participation of the great man in educating his daughters. He had instructed both of them; the elder, an abbess in Herforden, is that German Princess to whom, continuing his lessons while the Court had been in Magdeburg, he wrote the so popular *Letters* [1768/343; 344; 1772/417] on various subjects in physics and philosophy. N. F.
Fuss mentioned Brandenburg-Schwedt, a principality on the north of Brandenburg, now in Saxony-Anhalt. O. S.
38. Euler always recalled with grateful pleasure the favour with which the King had treated him and his family during their stay in Warsaw and journey across

Poland. And the feeling of deep respect with which the superb qualities of that prince had inspired him had been strengthened by their correspondence. I can offer my readers the best notion about it by adoring my eulogy with one of the King's letters:

Professor Euler, Sir, In replying to your letter of 4 August of last year, I would have wished to by able to confirm your opinion about the more favourable circumstances that your friendship with me dictates to express virtuously and sensibly. However, [Fuss deleted the next phrase].

Nevertheless, I am grateful for your goodwill with regard to that subject and I will now thank you for caring to acquaint me with the observations that the skilled astronomers of your Academy had made in Bender and near the mouths of Dniestr and Danube, and with the coordinates of some other places equally important for geography.

I will try to apply advantageously that information for perfecting that which is being done here with much application and success in spite of the troubles presenting a serious obstacle to the progress of science.

I am asking you to continue [your work] *both for the public benefit and my particular satisfaction and wish to have the opportunity to show you its effective proof. I am asking God to keep you under His sacred and worthy guard.*

Warsaw, 7 June 1772 King Stanislas Auguste

39. Youshkevich (1968, p. 81) quotes Lagrange's letter of 1773 to Euler. The French scientist was "especially" delighted by the part of the textbook dealing with indeterminate equations; he had not seen any other textbook containing their satisfactory description. O. S.

40. Fuss had rather thought here about the French edition published in 1770 "in Mitau [Yelgava] & Leipzig" since the first volume of the Paris edition only appeared in 1782. F. R.

41. The transit of Venus is itself considered as an eclipse. O. S.

42. Above (§ 36 and Note 18) I mentioned an official sign of respect and appreciation shown Euler by the King of France, so to say in the name of the French nation. A report about a similar righteousness expressed by another not less enlightened and generous nation about the merits of the great man can only be pleasant for his admirers. And so, a passage from a letter from the British Admiralty cannot be superfluous:

Admiralty Office London, 13 June 1765

Sir, The Parliament of Great Britain having, by an Act passed in their late sessions (a printed Copy of which I herewith transmit to you), having been pleased to direct, that a sum of money, not exceeding Three hundred pounds in the whole, shall be paid to you, as a reward for having furnished Theorems, by the help of which the late Mr. Professor Mayer of Göttingen constructed his Lunar Tables, by which tables great progress has been made towards discovering the longitude at sea. I am directed by the Commissioners of the Longitude to acquaint you therewith and to congratulate you, upon this honorary and pecuniary Acknowledgement, directed to be made you by the highest Assembly of this Nation, for your useful and ingenious labours towards the said discovery, etc.

43. The number of foreign members of the Paris Royal Academy is known to be equal to eight; men without outstanding merits rarely dare claim that honour. Euler, however, was elected when there were no vacancies; the circumstances accompanying that exception are very honourable for him, and I do not therefore hesitate to include here the following letter from the then Royal state minister, the Marquis d'Argenson.

Versailles, 15 June 1755

According to the wish of His Academy, the King had recently chosen you for filling the place of a foreign member of that Academy. And since it named at the same time Lord Maclesfield, the President of the Royal Society, for filling another such place made vacant by De Moivre's death, His Majesty had decided that the first such place vacated in future will not be occupied.

The extreme rarity of such an arrangement is too remarkable for especially bringing it to your notice and to assure you of my participation in promoting it. The Academy lively wishes to see you connected with its work and His Majesty was only able to express an evidence of the esteem which you merit so justly. Be assured, Sir, that it is impossible to be more perfectly devoted to you than I am.

That I have picked up this and some other letters from Euler's immense correspondence with persons of note hardly needs begging to be excused. They are documents that do not increase the glory of the great man but can prove that his time justly appreciated him.

The following fact not indifferent to Euler's honour can be added to the letter above: because of respect to Euler and taking into account his own merits the King appointed his eldest son a foreign member of the Paris Academy as his successor.

44. That information is not quite correct. In the *Avertissement* [Announcement] in vol. 9 of the *Recueil des pièces qui ont remporté les prix de l'Académie Royale des Sciences* that includes both prize memoirs, the first of them [1777/485] is attributed to L. and J. A. Euler, but the second one [1777/486], only to L. Euler. F. R.

45. Euler first wrote down those equations in the form mentioned back in 1749 (Wilson 1995, p. 97). O. S.

46. Elongation of the Moon is its maximal apparent distance from the Sun, and the elongation of other heavenly bodies is defined similarly whereas elongation of the Sun is a meaningless expression. The lunar parallax determines its distance and it should not have been mentioned between parameters of the lunar orbit. O. S.

47. Many books and manuscripts on the concept of the lunar theory were destroyed and the younger Euler took it upon himself to restore all that [but certainly not the books!] and to calculate everything once more. N. F.

48. Its last volume (for 1775) appeared in 1778. O. S.

49. In the *Miscellanea Berolinensia* and *Mém. Acad. Roy. Sci. et Belles-Lettres*. O. S.

50. The trade winds are the winter monsoons and Fuss' expression is not quite proper. O. S.

51. Why did Fuss dismiss Daniel Bernoulli? O. S.

52. The state itself always was the "entrepreneur" of a tontine. On that aspect of Euler's work see also [1767/335; 1770/403; 1785/599]. O. S.

53. It could have been thought that Euler's numerous discoveries possibly blunted his pleasure felt by the soul when new truths are being established and which mathematicians enjoy more distinctly and perhaps more frequently than any other scientists. Euler, however, invariably remained very responsive to such pleasure and demanded the same warm-heartedness from everyone. How often was he hurt when I reported him my little discoveries with an indifferent expression caused by modesty! N. F.

54. The earliest of these memoirs were since then published separately [1783/531; 1785/580] to the benefit of mathematicians avidly interested in Euler's work. N. F.

55. In 1781 William Herschel discovered Uranus. O. S.

56. This seems to mean that the influence weakened, see, however, § 60. O. S.

57. They were not mentioned at all. O. S.

58. Virgil's poem (a draft only). O. S.

59. This statement contradicts modern notions: it is the memory for recent events that first weakens. O. S.

The following fact deserves to be described as an extraordinary proof of the power of Euler's memory and imagination. As a hobby, during the last year he had been teaching four of his grandchildren the art of calculation and geometry. The extraction of roots made it necessary for them to find powers of numbers. A sleepless night prompted him to calculate the first six powers of all [natural] numbers under twenty, and he repeated them to us many days later without any hesitation. N. F.

60. Here, however, is Euler's private letter of 1763 (Youshkevich et al 1959, p. 221): "D'Alembert shamelessly defends all his mistakes". Those pertaining to probability theory are generally known. O. S.

61. I am glad to be able to inform the readers of this eulogy that the kings of Prussia, Sweden and Poland, the Crownprince of Prussia and the Margrave of Brandenburg-Schwedt lively regretted the loss experienced by the Academy due to the death of that immortal man and they had also informed in writing his eldest son about their pity. That was the greatest eulogy on his mind and virtues. N. F.

62. He rose to the rank of lieutenant general (Youshkevich 1968, p. 110). O. S.

63. A staff officers' rank existing in 1711 – 1797. O. S.

64. Actually, among academicians there were eight mathematicians who had benefited from being Euler's pupils: J. A. Euler, Kotelnikov, Rumovsky, Krafft, Lexell, Inochodtsev, Golovin and I. Three were absent and Lexell had meantime died to the greatest regret of the Academy and each admirer of real merits.

Oh, my friends, whom I saw while addressing you or rather when overflowing my sorrow, I saw your tears of most heartfelt emotion! I was only able to shake your hands because pain had stifled my voice, but I will never lose the memory of these sincere signs of your honest distress and I am publicly attesting to your hearty feeling being an expression of a justified affection toward our unforgettable common teacher.

At the academicians' expense Euler's bust of white Carrara marble was sculpted for which Her Highness Princess Dashkova donated a half-column of Italian marble. The bust will be installed in the Conference Hall of the Academy as a proof of that feeling for the posterity. N. F.

In his main text Fuss mentions five pupils being present; here, however, he adds three absent and one deceased, so how was it possible that in all Euler had only eight pupils among the academicians? O. S.

Some Personalities Mentioned by Fuss

For more detailed information on the first academicians of the Petersburg Academy see *Rossiiskaia* (1999)

Bernoulli Nikolaus II, 1695 – 1726. Died in the prime of life

Dashkova E. R., 1743 or 1744 – 1810. Princess, literary person. Director of Petersburg Academy from 1783

Dollond J., 1706 – 1761. English optician

Euler Chr., 1743 – 1808. Son of L. Euler, lieutenant general

Euler J. A., 1734 – 1800. Son of L. Euler, his disciple, academician of the Petersburg Academy, foreign member of the Paris Academy. List of his publications: Euler L. (1962, pp. 385 – 386)

Euler K., 1740 – 1790. Son of L. Euler, physician and astronomer

Golovin M. E., 1756 – 1790. Euler's disciple, academician of the Petersburg Academy, mathematician

Haller A., 1708 – 1777. Swiss natural scientist and poet

Hermann J., 1678 – 1733. Mathematician and mechanician, one of the first members of the Petersburg Academy

Inochodtsev P. B., 1742 – 1806. Euler's disciple, member of the Petersburg Academy, astronomer

König S., 1712 – 1757, mathematician. See Fellmann (1973) and [vi, note 7]

Kotelnikov S. K., 1723 – 1814. Euler's disciple, member of the Petersburg Academy, mathematician

Krafft (Kraft) V. L., 1743 – 1814. Euler's disciple, member of the Petersburg Academy, mathematician

Lexell A. I., 1741 – 1784. Euler's disciple, member of the Petersburg Academy, mathematician and astronomer

Loen J. M. von, 1694 – 1776, Prussian statesman

Nestor, legendary hero of ancient Greece, lived to a very old age

Orlov V. G., 1737 – 1798. Military man and statesman, Director of the Petersburg Academy

Rumovsky S. K., 1734 – 1812. Euler's disciple, member of the Petersburg Academy, mathematician

Segner J. A., 1704 – 1777. Professor of natural sciences and mathematics

German Place-Names and Rivers Mentioned by Fuss

Herford, town between Hanover and Münster

Jülich, now a town between Aachen and Cologne

Reichenbach, there are about 20 settlements thus named

Schönebeck, there are at least three settlements thus named. The largest is to the south-east of Magdeburg

Waldow, a settlement to the east of Berlin. Friedrich II wrote *Waldou*

Havel, a tributary of Elbe, flows through Brandenburg

II

M. J. A. N. Condorcet

Eulogy on Euler

Éloge de M. Euler. *Hist. Acad. Roy. Sci. Paris* 1786 pour 1783, pp. 37 – 68. Reprint: Euler, L. *Opera omnia*, ser. 3, t. 12. Zürich, 1960, pp. 287 – 310.

[**1**] Leonhard Euler, Director of the mathematical class of the Petersburg Academy and previously of the Berlin academy, fellow/member of the Royal Society of London, academies of Turin, Lisbon and Basel, and foreign member of the Académie Royal des Sciences, was born in Basel 15 April 1707.

His mother was Marguerite Brucker, and his father, Paul Euler, who in 1708 became the pastor of Riechen, a village near Basel, was his first teacher. Paul Euler soon had the pleasure of seeing how his expectation of the talent and glory of his son, so sweet for a paternal heart, was being born and fortified before his eyes and owing to his care.

The father studied mathematics under the guidance of Jakob Bernoulli; it is known that that illustrious man combined in himself a great talent for sciences and a deep philosophy that does not always accompany the former, but, [if it does], imparts it with additional scope and makes it more useful. During his lectures, he made his students feel that geometry was not an isolated science and presented it as the foundation and key to the entire human knowledge, as a science whose study allows to understand better the progress of the mind, whose culture most usefully exercises our faculties by endowing our intellect at once with power and precision. Finally, he explained that the study of geometry was just as important due to its numerous and various applications and owing to the advantage of acquiring a habit of a method of reasoning which could be used for investigating all kinds of truths and guiding us later in [everyday] life.

Paul Euler, having adopted the principles of his mentor, taught his son the elements of mathematics although preordaining him to study theology. When the young Euler was sent to Basel University, he found himself deserving to study by Johann Bernoulli. His application and cheerful disposition soon won the friendship of Daniel and Nikolaus [II] Bernoulli, the students and already the rivals of their father. He was even lucky in that the stern Johann Bernoulli took a liking to him and deigned to give him particular weekly lessons for clearing up the difficulties experienced by him during his studies. Euler occupied the other days of the week for preparing himself to be able to profit from that signal favour.

This excellent method prevented his nascent genius from exhausting itself by attempting to overcome invincible obstacles or from getting lost while attempting to open up new routes. It guided and fostered his efforts and at the same time required of him to deploy all his power strengthening with his age and the scope of his knowledge.

Euler did not enjoy that advantage for a long time: once he had obtained the degree of Master of Arts, his father who preordained his son to succeed him compelled Leonhard to quit mathematics for the sake of theology. Happily however that strong measure proved transient: it was not difficult for the father to perceive that his son was born to replace Johann Bernoulli in Europe rather than him himself as pastor of Riechen.

[**2**] In 1727, Euler's contribution [4] written at the age of 19 on the rigging of ships, a subject proposed by the Paris Academy of Sciences, won an *accessit* [a honourable reference]. This was all the more favourable since the young Alpine inhabitant was not aided by any practical knowledge and since he was only defeated by Bouguer, a skilful geometer then at the peak of his talent and being for ten years professor of hydrography in a maritime city.

Also at that time Euler stood for a chair at Basel University, but it was the lot that selected the candidate from those admitted to dispute such positions, and it was not favourable; we do not at all say *for him*, but for his fatherland that lost him a few days after that episode, and forever.

Two years previously, Nikolaus and Daniel Bernoulli were invited to Russia. Euler, who saw them off with regret, made them promise to attempt to provide for him the same honour which he not surprisingly had solicited.

[**3**] The splendour of the capital of a great empire, its shine that extended on the labours going on there and on its inhabitants and seemed to augment their glory, could have easily seduced youth and astonish a free but obscure and poor citizen of a small republic. Brothers Bernoulli remained faithful to their promise and exerted as much efforts to have such a formidable rival beside themselves as ordinary men would have spent for preventing this to happen.

Euler's voyage began in an evil hour: he soon found out that Nikolaus Bernoulli already fell victim to the rigours of the climate. And Ekaterina I died the same day that he set foot on Russian soil. The deceased monarch, true to the intentions of her husband [the late Peter the Great] had just established the Academy but her death seemed at first to threaten it with an imminent disbandment.

Unlike Daniel Bernoulli who was supported by fame and respect, and far from his homeland, Euler decided to join the Russian navy.

[**4**] One of the Admirals of Peter the Great had already promised him a post, but, happily for geometry, the impending storm had dispelled, Euler obtained a professorship and in 1733 succeeded Daniel Bernoulli when that illustrious scientist returned to his fatherland. The same year he married his compatriot, Miss Gsell, a daughter of a painter whom Peter the Great had brought to Russia when returning from his first voyage [abroad].

From that moment he understood that, to borrow an expression from Bacon [from which one of them?], he had become hostage of his destiny and that the nation where he could have hoped to be established with his family will of necessity be his fatherland. Euler was born in a nation where all governments have been maintaining at least an appearance and language of republican constitutions; where, in spite of more real distinctions than those between the foremost slaves of a despot and his last subject, all the forms of equality were being carefully preserved; and where due respect for the law, if sanctified by antiquity and public opinion, extended to most insignificant cases.

And now he found himself moved to a state where the monarch exerted unbounded authority; where the most sacred law of absolutism, the one that regulates the succession to supreme dominion [to the throne] became uncertain and contemptible; and where the Chiefs, being the slaves of the Sovereign, despotically ruled the enslaved populace. And that happened at the time when that Empire was governed by an ambitious foreigner, defiant and cruel, and moaned under Biron's tyranny[1].

For the scientists who came to its bosom to find glory, fortune and the liberty of peacefully enjoying the fruits of research the Empire provided a threatening and instructive spectacle. We can perceive all that which Euler had to feel at heart while being entangled in chains which he was unable to sever. Perhaps this aspect of his life led to habitual persistence in research and became his sole support in a capital only inhabited by flunkeys or enemies of the Minister, the former humouring, the latter attempting to evade his suspicions.

[**5**] For Euler, that impression proved so vivid, that it still persisted when, in 1741, the year following Biron's downfall and a more moderate and humane government replacing his tyranny, he quit Petersburg so as to establish himself in Berlin where the King of Prussia had invited him to.

He was introduced to the Queen Mother, the princess who was pleased to speak with educated people and to receive them in a noble free and easy manner that reveals in princes a sense of personal greatness independent from their titles and being a trait common to members of that royal family. Nevertheless, the Queen Mother of Prussia was only able to hear monosyllables from Euler and reproached him for that timidity, that embarrassment which, as she believed, she did not inspire.

Why don't you want to speak with me, she asked him. *Madam*, he answered, *because I came from a country where people are hanged for talking*.

[**6**] The time has come to provide an account of Euler's immense labours, and I feel that it is impossible to describe the details, to make known that pile of discoveries, of new methods and penetrating points of view scattered over more than 30 separate contributions and about 700 memoirs of which about 200 having been submitted during his lifetime to the Petersburg Academy were destined to enrich gradually the set [of yearbooks] it was publishing.

However, a particular trait seems to distinguish him from the illustrious scholars who followed his routes and glorified themselves, but did not eclipse him. That singularity was that he had embraced mathematical sciences in their universality consecutively perfecting their various parts and enriching all of them by his important discoveries thus producing a useful revolution in the manner of dealing with them.

I therefore believe that, if a methodical table of the various branches of those sciences were compiled and the progress of each of them with successful changes made due to Euler's genius recorded, then I would have provided to the best of my abilities at least a more correct idea about that celebrated man. Owing to the combination of so much extraordinary qualities, he represents, so to say, a phenomenon in the history of science unparalleled until now.

For a long time algebra only remained a very restricted science. This manner of only considering the idea of magnitude as abstractly as possible for the intelligence to attain; the rigour with which that idea had been separated from everything that with some imagination could have provided a certain respite for the mind; and finally the extreme generality of the symbols used by that science, – these circumstances render algebra in some sense excessively alien to our nature, too remote from our everyday conceptions for the human mind to find easily pleasure and get accustomed to it.

The very progress of algebraic methods repels even those most suited for such meditations. However simple is the pursued object, they are compelled to forget it entirely and only to think about formulas. The path to follow is clearly seen, but both the point to be attained and the point of departure disappear from the eyes of the geometer. And he has to be courageous to dare for a long time lose firm footing and expose himself by taking the new science for granted.

And so, when casting a glance on the works of the great geometers of the last century, even of those to whom algebra owes its most important discoveries, it is seen how poorly they managed to master the very arsenal that they perfected to such an extent[2]. It is therefore impossible to fail to perceive the revolution that rendered algebraic analysis a clear, universal, generally applicable and even simple method as the result of Euler's contribution.

After providing many new theories about the form of the roots of algebraic equations, about the general solutions of these equations, and eliminations [of the unknowns] and offering pertinent ingenious or deep views, Euler generalized his researches on the calculus of transcendental magnitudes.

[7] Leibniz and both Bernoullis [Jacob and Johann] share the glory of introducing exponential and logarithmic functions into algebraic analysis and Cotes provided the means for representing the roots of certain algebraic equations by sines or cosines.

A successful application of these discoveries led Euler to note the singular relations between exponential and logarithmic magnitudes and the transcendental functions generated by the circle [circular functions] and then to discover methods for leading to the disappearance of the imaginary terms which entangle calculations from the solutions of problems.

These terms are annihilated and Euler was thus able to express the formulas simpler and more conveniently and to introduce an entirely new form of that part of the analysis which is applicable to problems in astronomy and physics. All geometers adopted it, it became generally employed and revolutionized that part of the calculus almost to the same extent as the discovery of algorithms transformed the ordinary calculus.

And so, during certain stages, or after grand efforts had been made, mathematical sciences seem to have exhausted all the possibilities of the human mind by attaining a certain place in their progress. But then, suddenly, a new method of calculations is introduced to those sciences and change their face. Soon they are rapidly enriched by solving a great number of important problems with which geometers had not dared occupy themselves having been frightened off by difficulties and, so to say, by the physical impossibility of carrying through their calculations to real results.

Justice will perhaps demand from those who were able to introduce the new methods and render them useful to give over a share of their glory to all those who successfully applied them, but at least those latter ought to acknowledge such rights [of the former] which they cannot contest without being ungrateful.

[**8**] Euler engaged in analysing series during almost all periods of his life; and this subject indeed forms one of the parts of his writings most glittering with that subtlety, that insight and variety of means and possibilities which are characteristic of him.

Geometers seemed to have almost forgotten about continued fractions invented by Lord Brouncker, but Euler perfected their theory, multiplied their applications and provided an idea of all their importance. His investigations of infinite products, almost entirely new, ensured the necessary means for solving a large number of useful or curious problems. It is especially the opening up of possibilities of imagining and applying the new forms of series [?] not only as approximations with which one is so often compelled to be content, but also for discovering absolute and rigorous truths that made it possible for Euler to extend that branch of analysis, so broad today but restricted before him to a small number of methods and applications.

The integral calculus, that most fruitful tool that mankind had at its disposal for discoveries changed its face after Euler's work. He perfected, extended, and simplified all the methods applied or proposed previously; to him is due the general solution of linear [differential] equations [and] the first justification of those [?] approximate formulas, so diverse and useful.

[**9**] A pile of particular methods founded on various principles are scattered among his contributions and united in his treatise on integral calculus [1768/342; 1769/366; 1770/385]. Indeed, we see there a successful use of substitutions or a return to a known method resulting in reducing equations that seemed unyielding to such means to first differentials of equations of higher orders. By considering the forms of the integrals he deduced the conditions which differential equations can satisfy or by examining the kind of factors for compiling total differentials he formed general classes of integrable differential equations.

Sometimes a noted particular property of an equation provided him a means for separating the indefinite magnitudes which should have seemingly remained confounded. In other cases, an equation in which these magnitudes were separated, did not yield to known methods [of solution] but their unification led to the calculation of its integral.

At first glance the choice and the lucky results of these methods could have been thought to be in some ways caused by chance. But their success, so often and surely achieved, compels to acknowledge another cause, and it is sometimes possible to follow the clue that guided Euler's genius. When, for example, studying the forms of substitutions applied by him, one often discovers what exactly allowed him to foresee their success. And when considering the form of the factors in a certain equation of the second order which he proposed in one of his most elegant methods, it will be seen that he selected one of those particularly suitable for the equation of that order.

Actually, such a sequence of ideas that guided an analyst is a particular instinct of sorts difficult to describe rather than a method susceptible of being developed by him. It is often better not to study the history of his ideas, not to become exposed to the suspicion of concocting an ingenious novel and being wise after the event.

Euler noted that differential equations can have particular solutions not included in their general solution; Clairaut made the same remark, but Euler later indicated the pertinent cause and was the first to occupy himself with the appropriate theory since then perfected by many famous geometers including Lagrange whose memoir leaves nothing more to desire concerning either the nature of these integrals or their application for solving various problems.

[**10**] We will now mention another part of the calculus almost entirely due to Euler, that in which particular integrals for certain values of the unknowns included in the equations are looked for. This theory all the more important since general solutions are often absolutely unyielding to our investigations and in addition since the knowledge of the pertinent particular solutions can help when an approximate value of the integral is not sufficient for our purpose.

Actually, at least in some sense the exact value is known, and, together with the knowledge of the approximate value of the total integral it ought to be sufficient almost for each goal of the analysis. No one applied more comprehensively and successfully the methods providing ever more exact values of a magnitude determined by differential equations by issuing from a certain initial value.

And Euler likewise occupied himself by providing a direct method of immediately deducing from the equation itself a value so near the true one so that the higher powers of the difference between those values could be neglected. Without that innovation the methods of approximation used by the geometers could not have been extended on equations for which the observations or particular considerations did not furnish that initial value presumed to be known when applying those methods.

[**11**] What is said above is sufficient for showing to what extent Euler had deepened [the study] of the nature of differential equations, the sources of difficulties opposing their integration and the means for evading or overcoming them. His great work on this subject is not only a precious collection of new and general methods, it is also a fruitful source of discoveries which anyone endowed with some talent cannot

scan without finding out unsophisticated riches. It can be said about the mentioned part or many other parts of Euler's works, that it includes methods which will be applied for a long time after him for solving important and difficult problems and that his contributions will yet produce more than one discovery and establish more than one reputation.

[**12]** The calculus of finite differences was almost unknown; Taylor's work was an obscure but very penetrating exception. Euler transformed it into an important branch of integral calculus, provided it with a simple and convenient notation and successfully applied it in the theory of sequences [series] for investigating their sums or the expressions of their general terms, in calculating the roots of some equations and for easily determining approximate values of infinite products or sums of certain numbers.

The real discovery of the calculus of partial differences ought to be attributed to D'Alembert since to him is due the knowledge of the general form of their integrals [of the integrals of the equations in these differences]. But in his first works on this subject we rather see the result of his calculations than those calculations themselves.

Euler provided the notation and was able by means of a deep theory to render that calculus suitable in some sense for solving a large number of those equations; to distinguish the forms of their integrals for the various degrees and numbers of variables; to reduce some forms of equations to ordinary integrations; and to furnish a means for transferring by successful substitutions to these forms those equations that were remote from them.

In a word, Euler discovered many singular properties in the essence of equations in partial differences which render the general theory so difficult and exciting, the two qualities almost inseparable in geometry where the degree of difficulty of a problem often measures its interest and the honour attached to a discovery.

[**13]** For scholars to whom the pleasure of discovery a truth is always proportional to the efforts made **t**he influence of a new truth for the science itself or for some important application is the only advantage which could be compared with the merit of a difficulty surmounted.

Euler did not neglect any part of the analysis; he proved some of Fermat's theorems on indeterminate analysis [on analysis of indeterminate equations] and found many other propositions, not less curious or less difficult to discover. The moves of the knight around the chessboard and various other problems of situation [of the future topology] also excited his imagination and exerted his genius.

He alternated most important investigations with these amusements, often more difficult but almost useless either for the progress itself of science or applications tested until now. Euler was wise enough not to feel himself uncomfortable for being engaged for a long time in these purely curious researches, but at the same time too knowledgeable not to perceive that their uselessness was only fleeting and that the only means for changing the situation was to try to deepen and generalize them.

[14] The application of algebra to geometry had occupied almost all the geometers of the last century beginning with Descartes but Euler proved that it was not at all exhausted. We owe him new investigations about the number of points determining a curve of a given order, about the number of intersections of curves of different orders. We are also obliged to him for deducing a general and very remarkable for its exceptional simplicity equation of curves whose [evolute] and [evolutes] of the second, third, and actually of any order are similar to the generating curve [to the involute].

The general theory of curved surfaces was little known and Euler was the first to develop it in an elementary contribution. He adduced the theory of osculating rays […][3]

Euler also provided a method for determining developable surfaces and a theory of geographical [cartographical] projections of a sphere. The two pertinent memoirs include an application of the calculus of partial differences to geometric problems that can be extended to many other interesting problems the first idea about which was due to him.

These investigations of algebraically rectifiable curves on a sphere and of curved surfaces whose parts corresponding to parts of a given plane are equal to each other [are of equal areas?] led to a new kind of analysis. He named it *infinitesimally indeterminate* since, similar to the ordinary indeterminate analysis, magnitudes remaining arbitrary are subjected to certain conditions, and, like the latter, it can sometimes assist in perfecting algebra. Euler considered the new analysis as a science that sometime will be useful for advancing the integral calculus. Actually, these particular subjects do not touch on the methodical structure of mathematical sciences and neither do they bear on possible applications, but they should not be only regarded as a means for exercising the power or importing lustre to the genius of geometers.

In sciences, some isolated parts are at first almost always cultivated separately, but, as the consecutive discoveries are being multiplied, the connections linking these parts become noticeable and most often the grand discoveries heralding an epoch in the history of the human mind are due to the enlightenment ensured by those connections.

[15] The problem of determining the curves or surfaces on which certain indefinite functions are greater or less than all the other ones, exerted the efforts of the most illustrious geometers of the last century. The solutions of the problems of the solid of least resistance, of the curve of steepest descent, of the curve providing a maximal area with a given perimeter became widely known in Europe, but the general method of solving these problems had been concealed. This especially concerned Jakob Bernoulli who discovered the solution of the isoperimetric problem and thus overtook his brother. The latter, in spite of the chefs-d'oeuvres he gave birth to since then, was unable to forget that.

It was necessary to develop that method, to reduce it to general formulas, and this Euler had done in 1744, in one of the best monuments to his genius. For deriving these formulas he had to consider curves [1744/65] but 15 years later a young geometer, Lagrange, who from his first contributions showed himself as a worthy successor of Euler, solved the same problem by a purely analytical method.

Euler was the fist to admire that new effort in the art of calculation, himself described the new method presenting its principles and developing it with that clarity and elegance which glitter in all his works. Never did a genius receive and show a more pleasant respect, never did Euler stoop to petty passions which a loss of some glory makes ordinary people so active and violent.

[**16]** We conclude here our exposition of Euler's work in pure analysis and note that it would be unfair to restrict his influence on the progress of mathematics to the innumerable discoveries filling his contributions. Those links which he revealed between all parts of a science so vast; those general views which he often did not even indicate but do not at all escape an attentive mind; those paths to which he was content to find an entry and the first obstacles along which he smoothed, – all that had also enriched the sciences as much and will enjoy posterity perhaps forgetting the man who provided it.

[17] The treatise on mechanics [1736/15 and 16] is the first grand work in which analysis is applied to the science of movement. The number of subjects either new or presented there in a new manner would have amazed geometers had not Euler already made public the greatest part of them separately. In a large number of writings on the same science he always remained faithful to analysis, and its successful application earned the method [application of analysis] the preference to all the others which it finally obtained.

The solution of the problem in which the motion of a body hurled into space and attracted to two fixed points is studied became famous because of Euler's skill of foreseeing so well the form of the substitution. He thus came to reduce to quadratures the equations which could have been regarded insoluble due to their complexity and form.

He applied analysis to study the movement of a solid body of any given form and it led him to the elegant theorem already provided by Segner, that *a body of any form can revolve freely and uniformly about three mutually perpendicular axes*; to the knowledge of many singular properties of those principal axes; and finally to general equations of motion of a body of any figure and any law of accelerating forces acting on its elements and some of its parts.

He subjected the problem of vibrating strings and all those that belong to the theory of sound or the laws of the oscillation of the air to analysis by new methods with which he had enriched the calculus of partial differences.

[18] The theory of movement of fluids based on the same calculus surprised readers by the clarity which he extended on such delicate subjects and the facility with which he was able to apply methods founded on such a deep analysis.

All problems of physical astronomy dealt with in this century were resolved by Euler's particular analytical methods. His calculation of the perturbations of the terrestrial orbit, and especially his lunar theory [1770/399; 1772/418] are specimens of simplicity and precision with which it is possible to apply those methods. And, when reading the work on that latter subject, one is not less surprised to see the extent to which a man of great genius, animated by a desire to leave nothing undone in an important subject, can try his patience and manifest his persistence.

[19] Astronomy had only applied geometrical methods but Euler imagined all that could be expected with the assistance of analysis, and he proved his point by examples which many celebrated scientists have imitated since then and which can sometime provide a new form for that science.

He embraced the naval science in a grand work [1749/110] based on scientific analysis where the most difficult problems were subjected to general and fruitful methods which he knew so well how to create and apply. A long time afterwards he [1773/426] published an elementary and short exposition of that treatise inserting in the most simple form all that can be useful in practice and should have been known by those who devoted themselves to the sea.

Although the author only intended his contribution for schools in the Russian Empire, it earned him a financial reward from the King [of France] who decided that works useful for all people were entitled to be recognized by all Sovereigns and wished to show that even at European extremities such rare talents cannot avoid either their attention or their good deeds. Euler appreciated that token of esteem by a mighty King made more valuable in his eyes by the hand of Turgot, the intermediary. He was the minister respected in Europe for his enlightenment as well as virtues, born for directing an opinion rather than obeying it, whose voice always dictated by truth and never by a desire to attract to himself public approval could have flattered a sage too accustomed to glory for lending an ear to rumours of his celebrity as well.

For men of superior genius an extreme simplicity of character can go with the qualities of the mind and thus apparently all the more testify to mastery or subtlety. Therefore, Euler, although simplicity never left him, was able to distinguish acutely, but, to say the truth, always leniently, respect showed by an enlightened admiration from irrepressible vanity directed to great men in order to ensure at least the merit of enthusiasm.

[20] Euler's work on dioptrics is based on less deep analysis and we may venture to be grateful to him for that as though for a sacrifice of sorts. In one and the same medium the various rays that form the solar ray experience different refraction. When being separated from neighbouring rays, each will become isolated or at least less intermingled with others and provide a sensation of the colour proper to it.

For each ray, this refrangibility varies in different media and, moreover, according to a law which is not the same as for the mean refraction in the appropriate medium. This fact makes it possible to believe that two combined differing prisms of different substance can deflect a ray from its path without decomposing it, or rather that it is possible to shift an elementary ray to a parallel route by triple refraction. The verity of this conjecture for optical instruments can depend on the destruction of the rainbow that colours the objects passing through the lenses.

The following metaphysical idea convinced Euler in the possibility of success: *The eye is only composed of various fluids with the intention of destroying the effects of the aberration of refrangibility* [of chromatic aberration]. Since it was thus only necessary to imitate nature, he proposed the means in accordance with a theory he himself provided. His first essays persuaded physicists to occupy themselves with a subject that seemed to be neglected, but the experience they gained did not at all conform to Euler's theory although corroborated his opinion about perfecting optical instruments.

Being thus acquainted by them with the laws of dispersion in various media, he abandoned his first ideas, subjected the results of their experience to calculation and enriched dioptrics by simple, convenient, general analytical formulas applicable to all possible instruments [1769/367; 1770/386; 1771/404].

Euler also published several essays on the general theory of light in which he attempted to coordinate the pertinent phenomena with the laws of oscillations of fluids since, as it seemed to him, the hypothesis of emission of rays along straight lines was fraught with insurmountable difficulties. The theories of magnetism, of the propagation of fire, of cohesion and friction of bodies became for him an occasion for scientific calculations regrettably based on hypotheses rather than experience.

[21] The calculus of probability and political arithmetic also became objects of his tireless work. We will only mention his investigations of mortality tables […], his method of choosing a mean of observations, his calculations for establishing a loan bank […][4].

[22] It is seen in the Eloge on Daniel Bernoulli that he only shared with Euler the glory of being awarded 13 prizes of the [Paris] Academy of Sciences[5]. They both often worked on the same subjects and each had to share the honour of overcoming his rival which nevertheless had never checked the reciprocal testimonials of esteem or dampened the feelings of friendship.

When examining the subjects for which one or the other got the prize, it is seen that the success especially depended on the essence of their talent. If the problem demanded skill in the manner of considering it, a successful application of experience or ingenious and new physical views, the advantage was on Daniel Bernoulli's side. If, however, it was only necessary to surmount great difficulties of calculation and create new methods of analysis, it was Euler who carried off the prize. And for those brave enough to compare them with each other, the judgement should have been pronounced in accordance with the kind of the mind, the manner of applying genius rather than with the personality.

We would have provided not more than a very imperfect idea of Euler's fruitfulness without mentioning in our weak sketch of his work that there was only a small number of important subjects to which he had not returned and rewritten even many times his first contribution. Sometimes he substituted a direct analytical method instead of an indirect method; sometimes he generalized his first solution to include a previously missed case. Almost always he added new examples which he knew how to choose with a singular skill among those providing a useful application or admitted of some curious remark. The intention itself of presenting his work in a more methodical form, to throw more light, make it simpler would have sufficed for the immensity of his works. Never was geometry described in such a way, and never had anyone achieved such a degree of perfection in his works. When he published a memoir on a new subject, he described his route in a simple manner noting the difficulties or roundabout ways.

Having thoroughly described for his readers the course of his thoughts in his first sketches, he then showed how he managed to find a simpler route. It is seen that he preferred to instruct his followers to the petty satisfaction of surprising them and that he believed that he would not have done enough for science without adding an unsophisticated exposition of the ideas that directed him to the new truths with which he enriched it.

[23] This method of embracing all the branches of mathematics, of invariably presenting, so to say, all the questions and theories to the mind, was for Euler a source of discoveries, accessible to him only but shut off for almost all the others. Thus, in the sequence of his works a special method of integrating equations by differentiating them became known to him; sometimes a remark about an issue in analysis or mechanics directed him to a solution of a very complicated differential equation that evaded direct methods; or he at once solved a problem apparently very difficult by a very simple method.

Or, to the contrary, a solution of a seemingly elementary problem met with difficulties that could only be surmounted by putting in great efforts. Another time a combination of special numbers or a new form of a series presented sharp questions or directed him to unforeseen truths. Euler diligently indicated that he owed the discoveries of that kind to chance. This does not diminish their merit since it is easily seen that [happy] chance can only come to a man who possesses vast knowledge and most rare insight. And shall not we commend such candour even if it costs a loss of some glory? Men of great genius rarely resort to those petty tricks of self-respect which only belittle them in the eyes of enlightened judges and exalt them in the opinion of the crowd. Either a talented man feels that he will never become grander without displaying himself in a natural way or that opinion does not exert such power on him as on other people.

When acquainting ourselves with the life of a great man, either the conviction in imperfection rooted in human feebleness or the low degree of justice that we are capable of do not even allow us to recognise a superiority in other mortals in which case nothing will console us; or finally the idea of another person being perfect wounds or humiliates us even more than the idea of grandeur itself. It seems that we need to reveal a weak link and look for some defect that can elevate us in our own eyes and we are unwittingly led to doubt the sincerity of an author if he does not show us such a link, does not at all draw aside the disagreeable veil concealing his defects.

[24] It seems that Euler sometimes only occupied himself for the pleasure of calculation and regarded a certain examined point of mechanics or physics as a sole occasion for exerting his genius and abandoning himself to his dominant passion. And scientists had therefore reproached him for having sometimes wasted his analysis on physical hypotheses or even metaphysical principles without sufficiently examining either their likelihood or reliability. He was also reproached for excessively basing himself on the possibilities of calculation and neglecting resources that could have examined those very problems which he intended to solve.

We acknowledge that the first reproach is not unfounded; we agree that Euler the metaphysician and even the physicist is not as grand as the geometer. It should be undoubtedly regretted that many parts of his contributions, for example of those pertaining to the naval science or artillery, are almost only useful for advancing the science of calculation[6]. However, we believe that the second reproach merits much less. All the works of Euler testify that he occupied himself with enriching analysis, with extending and multiplying its applications, and it is also seen that at the same time when employing his unique tool, he wished to make it universal. The natural advance of mathematical sciences will bring about that revolution, but he had seen how, owing to his genius, it was developing as though before his eyes, being the fruit of his efforts and discoveries. He apparently abused analysis and exhausted all the hardly noticeable methods for solving a problem whereas some considerations alien to analysis could have provided the answer simply and easily. Nevertheless, he often only sought to prove the power and possibilities of his skill, and he should be excused if sometimes, being seemingly occupied by another science, it still was the propagation of analysis that his works were devoted to. The revolution which was the fruit of those efforts provides one of the first rights to be generally recognised and one of the very best foundations of glory.

[25] I did not feel myself obliged to interrupt the details of Euler's work by relating the very simple and rare events of his life. He established himself in Berlin in 1741 and stayed there until 1766. The Princess of Anhalt-Dessau, a niece of the King of Prussia, wished to receive from him some lessons in physics. They are now published [1768/343 and 344; 1772/417] and are valuable because of the special clarity with which he described the most important facts of mechanics, physical astronomy, optics and the theory of sound, and also because of the ingenious views, less philosophical but more scientific than those that can be perceived in Fontenelle [1686] with his system of vortices.

Euler's name is so grand in sciences, and the sublime idea that forms from his contributions destined to develop the most difficult and abstract in analysis is indicated in those *Lettres* so simply, so easy in a specially charming manner, so that those who have not studied mathematics are surprised and perhaps flattered, and therefore grateful by being able to understand Euler's writing. And the elementary details of sciences acquire a grandeur of sorts by being moved nearer to the glory and genius of the illustrious man who traced them.

The King of Prussia employed Euler for financial calculations, planning the water supply in Sanssouci, and examining [the expediency of] many navigable channels. That monarch was born to believe that great talents and deep knowledge will never become superfluous or dangerous and that the happiness of being useful is a benefit which the nature did not reserve for the ignorant and mediocre.

In 1750 Euler travelled to Frankfurt [am Main ?] to meet his mother, already a widow, and take her to Berlin, and until 1761 he was happy to retain her. For eleven years she enjoyed her son's glory as a mother's heart can and was perhaps even happier by his tender and thoughtful care with its value augmented by that glory.

[26] It was during his stay in Berlin that Euler, being obliged by gratitude to Maupertuis, considered it his duty to defend the principle of least action on which the President of the Académie de Prusse based his expectations of general recognition. The means chosen by Euler [1751/199; 1753/186] could have been only applied by him. Indeed, he set forth to solve some of the most difficult problems of mechanics by issuing from that principle. Thus, in times immemorial God saw fit to create weapons for his favoured warriors impenetrable to the blows of their enemies[7].

We would have wished that Euler's gratitude were restricted to that protection of Maupertuis, so noble and worthy of him, but we cannot conceal that his answers to König were too tough. And we ought to reckon regrettably the great man among the enemies of that miserable and persecuted scientist. Happily though Euler's entire life is free from more serious suspicions; without considering his simplicity and invariably manifested indifference to glory we could have believed that the pleasantries (later consigned by Voltaire himself to just oblivion) of a famous partisan of König had altered the character of the sage and peaceful geometer. The only fault that may be attributed to him is just an excess of gratitude. And only once during his lifetime because of that valid sentiment he acted unjustly.

[27] In 1760 the Russians penetrated the mark of Brandenburg and plundered Euler's estate situated near Charlottenburg. However, general Tottleben[8] had not come to make war with science. Once being acquainted with the loss incurred by Euler, he was quick to indemnify him much in excess of the real damage. He also informed the Empress Elizaveta [Petrovna] about that unintentional lack of respect and she presented Euler four thousand florins above the much more than sufficient indemnity.

This deed is completely unknown in Europe and we enthusiastically cite a few similar acts which the ancients have mentioned. The difference between our judgements, does it not prove the happy progress of humankind which several authors yet obstinately deny, apparently to avoid being accused of involvement[9].

[28] The Russian government never treated Euler as a foreigner and he always partly received his salary in spite of being absent, and in 1766 the Empress [Ekaterina II] invited him to return to Petersburg and he agreed. In 1735 the efforts of an astronomical calculation for which other academicians demanded many months but which he completed in a few days caused an illness with loss of one eye.

He had reason to fear total blindness if experiencing once more the harmful influence of an unfavourable climate, but the interests of his children prevailed[10]. And, when recalling that research had been Euler's exclusive passion we will undoubtedly decide that only a few examples of paternal love can better prove that it is the most powerful and delicate of our affections.

In a few years Euler did experience the foreseen misfortune although luckily for himself and science he still retained the possibility of distinguishing large characters written on a blackboard. His sons and students copied his calculations, wrote down his later memoirs from dictation. And, judging by their number, and often by the talent acquired anew, we may believe that the still more complete absence of any distractions and the new energy that the ensued concentration thus brought about to all his faculties had rather benefited him and compensated his weakened eyesight and did not allow him to lose either the possibility or means of working.

[29] However, by the nature of his genius and habits of life he was even involuntary endowed by unusual resources. When examining those enormous analytical formulas, so rare previously and so frequent in his memoirs, whose combination and elaboration ensue (dont la combination & le développement réunissent) such simplicity and elegance, whose form all by itself is pleasant to eye and spirit, we perceive that they were not only created by calculation written down on paper, but that they were entirely produced in the mind by imagination equally powerful and active.

In analysis, there exist usual and almost daily applied formulas whose number Euler had essentially multiplied. He invariably had them at the back of his mind, knew them by heart, cited them in conversation. D'Alembert, when seeing Euler in Berlin, was astonished by the power of his memory that implied as much clarity as vigour of mind. Finally, Euler's ability of mental calculation was brought to a hardly believable degree of perfection had not the history of his work accustomed us to wonders.

It is known that to exercise his grandson in extracting roots he compiled a table of the first six powers of all the [natural] numbers from 1 to 100 and kept it exactly in memory. Then, two of his students calculated the [sum of the first] 17 terms of a very complicated convergent series. Their results, although arrived at on paper, differed by a unit in the 50th digit. They entrusted that dispute to their mentor who mentally calculated the sum anew and his result occurred to be correct[11].

Since losing his eyesight Euler could only amuse himself by producing artificial magnets and giving lessons in mathematics to one of his grandsons who seemed to be favourably disposed to that science.

[30] He still went sometimes to the Academy, mostly when circumstances became difficult if believing that his presence might be useful for maintaining the freedom [of research]. It is known how a perpetual President, appointed by the Court, can disturb the tranquillity of an academy and all that should be feared if he is not selected from the class of scientists[12]. Euler's reputation depended on the voice of his colleagues, but even that did not stop him.

Those, only occupied by their peaceful work, only able to speak the language of science, being so alien, isolated, far from their homelands, and wholly dependent on the government, – how could they demand it to defend them against the Chief appointed by that very government?

There exists, however, such a degree of glory that delivers from fear. This will happen if all Europe rises against personally injuring a great man, and therefore without running a risk he can set off his authority and popularity against injustice and give his voice that it was impossible to drown out in favour of sciences. Euler, simplicity itself, modest as much as possible, felt his power and successfully applied it more than once.

[31] In 1771 Petersburg endured a terrible fire, and the flames engulfed Euler's house. Pierre Grimm, an inhabitant of Basel, whose name undoubtedly merits to be remembered, understood the danger threatening his illustrious compatriot, blind and sick, and rushed through the flames, shouldered him and saved in peril at his own life. The library and the furniture were burnt in the fire but the fervent efforts of Count Orlov rescued the manuscripts.

Such consideration shown in the midst of commotion and horrors of that awesome disaster represents respect of a most sincere and flattering kind that public authorities had never before rendered to a genius of science. Euler's house was the Empress' good deed, and [her] new charitable act soon retrieved the situation.

[32] Euler had 13 children by his first wife of which eight died very early. Three sons survived him but unfortunately both daughters died during the last year of his life. From 38 grandchildren 26 were still alive at the time of his death. His second wife was [Salome Abigail] Gsell, a consanguineous sister of his first wife whom he married in 1776. Following the example of his paternal home, Euler kept all the simplicity of morals. Before losing his eyesight, come evening, he gathered all his household, – grandchildren, domestics and students living with him, – for a communal prayer, and read out a chapter from the Bible. Sometimes he accompanied that lecture by an exhortation.

He was very religious. He provided a new proof of the existence of God[13] and spirituality of the soul with many theological schools adopting the latter. He scrupulously kept to strict Calvinism, the religion of his homeland, and, unlike most Protestant scientists, apparently did not allow himself either to adopt particular opinions or form his own system of religion.

Euler's erudition was very broad, especially in the history of mathematics. It was thought that he extended his curiosity up to studying astrological courses of action and rules and that he even applied it several times. Nevertheless, when in 1740 Euler was ordered to compile a horoscope for Tsarevich Ivan [VI, son of the Tsar, born the same year], he explained that that was the duty of Kraaff [Krafft or Kraft], the court astronomer.

It astonishes to see such credulity manifested at this time by the Russian Court; it was common to all European courts a century earlier, but their Asian counterparts have not yet cast off that yoke. It ought to be recognized that until this very day, apart from the general principles of morality, not a single truth has been generally acknowledged as long as many ridiculous and pernicious mistakes.

[33] Euler studied almost all branches of physics, anatomy, chemistry and botany but his superiority in mathematics did not allow him to attach even least importance to his knowledge of those other sciences which would have been sufficiently broad for someone more susceptible to the pettiness of self-importance and claiming universality of sorts.

The study of ancient literature and the scientific languages [Greek and Latin] was part of his education and his liking for those subjects lasted all through his life. Euler forgot nothing learned previously, but never had either time or desire to augment his initial knowledge; he did not read modern poets, but knew *Aeneid* by heart. Nevertheless, Euler had not been losing mathematics out of sight even when reciting Virgil. Everything was suitable for recalling to him that science, almost the only one in his thoughts; we even find among his writings a scientific memoir on a problem of mechanics about which he recounted that it was a verse from the *Aeneid* that had given him its first idea.

It is said about men of great talent that for them the pleasure of work is even sweeter than glory. If that maxim needs to be corroborated by examples, Euler's behaviour does not permit to doubt it anymore.

[34] Never in scientific discussions with famous geometers did he let fall even a single word that could have given cause to suspect that he cared about self-interest. He never referred to any of his discoveries, and if someone claimed something contained in his memoirs, he hastened to eliminate the unintentional injustice even without duly examining whether strict fairness demanded him to abandon absolutely his share.

If someone had noted his [imagined] error and reproached him groundlessly, he forgot about it. Otherwise, he corrected the mistake without even thinking that the merit of those boasting to have revealed his faults often only consisted in easily applying the methods which he himself had taught them and theories whose greatest difficulties he had previously smoothed over.

Mediocre people almost always try to enhance their reputation by severity proportional to the elevated idea that they wish to attribute to their judgement or genius, relentless to everything excelling them and do not even spare inferiority. It is thought that some concealed feeling convinces them that it is necessary to humiliate all the others.

On the contrary, Euler was impelled to glorify talents as soon as some successful essays surprised him, without awaiting public opinion to ask him about it. We see that he spent his time to remake and clarify his works and even to solve already solved problems which only earned him more elegance and method with the same passion and patience as when looking for a new discovery that will increase his fame. Moreover, had an ardent desire to achieve glory existed in his heart of hearts, the sincerity of his character would not have permitted him to conceal it.

[35] The glory, about which he thought so little, itself sought him out. The special fruitfulness of his genius astonished even those unable to appreciate his works. Although it only considered geometry, his reputation extended to those most alien to that science. For all Europe he was not only a grand geometer but a great man. In Russia, it is usual to confer military rank to those very remote from the armed forces thus paying tribute to the prejudiced opinion that only the military profession is noble, but at the same time indicating how false is that view. Some scientists obtained titles up to Major General, Euler however neither had, nor wished any. And indeed, what title would have honoured his name? Respect to the observance of the natural rights of men obliges us in a sense to provide this example of being wisely indifferent to the rattles of human vanity, so infantile but also so dangerous.

Most of the European (du Nord) monarchs who knew him personally displayed him tokens of their respect or rather veneration impossible to deny to a combination of such a simple virtue and such a vast talent. When the Royal Prince of Prussia travelled to Petersburg, he visited Euler and passed a few hours at the bed of that illustrious old man coupling hands with him and holding on his knees one of his grandsons whose early developed inclination to geometry rendered him a particular subject of paternal tenderness.

[36] All the presently living famous mathematicians are Euler's students. Everyone had developed by reading his works, obtained from them the formulas, the method he had applied, was guided and supported in his discoveries by Euler's genius. He owes this honour to the revolution he produced in the mathematical sciences by subjecting all of them to analysis; by the power of his works that permitted him to embrace the entire scope of those sciences; by the methodical order he was able to introduce into his grand contributions, the simplicity and elegance of his formulas, clarity of methods and demonstrations even more augmented by the multitude and choice of examples. Neither Newton nor even Descartes whose influence is so powerful earned that glory which Euler possesses to this day among geometers without sharing it with anyone.

[37] However, as a professor Euler prepared students who more particularly belong to him. Among them we mention his eldest son whom the Paris Académie des Sciences had chosen to replace him [as their foreign member] without fearing that such a honourable succession accorded to the name of Euler, as also was the case with Bernoulli, will become a dangerous example. His second son now devoted to the study of medicine had in his younger years won the prize of our Academy for [examining] the alterations in the mean movements of planets.

Then, Lexell whose premature death carried him off, away from science. And finally Fuss, the youngest of his students, companion of his last works. Daniel Bernoulli sent him from Basel to Euler and he proved worthy of Bernoulli's choice and Euler's lessons by his own works. After honouring his illustrious mentor [by an Eloge] at the Petersburg academy he has just married [one of] Euler's granddaughters.

Euler prepared eight out of the sixteen professors attached to the Petersburg academy. All of them are known by their works, crowned with the status of academicians and additionally glorified by the honour of being Euler's students.

[38] Euler had preserved all his faculties and apparently all his power; no change had announced that the sciences were threatened with his loss. 7 September 1783, after amusing himself by calculating on a blackboard the laws of the ascending movement of hot-air balloons[14] whose recent discovery had aroused the interest of all Europe, he had dinner with his family and Lexell and spoke about the planet discovered by [William] Herschel and the calculations that determined its orbit. Soon afterwards he called his grandson and joked with him, drank a few cups of tee, but then his pipe suddenly fell out of his hand and he quit calculating and living.

Thus ended the life of one of the grandest and most extraordinary man whom nature had ever produced; whose genius was equally capable of greatest efforts and most incessant work; who multiplied his contributions beyond any conceivable confines being nevertheless original in each of them; whose mind was always occupied and soul remained always calm[15]; and, finally, whose regrettably extremely rare destiny had been combining an almost cloudless happiness with glory never to be contested.

Even [?] in the nation where he lived his death was regarded as a public loss. The Petersburg academy marked it by holding a meeting of solemn mourning and sculpting at its own expense his marble bust to be placed in its conference hall. During Euler's lifetime the Academy had rendered him a more special honour. On an allegorical table Geometry rests on an engraved plate covered with calculations, – with formulas of his new lunar theory inscribed there by the Academy's order.

And so, a country that at the beginning of this century we considered yet barbaric, shows the most enlightened European nations how to honour the life of great men and revere their memory (mémoire récente). It offers those nations an example which will perhaps compel many of them to blush with shame since they could not have either foreseen or even imitated it.

Notes

1. This description is superficial. Peter the Great issued the edict on the succession in 1722, and it was confirmed in 1731 and again in 1761; the Emperor was to choose a member of the imperial family to succeed him. So how should we understand that "the most sacred law … became uncertain and contemptible"? See *Great Sov. Enc*., 3rd edition, vol. 3, 1970, articles *Biron* and *Bironovshchina*, and vol. 22, 1975, article *Romanovy* (English edition of that source, same volumes, 1973, p. 344 and 1979, p. 238). The original spelling of the family name Biron was Beron, and that is how Condorcet had written it. O. S.

2. After recalling the later development of mathematics the author's indignation seems mistaken although the work of the group of mathematicians collectively calling themselves Bourbaki did come under criticism. O. S.

3. Beginning with *osculating ray*, this subject (the curvature of a surface element) as described by Condorcet is difficult to understand. O. S.

4. Condorcet described this subject inadequately. The term *political arithmetic* was then being superseded by *statistics*; Euler never studied mortality tables; Condorcet interrupted his account by mentioning the treatment of observations (but saying practically nothing about it) and allegedly described Euler's memoir [1776/473]; actually however, the memoir of Fuss (1776). O. S.

5. The author mistakenly referred to himself [iv, § 12]: he mentioned ten prizes there. O. S.

6. How then can we understand his § 19 (financial reward from the King of France)? O. S.

7. I can only refer to Ephesians 6: 13 – 20: "Having put on the breastplate of righteousness … above all taking the shield of faith with which you can quench all the flaming darts of the evil one". Cf. also the *Book of Mormon* (1 Nephi 16: 29): "by small means the Lord can bring about great things".

Then, why did Condorcet mention König (below)? Concerning that scientist see Wolf (1859, pp. 147 – 182), Fellman (1973) and Speiser (2008, pp. 261 – 265) who is very critical about König and, as far as scientific work is concerned, called him "confused". O. S.

8. General Tottleben (1717 – 1773) was in command of some of the Russian troops during the Seven Years' War of 1756 – 1763. Charlottenburg is now a district of Berlin. O. S.

9. Much remains unexplained. The "similar acts" are not mentioned and, moreover, according to the text they should have been dissimilar. And what did Condorcet mean by "involvement"? O. S.

10. The author apparently thought that young men had better opportunities in Russia, cf. Fuss [i, § 61]. However, Fuss mentioned quite different causes of Euler's decision; see also my Foreword, § 1. O. S.

11. This is a fairy tale. Who needed calculations up to the 50th digit? And even Euler would have been unable to accomplish the feat mentioned, and in any case who and how could have been able to check those calculations? O. S.

12. The presidents of the Petersburg Academy are listed in the *Great Soc. Enc*., 3rd edition, vol. 1, article Academy of Sciences of the USSR, history (see the English edition of that source) and during 1746 – 1798 the president was indeed a foreigner. However, a later source (*Rossiiskaia* 1999) names Razumovsky for the same period. Anyway, presidents had been certainly appointed; that practice was stated even in the Academy's *Regulations* of 1803 signed by Aleksandr I, see its reprint, in Russian and French, in *Protokoly* (1897 – 1911/1911, vol. 4/1, p. 1145): "Le Président de l'Académie sera nommé par NOUS et choisi parmi les personnes des quatres premieres classes de l'Empire" (§ 24).

On these classes see *Table of Ranks* (*Great Sov. Enc*., English edition, vol. 25, 1980, pp. 270 – 271). O. S.

13. Such proofs continue to appear. However, here is what the late Pope Jean Paul II had to say about them on 10 July 1985 (Vikipedia, SFBay Catholic. Pope JP2 of God's existence): "To desire a scientific proof of God would be equivalent to lowering God to the level of beings of our world …" O. S.

14. During the last day of his life Euler perhaps did amuse himself by those calculations, but somewhat previously he considered the mentioned subject quite seriously, see his memoir [579]. O. S.

15. Almost cloudless happiness in spite of eight of his children dying very early (§ 32), to say nothing about his various troubles in Petersburg (during both periods of his stay there) and Berlin (clashes with Friedrich II, see my Foreword, and Lambert, see [ix, § 11])! O. S.

Joint Bibliography to Fuss [i] and Condorcet [ii]

A few general references not mentioned in the text are added

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III

Daniel Bernoulli

Autobiography

Daniel Bernoulli, *Gidrodinamika*.
Leningrad, USSR Academy of Sciences, 1959, pp. 427 – 433.
This book is a translation by V. S. Gochman
of the author's *Hydrodynamica*. Basel, 1738.

Daniel Bernoulli, Doctor of philosophy and medicine, ordinary and perpetual (perpetuus) professor of physics at Basel Academy.

Born 29 January 1700; his parents were Ivan [Johann I] Bernoulli, then adorning the chair of mathematics in Groningen and Dorothea Falkner, also descended from a renowned and very old Basel family. On his sixth year his parents returned him to [their] homeland; there, in 1713, having learned German and concluding a usual course of education at a gymnasium in Basel, he was certified worthy of being intended for reading [attending] university lectures.

So as to strengthen his practical knowledge of French, he was sent for a year to be educated by some French priest in Courtelary [in the Berner Jura]. He returned home in a year and in 1716 earned the title [the degree] of Master of Philosophy. He then joined those studying medicine and diligently heard the physicians then teaching in Basel. In 1718 moved to the Heidelberg Academy; there, under the guidance of Nebel, a most distinguished physician, thoroughly studied all branches of medicine. Next year he moved to the dwelling of muses in Strasbourg where most distinguished men were labouring in the field[s] of anatomy and surgery. In 1720, upon returning to the homeland and defending a dissertation on breathing [1721/1], was declared Lizentiat of medicine.

However, the example of the members of his family, namely of his father and elder brother Nikolaus, as well as the inclinations of his own soul attracted him to mathematical sciences and study of nature. He almost wholly surrendered himself to these pursuits although not entirely abandoning medicine. In 1723 he therefore arrived in Venice to strengthen himself in the practical knowledge of medicine under the guidance of a most celebrated physician, Michelotti. In 1724, a noble Venetian, a friend of the author, published in Venice at his own expense a few copies of a small and mostly polemic writing [1724/4].

After that our Bernoulli remained alien to any scientific wrangles whatsoever as his various later published writings testified with perfect clarity[1]. At that time the Paris Academy of Sciences arrived at a really commendable idea of proposing problems to scientists and promising very generous prizes [for solving them] which greatly fostered the development of geometry, astronomy and medicine. The first of these prizes having a practical goal invited scientists to express their opinion about the best manner of constructing sand clocks (clepsydra)[2] so that seafarers can conveniently make use of them. Bernoulli undertook to solve that problem and sent his investigation to the Academy of

Sciences. It won the prize, as decided by the judges, and was later published [1725/8].

Then, being attracted by the glory of the very first man, Giovanni Battista Morgagni, he came to Padua. However, he was taken ill with a most severe fever just after crossing its boundary. The zeal of the most distinguished physicians, Valisner, Morgagni and Cogross[3], overcame its power, but the illness had undermined his strength to such an extent that he barely rehabilitated after a six-month stay at the Padua muses and had been unable to manifest any success in studying the sciences.

During his stay in Italy, Bologna, however, conferred on him a new honourable degree. In that city, a glorious institution for developing science had then changed into a scientific academy and in 1724 Bernoulli was entered on a list of its members. Just the same, the idea of establishing a similar scientific society occurred in the famous [Most Serene] Republic of Genoi, and its leadership was offered to Bernoulli. He, however, held a more modest opinion about himself, and, not being sure of his powers, thought that that pursuit would have been too difficult.

He therefore remained at first irresolute, but, while he hesitated, divine Providence offered him another lot. Not long before, Peter the Great, the Emperor of the Russians, established the most celebrated Petersburg Academy of Sciences[4], and in 1725 Bernoulli was invited to Petersburg with his brother Nikolaus, who, after only eight months, quit his worldly existence and departed from life.

As to Daniel, after five years devoting his work to the Academy, owing to his poor health he decided to ask to be discharged and to return to his homeland. Because of some causes the Academy retained him which only additionally served to glorify him since his salary was augmented by half and accompanied by a grant of a lifelong pension, the title of honorary professor of the Academy
and received permission to stay in Petersburg as long as it will be useful to him.

These manifestations of the imperial favours retained him like fetters and he therefore remained in the Petersburg Academy for three years more, until the unstable state of his health did not finally compel him to think seriously about returning to his homeland. However, all the time that he stayed in that country, he made every effort to come nearer to the result to attain which had aimed he of glorious memory, the founder of the Academy[5].

The most benevolent patron (Princeps Optimus)[6] ordered the academicians to investigate and treat some subject useful for the human society and yet insufficiently studied. Bernoulli therefore compiled his *Hydrodynamica* and submitted it to the Academy before his departure. Later, he published it in an extended form in Strasbourg (1738/31). In addition, shortly before his departure, he found out that a double prize will be awarded to that person who solved the problem of the mutual inclinations of the planets proposed by the Paris Academy of Sciences in 1732, when no solution worthy of the prize was submitted. He decided that he also ought to participate in that competition. The glory of that prize seized the minds of scientists to such an extent that, as the Academy announced in a special declaration, it had to refuse with much regret the prize to very many of them. Admittance to the Academy was opened for three scientists[7] and certificates of merit [*accessit*] given them and the awarded prize was shared between two scientists. After the sealed envelopes with [the names concealed by] the mottos were opened, it occurred that the prize was awarded to the Bernoullis, father and son. Daniel Bernoulli therefore translated his piece written in Latin into French and the Academy published it in both languages (1735/24).

In 1733 he departed to his homeland accompanied by his younger brother Ivan [Johann II] who had been then travelling for the aim of educating himself (in itinere literalia versantem). After a dangerous sea journey they were blown to Danzig from where they set out to Holland, then to Paris.

At the same time the Academy of Daniel's home town conferred on him the title of public professor of anatomy and botany. At that time he entirely devoted himself to the science of nature and mathematical science and had been therefore only to a small extent occupying himself with studying medicine. Nevertheless, prompted by love for his fatherland, he accepted that invitation and returned to his previous occupations.

At the end of 1733 he arrived in Basel, accepted, according to the custom of [his] forefathers, the rights and honours of Doctor of Medicine and took up the duties entrusted him. However, after some time, when the chair of physics in the same home university became vacant, he changed, by the consent of both the highest officials of the republic and the high academic council, the chair of medicine for that chair, nearer to his heart.

At the same time, since he obtained the title of honorary professor of the Petersburg Academy accompanied by the appropriate salary, he made every effort to fulfil properly those duties by sending his writings, mostly on the subject of mechanics, to Petersburg. He continues this work up to the present day[8]. He preferred scientific pursuits at leisure and love for his fatherland to all other very beneficial and splendid assignments entrusted him by scientific institutions of all nations. Nevertheless, he invariably corresponded with foreign scientists among which in preference to others Maupertuis, Bouguer, Leonhard Euler[9], Clairaut and Johann-Albrecht Euler deserve to be mentioned.

Apart from those two pieces mentioned above for which the Paris Academy of Sciences awarded prizes to Bernoulli, he wrote many other prize winning contributions (1737/28; 1741/33; 1748/39; 1750a and 1750b/42a and 42b). Then, a double prize was awarded to contribution (1769/44) and two more pieces (1769/47 and 1771/48) also won prizes[10]. All these writings are published.

It ought to be mentioned that his compatriots from his home university variously honoured Bernoulli and conferred on him several titles. Thus, he was twice elected Rector of the Academy [of the university] (1744 and 1756) and in 1754 was appointed Dean of the Basel St. Peter *capitulum*[11] after being its member the previous year.

In addition to the scientific institution in Bologna which, as we mentioned above, he joined in 1724, and the Imperial Petersburg

Academy, whose honorary professor he was appointed in 1730, he was elected member of many other most celebrated foreign academies and scientific societies, and in particular, in 1747, of the Royal Berlin Academy of Sciences. In 1748, he was elected to the Royal Academy of Sciences in Paris which only has eight seats for foreign members; in 1750, to the London Academy [to the Royal Society]; in 1762, to the Bern Economic Society; in 1764, to the Zürich Economic Society; and in 1767, to the Kurpfälzigsche Akademie [Academy of the Electoral Palatinate].

It cannot be passed over in silence that Bernoulli was among those seven foreign scientists whom the present most gracious and most powerful Empress of the Russians granted a copy, inscribed with his name, of the famous gold medal recently struck in commemoration of the glorious peace made with the Turks [in 1774]. Impatiently and with deepest gratitude he is awaiting that most valuable token of his happiness which will arrive in the next few days. [The autobiography suddenly comes to an end here.]

Notes

1. Bernoulli did not mention his wrangles at the meetings of the Petersburg Academy. O. S.

2. At least initially *clepsydra* meant a water clock. O. S.

3. The spelling of the first and third names can be wrong. O. S.

4. It was his widow, the Empress Ekaterina I, who officially established the Academy. O. S.

5. This is difficult to understand. O. S.

6. An anonymous note apparently inserted by Gokhman or Smirnov explains that that patron was Biron. It is known however, that he introduced an extremely reactionary regime in Russia, see *Great Sov. Enc.*, English translation of its third edition, vol. 3, 1973, p. 344, articles *Biron* and *Bironovshchina*. O. S.

7. This is also too difficult to understand. O. S.

8. As seen in the *Verzeichnis* of his publications [vii], Bernoulli did not submit anything to the Petersburg Academy for more than 20 years. O. S.

9. As noted by Michajlov [vi, § 2], Bernoulli's correspondence with Leonhard Euler had been interrupted for twelve years. O. S.

10. Bernoulli mentioned three prize winning publications on the theory of magnets whereas the list of his publications includes only one (1748/41) joint piece (by Daniel and Johann II Bernoulli). See, however, [iv, Note 24]. O. S.

11. A capitulum is a college of ecclesiastics and their members are called canons. O. S.

Smirnov, V. I. (1959), Daniel Bernoulli, 1700 – 1782. In Bernoulli (1738/1959, pp. 433 – 501). In Russian.

IV

M. J. A. N. Condorcet

Eulogy on [Daniel] Bernoulli

Éloge de M. Bernoulli
Hist. Acad. Roy. Sci. Paris pour 1782, 1785, pp. 82 – 107
M. J. A. N. Condorcet, *Oeuvres*, t. 2. Paris, 1847, pp. 545 – 580

German translation by Daniel II Bernoulli:
Lobrede auf Herrn Daniel Bernoulli. Basel, 1787.

Foreword by Daniel II Bernoulli

An eulogy ought to meet two main demands: reliability and the use of skilful and sensitive expressions. And it is seldom that an author of an eulogy is fortunate enough to combine these two qualities and thus to achieve the aim of perfection. A few years ago, on the memorial day a year after the death of the late Daniel Bernoulli, I read a scientific address in Latin to an impressive meeting and thus fulfilled my duty of most sincere deep respect, love and gratitude towards my praiseworthy uncle and benefactor. Later, as demanded, I gave over the text of my address for publication (1783).

I compliment myself for managing to meet in full the first of the two demands, but I never claimed the glory of satisfying the other one. In spite of this advantage anyone should have followed the *Eulogy* that the learned and eloquent Marquis de Condorcet, being the perpetual secretary of the Royal French Academy of Sciences, read out at its public meeting. On the other hand, it seems that the author of that *Eulogy* had picked up the various described anecdotes and circumstances from perhaps not quite reliable sources without examining their genuineness[1].

Although the second main demand ought to be held much less important than the first one that concerns matter, because it only has to do with the form of the eulogy, I have nevertheless resolved to acquaint better my compatriots and the entire German public with the merits of our immortal Daniel Bernoulli as a counterpart to the German *Eulogy* on our great compatriot Leonhard Euler, friend of Bernoulli, and an equally enthusiastic fellow citizen, recently published by Fuss in Petersburg.

Rather than undertaking the unpleasant (at least for me) task of translating my own piece, I have chosen the French E*ulogy* as that counterpart and every now and then commented on both the noticeable mistakes and on places where I thought it advisable. However, since I have never executed the certainly not easy work of a translator, and since, in addition, a translation always loses much as compared with the original text, I do not at all imagine to have come near to the latter’s beauty.

I only wish that my work here presented would not be quite unworthy of Condorcet's *Eulogy* and that my diligence will to some extent compensate for my lack of experience! Then I will consider myself lucky enough to have turned the attention to the merits of my unforgettable uncle of many of my compatriots not yet sufficiently knowing about them, and to have offered those who already were his admirers not quite a trivial present.

Basel, 1 June 1787

[1] Daniel Bernoulli, ordinary professor of physics and extraordinary professor of medicine at Basel university, foreign member of the Royal Academy of Sciences in Paris, of the Royal Society in London, Institute of Bologna, Academies in Petersburg, Berlin, Turin and Mannheim and of various other scientific societies, was born on 9 Febr. 1700 in Groningen. His father was Johann Bernoulli, then professor of mathematics at the university of that city, and his mother, Dorothea Falkner, from one of the most ancient and most illustrious families in Basel.

Since he was a son and nephew of two famous mathematicians whom the voice of their contemporaries placed alongside Newton and Leibniz, it could be thought that the young Daniel Bernoulli, mathematically educated by his father from his childhood[2], becomes a geometer for following in some sense the calling of his family, and that nature has fortunately assisted that which his birth had prepared by chance.

Nevertheless, at first Daniel was destined for commerce. From his youth, however, his eyes were accustomed to the sparkle of glory and it was impossible to urge him to stoop to fortune, so then he was obliged to take to studying medicine, an occupation at least more agreeable to his taste and genius. In truth, a few lessons in mathematics were not neglected to be given him. His father, Johann Bernoulli, regarded mathematical sciences as the foundation of all the others, as a tool useful for all professions of life. However, the manner of teaching it by demanding too much from his pupils would have repelled any child not born for them.

To test the capabilities of his son, the father once put him a simple problem. Young Daniel took off to his room, examined and solved it, returned thrilled with joy to report to his father and get the expected approval. The only response was, however, "Shouldn't you have solved it at once?" That answer, although perhaps more flattering than humiliating, the tone, the accompanying gesture distressed the young man, and the memory of that first grief did not ever erase from his memory.

Finally, the natural instinct that inspired Daniel Bernoulli prevailed over his parents' projects and in spite of itself his family obtained the honour, unique until now, we will not at all say in the history of science [why not?], but in the annals of the world of producing three great men in only two generations. Had not Daniel Bernoulli's elder brother [Nikolaus II] died prematurely, the miracle would have been even more surprising with Europe reckoning two brothers Bernoulli twice in succession among geniuses of the first rank[3]. The generation that enjoys their work distributes its admiration among them and leaves the right to range them solely to the posterity[4].

[2] The life of Bernoulli provides a small number of special events. He stayed a few years in Venice and Padua where he went to further himself in medical sciences under the two then famous physicians, Michelotti and Morgagni, but was unable to resist occupying himself for most time with mathematics.

He left Italy crowned with scientific honours after refusing, being 24 years old, to become president of the academy that the [Most Serene] Republic of Genoi proposed to establish.

[3] Next year Daniel was invited to Petersburg together with his elder brother Nikolaus II[5] who died there prematurely after nine months. Although enjoying a fortune above his desires from the blossoming Academy, he invariably yearned for his fatherland, for its republican equality rendered still dearer by the sight of the court, stormy and brilliant. He dreamt of leaving Russia, but the Court, wishing to retain him, raised his salary and granted him a half of it as a lifelong pension with the liberty to retire.

That manner of retaining him without denying that liberty, which they apparently [?] left him was really noble. Daniel remained in Petersburg for three years more and only returned home after concluding the work[6] with which he wished to pay homage to his benefactors and when his health did not permit him to extend his sacrifice anymore.

[4] He only returned to his fatherland[7] in 1733 to establish himself there[8] and at first occupied the chair of medicine in Basel university, then exchanged it for the chair of physics[9]. From that moment onward, the history of his life was the same as the history of his works.

The number of his memoirs on mathematics, published in the collected works of the various academies whose member he was, is very considerable. Although all of them are very short, there is almost none not meriting a separate item in this *Eulogy*. However, had any one of them been the only contribution of an author, it would not be sufficient for considering him a man of genius.

The story concerns those rare men who trace their career by the considerable advance of the sciences achieved at their hands, and it is that advance rather than the details of their works that ought to occupy us. Therefore, rather than presenting here their long list, we will restrict our attention to indicating his discoveries enriching the sciences and discuss their influence on those different branches of mathematics with which he busied himself.

There are occasionally mediocre scientists who with a ridiculous pride rank men of genius and thus announce that they place themselves in the same class, and indeed prove by that same temerity how far they are from having such a right. When authors equal to those great men assume themselves capable of achieving that, they run the risk of being mistaken.

The differences between those of the first rank really much less depend on real superiority than on the character of the mind that distinguishes them. And each of the judges (supposing them impartial and reasoning in good faith) will necessarily decide in favour of that scientist whose genius conforms more to his own. Having Bernoulli in mind, I do not at all attempt to appreciate him and still much less to pronounce an opinion about the difference between him and his illustrious rivals. I will not at all be arrogantly judging those of whom I would have been honoured to be a student and will only try to indicate in Bernoulli's works the particular character of his genius that distinguishes him from those whose reputation places them alongside him. This is at once the only just and the only possible useful manner of considering a great man.

[5] Bernoulli reluctantly allowed the publication of his first work [1724/4]. Some letters written for enlightening and defending certain places in the contributions of his father seemed to him not sufficiently worthy of the name whose glory that he should have preserved or rather augmented. But the scientific world thought otherwise, and the solution of the Riccati celebrated equation there contained ensured at that time a place for the young Daniel Bernoulli among inventive geometers.

These considerations published in Italy appeared after having been approved by a censor and such a procedure necessarily obeyed at that time in whole Italy except Naples[10] should have seemed bizarre for an author born free and Protestant and perhaps to some extent became a cause for his invariable refusal to settle in Italy.

On the frontispiece of his book Daniel Bernoulli only indicated one title, the only one which he then had, of being the son of Johann Bernoulli (*Joh. Fil.*), and he continued to adduce it to his name at the beginning of all his memoirs [even] when he became able to indicate his own very honoured titles and did not need to borrow any alien glory.

That same work [1724/4] included reflections about the so-called recurrent series the general theory of which he was the first to offer a few years later[11]. It led him to a method of approximation very ingenious and very convenient for equations with a finite number of terms (équations déterminée) which he extended over to those consisting of infinitely many terms and problems depending on the recurrence of series. These theories became almost elementary due to the immense progress of mathematical sciences in our time, but in those days they combined the merit of innovation and elegance.

The theory of series is the branch of mathematics richest in special paradoxes that, offering an apparent contradiction between the results of calculation and a proposition evident in itself, will disgrace geometry if the calculation made by able hands does not discover the truth of those same results which seem to contradict it.

When beginning to study series, Bernoulli had already noted some of those paradoxes, and was able to explain them, but, being still young, he did not dare publish his findings. He waited until his age and his glory provide him more authority in sciences which is a kind of shyness common to all powerful minds once the train of their ideas leads them to extraordinary results.

There are series whose sums are periodic, return to their previous values after a certain number of terms. If this number is determined, it is easy to calculate that sum since it is known to which term of the period does it correspond. However, if there are infinitely many of such terms, what should the sum of the series be? It is impossible to say that those infinitely many terms correspond to one, or to another term of the period, whether, for example, that term is rather even than odd.

It is from that same difficulty that Bernoulli got the principle which, as he believed, will resolve it. Since, as he thought, *There is no sufficient reason to prefer one case to another, we ought to suppose them equally possible and assign the mean value resulting from that assumption to the series.*

This means that not only the metaphysical principle of sufficient reason which Leibniz made so famous[12], but even the principles of the calculus of probabilities was applied to pure mathematics and that the results that ought to be necessarily true were, so to speak, entrusted to chance.

Nevertheless, that method succeeded in all the examples proposed by that acute inventor; it completely corresponded with the results provided by the direct methods, but until now that agreement was only proved by facts. Therefore, a geometer who applies that [stochastic] method to problems impossible to solve by a rigorous method, only has a probable assurance of obtaining a result conforming to the truth, a circumstance that can seem to be most unusual in mathematics.

[6] In the first memoir on mechanics published by Bernoulli where he examined its fundamental principles, he provided a simple and ingenious proof of the parallelogram law of forces consisting in essence in showing the absurdity of any other premise[13].

The same elegance is seen in another memoir on the relations between the centres of gravity, of oscillations and of the centres of forces. He proved that the oscillations of a body are of the shortest possible period when the point of suspension coincides with the centre of forces[14] and that that singular property was valid for any form of the body.

After that Bernoulli occupied himself with newer and more important issues. He examined the movement of two bodies attached to a flexible thread oscillating about a fixed point. At first he determined the oscillations of the body nearest to the point of suspension supposing that the other body descends as though nothing altered it movement. Then he imagined a force that restored the length of the thread and changed the place of both weights. The application of that very simple principle led Daniel to the calculation of the movement not only of those weights, but of infinitely many equal or not weights situated along the thread, and finally of an oscillating heavy chain, whether of a homogeneous thickness or not.

It is known that when the centres of gravity and the point of contact of two bodies at the moment of shock are not situated on the same straight line, the entire body acquires a compound movement in space with all of its particles rotating. However, no method was known of decomposing those movements, of reducing one of them to the movement of the centre of gravity and the other to uniform rotation about an axis passing through that same centre, or of determining the direction and velocity of both these movements, and this is what Bernoulli accomplished. D'Alembert later provided general principles of the theory of the movement of bodies of any form, then it was developed by him, Euler and Lagrange and had become one of the most daring structures that the human mind has built in this century.

Nevertheless, it is impossible to deny Bernoulli the glory of formulating its first foundation.

[7] In 1747 D'Alembert solved the problem of a vibrating string. He was the first to provide the appropriate integral equations in their correct form and his solution was as general as was possible given the essence of that problem. A short while after[15] that Euler furnished a solution based on the same principles and arrived at the same results by a similar method. These two great geometers only differed in the manner of subjecting the arbitrary functions, introduced in the integrals by the calculation, to the law of continuity.

Bernoulli maintained that the Taylor method, the first by which the problem of the vibrating string was solved although under a particular hypothesis, was by its essence as general as the new method. He thus reduced the merit of the new solution to applying a completely new mathematical tool, to analysing equations in partial differences. That dispute included two really different issues: the generality of the methods themselves, and on that point only a small number of geometers sided with Bernoulli, and on the real scope of those methods when applied to phenomena that might occur in nature.

Bernoulli's simple hypothesis of decomposing the real movement of a string into isochronous and regular vibrations of the whole string and its aliquot parts served him to render the Taylorian solution all the generality he needed. He applied that principle for explaining the different sounds which the same string can emit either consequently or at once, and the more or less low tones of the same pipe depending on whether the air was blown into it with more or less force and velocity. Euler extended that solution on vibrations of sonorous bodies, air and strings of unequal thickness whereas Bernoulli simply and elegantly solved the same problems by applying his principle and thus balanced the merits of the profound analysis of his illustrious colleague.

By means of his principle Bernoulli also solved the problem of the vibration of an elastic sonorous band and Euler provided his own solution of the same problem by applying his analysis. Finally, Bernoulli considered the vibrations of a string consisting of two parts of unequal thickness, each of them being of the same thickness throughout all its length.

He managed to determine these vibrations by supposing, first, that each part vibrates alone and one of its ends is fixed and the other one restrained by a flexible non-elastic thread of a given length. Then, it only remained for him to determine the length of that thread for the string to have the same movement which it should have when both its parts are combined. If that solution presented a challenge of sorts, Bernoulli had chosen it well: at the point where the two parts met the law of continuity was violated and it was easy to foresee that another difficulty thus occurred for a purely analytical method. Nevertheless, Euler's analysis triumphed effortlessly.

Pleasurably and with surprise and respect we see in that long and glorious battle two men of genius, one of them displaying all the power of the analysis, the other, in order to avoid it, applying all the skill and sagacity of an inexhaustible mind armed with a supply of means. One of them infinitely capable of efforts and calculation since they do no cost anything to his genius equally fruitful and tireless, the other always attaining his aims simply and elegantly and gaining his glory by achieving much with little effort without fearing accusations of being enfeebled. They were equally sure to be admired by a small number of those who can understand and judge them and share this approval among both of them.

Bernoulli extended this method of reducing compound and irregular movements of a string to isochronous and regular vibrations to the movements of a thread loaded with weights. He applied that method for exactly determining the true length of a simple pendulum whose swings agreed with the vibration of a weight suspended on a flexible thread of a given length. It was usually supposed that the length of that pendulum was equal to the distance of the point of suspension to the centre of oscillations; Bernoulli proved, however, that that hypothesis was not only not rigorously exact, but that it could have even led to noticeable errors in case of very delicate determinations [measurements].

And again by applying that principle he discovered the laws of the movement of a pendulum allowing for the vibrations communicated to its support and bodies on which it acted. He proved that the less movement a clock received from the swings of its pendulum, the more the simple pendulum isochronous with it augmented its length and approached that which it would have in case of complete immobility. He thus explained that clocks were quite considerably slow only because, wishing to render their motion more regular, they were fixed on more solid supports.

We also find the same principle in Bernoulli's memoir on the determination of the movement of an elastic plate (lame, Platte) hit perpendicularly in its centre. The shock sets the plate in motion in its direction, but, in addition to that common movement, all parts of the plate will vibrate. When determining both these motions he came to a singular conclusion: that shock will force the plate's extremities to move in the opposite direction. The centre of the plate advances, but its extremities retreat beyond their former position. Mariotte and Leibniz had previously observed that phenomenon and Bernoulli experimentally confirmed its existence. It followed from his theory that since ordinary laws of shock concerning elastic bodies neglect that double movement they do not rigorously agree with nature, and experience once more corroborated the results of calculation.

Finally, it is seen that in many places of his memoirs he considered it possible to explain the most singular phenomena of light by that same principle, but apparently he did not dare touch so delicate a subject and restricted himself to indicating from afar a route for his successors which he feared to follow.

[8] Those geometers who studied Bernoulli's memoirs would have noticed that we only thought to extend the description over such that can best convey the distinctive character of his mind. Thus, we did not discuss either his application of the principle of conservation of live forces to the movement of bodies attracted to certain mean points or mutually attracted, or his investigation of oscillations or of paths described in resisting media, or, finally, his discovery of the principle of conservation of gyroscopic movement that d'Arci later provided along with new applications, see Condorcet [1847].

Bernoulli published only one great separate contribution, his celebrated treatise *Hydrodynamica*. The theory of the movement of fluids had occupied the most illustrious geometers of the 17th century, but all their efforts were barely useful for something more than a better understanding of the phenomena that needed to be explained, of the questions that ought to have been answered, and in the first place of the difficulties encountered.

Our Daniel earned the glory of being the first to provide that theory in a general manner and according to the principles which if not rigorous at least seemed only to deviate slightly from the truth. One of them is the principle of conservation of live forces only subjected to exceptions when the law of continuity ceases to take place in the phenomena[16]. The second principle consists in separating the flowing fluid into parallel channels and supposing that a common movement with the same velocity and direction takes place for all the particles in each channel.

It is by applying these two principles that Bernoulli resolved all the problems on the flow of a fluid from a vessel either through an opening or many pipes, either if the vessel was being emptied or always remained full. He employed those principles just as successfully to the movement of fluids from vessels of any form; to their pressure on the walls of those channels which contained them; to the laws of the oscillation of fluids in siphons or vessels connected by openings; to the shock exerted by the fluid on the planes exposed to their action; to the theory of the air and other elastic fluids; to the singular force exerted by water flowing through a hole pierced in the wall of a vessel on the opposite walls.

That repulsive force tends to move the vessel in the opposite direction, and Bernoulli thought that it can be advantageously used for sailing small boats upstream or supplementing the action of the wind when sailing large boats[17]. He later determined, once more by applying his method, the different states of equilibrium and infinitely weak oscillation of bodies submerged in fluids.

Some questions treated by Bernoulli had apparently eluded the principles he applied; however, he was able to return to them by equally ingenious and plausible physical considerations and so skilfully that it seems to stand up to a miracle. And the principles making it possible to determine the movements of fluids by the nature of forces applied to each of their particles only supposed to be subjected to the law either retaining the same volume or changing it according to a given rule, – by the time Bernoulli published his *Hydrodynamica*[18] D'Alembert had not yet discovered those direct principles. That contribution will therefore always be regarded as one of those monuments constituting an epoch in the history of sciences.

[9] The analysis of probabilities is a branch of mathematics towards which our Bernoulli should have been stronger attracted owing to the unusual nature of its results, its usefulness, and above all because it offers many topics for practising sagacity independently from the methods of calculation. In his first memoir on that theory he [1738/22] examined one of the fundamental rules of that calculus which prescribed for evaluating the expectation of each interest the multiplication of its value by the probability of its occurrence.

He showed that that rule, when applied to ordinary life, led to absurd results and proposed to amend it by substituting the absolute value of that expectation by the value which might be called *relative expectation*. According to Bernoulli, the expectation of gaining a certain sum should be expressed not by the sum itself, but by its ratio to the fortune of the person in question. As a result, each consecutive loss in a continuing game, although all of them being the same, should be regarded the greater, the more the fortune of the loser diminished, and the gain of the winner the lesser, the more his fortune augmented.

By applying that method [that principle] he concluded that for each of two gamblers of equal fortune playing a just game, the value of loss is much greater than the value of the expected gain. Calculation had thus led Bernoulli to conclude that a reasonable man never plays for high stakes.

However ingenious was Bernoulli's idea, it was not sufficient for resolving all objections levelled against the rule proposed by Fermat, Pascal, Huygens, Jakob Bernoulli and later adopted without examination by many geometers. It fell to D'Alembert to elaborate on all the difficulties and show that the previous rule should be replaced by another or only admitted with restrictions, or, finally, applied in a new manner[19].

In 1760, Bernoulli [1766/51] applied the calculus of probabilities to inoculation of smallpox. He considered that issue as a statesman and it could not be denied that in a victorious manner he established by an extremely delicate analysis the advantages of that procedure for the state in which it is generally adopted. However, he did not at all envision its action on an individual.

From that point of view, the situation is changed; actually, if a large number of people are inoculated at once, it is of little general interest that a small part of them risk to lose their life in a few days because at that price the state ensures a certainty of sorts of retaining for a longer time those who had escaped that low danger. It is not the same for each of them separately; for an individual, the essence consists in comparing a very low but immediate and restricted to a very short period risk with a higher but remote risk spread over his whole lifetime. Bernoulli however only calculated the effects of inoculation as a republican [or monarchist] in whose eyes the state is everything and the individuals are only citizens[20].

The calculus of probabilities leads to very complicated results when a set of many combinations ought to be considered, and this occurs almost always in its application to natural phenomena. Bernoulli therefore proposed to regard the changes in the formulas occasioned by a unit change of numbers as infinitely small and to replace the calculus of combinations by infinitesimal analysis. He proved by a large number of examples that this assumption only quite insensibly altered the results.

It is by means of that method that he determined, how many marriages subsist, and how many men and women will be widowed after some years out of the known number of marriages supposed to be contracted at the same time between persons of given age. He applied the same method for determining the boundaries within which will probably remain the difference between the number of newly born boys and girls out of a given number of births assuming that the birth of infants of both sexes are either equally probable or not (as most registers of births seem to prove).

These investigations show how to distinguish very improbable items compelling to suppose that nature had withdrawn from its laws and reject them from summaries of results, or at least to establish the truth of their evidence by an almost unimpeachable authority.

[10] Astronomers, whom their observations provide differing determinations, usually form a mean value by dividing […]. Bernoulli warned that that rule can only be valid if the observations are supposed to be equally probable and that such an unjustified hypothesis can only be established if it were absolutely impossible to know the ratio to each other of the different probabilities possibly characterizing observations apparently made with the same precaution. Then he attempted to determine those ratios only issuing from the known more or less large differences between the observations[21].

If the principles applied by him can seem a bit arbitrary, we should at least acknowledge that he let the geometers appreciate the need to study anew the rule admitted before him by everyone who treats observations of any kind, and many celebrated mathematicians did not regard this issue unworthy of their researches[22].

[11] Even the best manufactured clocks are exposed to derangements either depending on physical causes or apparently absolutely irregular; only the latter are the object of the calculus of probabilities[23]. Bernoulli supposed that each swing [of the clock's pendulum] can be with equal probability either slow or fast, and he examined the probability that after a day these errors either exactly compensate each other or do not [their sum does not] at all exceed some boundaries. He finally proved by examples that these investigations are not in the least useless and no one had since thought of giving them up. It is at least true that they are necessary for each observer to estimate the exactness of his clocks. That memoir containing a singular, new and useful application of the calculus of probabilities terminated Bernoulli's glorious carrier.

[12] Bernoulli ten times earned or partly earned prizes of this [Paris] Academy[24] in competition with the most illustrious European geometers. Until now, only one scientist was able to be his equal and to collect for himself the same number of prizes, – Euler, his countryman, student, rival and friend[25].

Bernoulli won his first prize being 25 years old for constructing a sand clock capable of exactly measuring time at sea. He proposed an ingenious and simple means for ensuring regularity of these clocks in spite of the external movements experienced by them[26]. In 1734, he shared the prize with his father[27]. It was asked to explain the physical cause of the larger or smaller considerable inclinations of the planetary orbits relative to the solar equator.

At first Bernoulli proved by invoking the calculus of probabilities that the boundaries within which those inclinations were contained justified the assumption that a certain physical cause prevented the planets from being more inclined relative to each other. Then he looked for that yet unknown cause and thought to have discovered it in the effects of the planetary atmospheres. It should be, however, admitted that his explanation was only ingenious.

Johann Bernoulli sorrowfully saw that in a sense his son became his equal as judged by a society whose favourable decision he himself had so many times aspired to and deserved. Paternal love, the strongest and perhaps the least personal of all that people can experience, yielded in his heart to his indignant glory. Little touched by seeing his family obtaining by that sharing a still unparalleled honour, insensible to the pleasure so sweet for a father to feel that his son is worthy of him, he only saw that son as a rival, and his success only as lack of respect with which he for a long time bitterly reproached Daniel.

There were perhaps other causes as well for that mood because his son's piece was better than his own and Daniel had imprudently hinted that he indeed thought so and his father was unable to conceal from himself that that opinion was justified. Finally, the son dared show himself as a Newtonian and abandoned Cartesianism still only supported by the name of Bernoulli. Daniel Bernoulli's admission was the last triumph previously lacking in Newton's glory that Daniel's father had the misfortune of struggling with all his life[28].

In 1740 Bernoulli shared the prize concerning the ocean tides with Euler and Maclaurin and each of their pieces was meritorious in its own way. Bernoulli treated all aspects of the proposed problem with that sagacity and in the same methodical way which are characteristic of all his writings. Maclaurin's memoir was based on the celebrated theorem on the equilibrium of ellipsoids; it bears his name and will immortalize him. Euler provided a method of integral calculus, then new, for solving the fundamental equation of almost all the problems about the motion of celestial bodies.

At the same time the Academy crowned yet a fourth piece whose sole merit consisted in being Cartesian, and that was the last public act of the cult bestowed [by the Academy] perhaps for too long on the system of vortices.

[13] Bernoulli won the next prize for 1743 for an inclination needle. The calculation of the error that the different types of friction can cause on the inclination of a metal needle movable on journals and subject to the action of magnetic force and weight; also, the even more delicate calculation of the change that the inclination of the needle and the curvature occasioned by its weight must cause in the position of its centre of gravity; the ingenious means for exactly finding out by experiments aided by calculation the veritable inclination whereas the direct observation of the needle was always erroneous, – those were the issues treated in that piece, which was one of Bernoulli's writings where he displayed most perception. Yes, when discussing his work, it is impossible to avoid that expression which seems so alien to the issues he treated.

In 1747 he shared a prize with an anonymous author for the best method of finding out the time at sea when the horizon is invisible (see Note 27). We see in his contribution excellent observations on the means of ensuring a correct motion of clocks regulated by a pendulum or a spring balance-wheel. Bernoulli explained the singular paradox consisting in that without air resistance the weight or the spring persistently augmented the oscillations of the pendulum or the irregularity. And that resistance which with respect to other circumstances harms the regularity of motion, is at the same time the veritable cause of the possibility of obtaining it.

Proposing to know the position of an unobservable horizon, when everything placed before your eyes sways with the ship and cannot preserve a constant direction, seems at first glance to be utterly impossible, but nothing is beyond Bernoulli's sagacity. He issued from a general principle to which he often returned in his works and based both on theory and experiment. The alternative irregular movements transmitted to a certain number of communicating bodies tend to a regularity of sorts and finally reduce to a subsisting rather than diminishing system of isochronous and simultaneous movements. It is seen with some surprise that an order is established by itself as the sole effect of necessary mechanical laws. That principle led Bernoulli to determine the veritable vertical direction [and therefore the horizon] by observing many pendulums of various lengths and combined in various ways although the ship's movement continually and apparently quite irregularly alters the effect of gravity.

Bernoulli's piece on ocean tides that earned him a double prize of 1751, is especially devoted to prove how the rotation of the Earth ought to produce a regular current on the surface of the sea south of the equator and how that current arrested by a continent produces another weaker current moving in the opposite direction.

There also we find the first indication of the property of a fluid to vaporize in vacuo whereas the same fluid (if contained in the atmosphere) remains at the same temperature unchanged.

[14] As the subject for the prize for 1753 the Academy proposed to investigate the manner of supplementing/compensating the lack of the wind's action on big boats, and our Bernoulli once more earned it. Abandoning the means that he proposed in his *Hydrodynamica* to employ the reaction of water, he subjected to analysis the action of oars. At first he examined human power and formulated a new principle that the total effort a man was able to exert during a day remained almost the same whether he fulfilled his task in a few hours (but did not exceed too much a certain limit of either the effort or the speed of work) or decreased the intensity of his work and proportionally extended the time needed. This rule agrees with nature and, as Bernoulli remarked, offers some kind of a principle of conservation of live forces applied to animal economy.

If a moving body experiences resistance proportional to the square of its velocity, the work necessary for continuing the motion ought to increase as a cube of that same velocity so that there exists a boundary after which the increase in the number of oarsmen almost does not augment the speed of the body that they wish to move. Finally, each time that the force acts not at a fixed point, but on a moving body, a part of it applied to move that body is lost as far as its proposed aim is concerned. Therefore, a useful and a useless part of the applied force should be distinguished, and Bernoulli indicated how to establish their ratio to each other in different cases and explained how was it possible by increasing the surface of oars to diminish the ratio of the latter to the former with all other circumstances remaining invariable.

Bernoulli won his last prize for answering how to decrease pitch and roll of ships without damaging their other qualities. After determining the form that ought to be provided for a ship to ensure its greater stability, whether in a calm state or more or less inclined due to the action of wind or waves, the author examined the means for preventing the causes such as waves or gusts of wind acting at more or less irregular intervals from continually increasing the ship's oscillations and exposing it to the danger of capsizing.

That part of the theory was absolutely new; it led to the paradox that in case in which accidental causes tending to communicate new movements to a ship act repeatedly with short intervals the danger of capsizing becomes more serious but the stability is increased. That danger, however, only exists when those intervals are shorter than the period of the ship's oscillations; happily in practice it rarely poses any threat at all and in all other circumstances it is useful to increase stability as much as possible.

[15] These details, perhaps too lengthy, suffice to acquaint ourselves with Bernoulli. It is seen that his manner especially directed him towards examining problems that present more difficulties for reducing them to analysis than to solve them when that is accomplished.

In the essence of such problems themselves, Bernoulli attempts to find the means for simplifying it, for reducing it to its simplest form only leaving for analysis that which cannot be taken away from it. It is seen that above all he wishes to apply theory for penetrating nature; to use mathematics not only in speculative mechanics, in studying the laws of abstract movement of bodies, but in physics, when examining phenomena in the universe in their real states, and according to the manner in which observations presented them.

No one discovered more analytical means for subjecting to calculation all the circumstances of a phenomenon, no one was able to arrange better an experiment to apply it either for confirming the results of a theory or for serving as a basis for calculation. He was invariably a philosopher and physicist as well as a mathematician.

Perceptiveness seems to have been the dominant quality of his mind and he possessed it to such a large extent, and applied it so fortunately, and it served him so well, that in essence it became majestic and led to admiration and surprise that seem to be reserved for miracles caused by the force and depth of genius.

[16] In 1748, Daniel Bernoulli replaced his [deceased] father at the [Paris] Academy of Sciences. His brother, Johann II, succeeded Daniel at the same seat [of foreign membership] which, since it was created in

1699, that is, for 86 years, had been occupied by scientists of that family[29]. This is a glorious succession indeed since it proved that in that really reputable family talents are not less heritable than titles. If the pride of birth can be not a childish weakness, we may attempt to excuse it when it is supported by such a brilliant example rather than by unimportant genealogical tables where shameless vanity so often displays pretensions founded on fables, brilliant prerogatives bought by baseness, great dignities demeaned by disgraceful actions and a hundred honourable titles hoarded behind a disgraced name.

Bernoulli was simple and upright, lacked vanity and false modesty. His society was agreeable, he did not make use of any tricks except for prompting others to speak about what they knew best. He only recalled the superiority of his genius or the glory for attempting to excuse himself and scorned success in society so humiliating for others and so negligible and easy for himself.

He never married. In his youth, a very advantageous catch was proposed him, but her extreme thrift soon made him decide to break up with the acquaintance. After that, he only thought about marriage to recall that he had once been on the brink of losing his freedom and peace, and to fortify his decision to avoid exposing himself to the same peril. He was decent in his lifestyle but not austere, did not defy the honour of general opinion but did not, however, sacrifice anything that could have strengthened the pleasures of life.

Although he respected the religion of his country both in his speech and writings, and even practised it, which in truth was not cumbersome. Some sour pastors and secular people strongly suspected that he only respected it outwardly and especially accused him of carrying the freedom of thought too far. He never allowed himself anything that could have confirmed that opinion, but never attempted to dispel it.

Out of all kinds of pleasure those promising the most are not those that always provide the most. The enjoyment of pride, the sharpest test of a famous man, is often due not to his grand work or most brilliant success. Our Daniel, sincere enough for admitting that he is familiar with these pleasures, and therefore liked to describe to his friends two insignificant adventures that, as he said, were more flattering than the honours and literary garlands with which sovereigns and scientific societies showered him.

His conversation aroused the curiosity of a learned fellow-traveller who expressed his wish to know the name of his companion in voyage. *I am Daniel Bernoulli*, was the plain and simple answer[30]. *And I am Isaac Newton*, replied the unknown person who thought that Bernoulli was making fun of him and did not want to believe Bernoulli until hearing an authentic proof that a person with a face so young[31] and an appearance so simple was that same man already so famous in Europe.

Another time Sam. König, a skilful mathematician [see [ii, § 26!]], had diner at Bernoulli's place and told him somewhat indulgently about a rather difficult problem that he only solved after a long while and much effort. The host continued to do justice to his diner, but, before leaving the table, Bernoulli presented König with a solution of his problem, much more elegant than that which gave him so much trouble[32].

Some of those people quick to judge that which they know the least dare remark that it is really possible to be much gifted for science but have not enough brains. That observation is unjustified: say what you will, a person of a really low intelligence is only averagely talented and had usurped his reputation. Or, if he, who is really gifted, seems to be witless, it is because he neglects to show his intelligence, is remote from the topics discussed in his presence and keeps silent or speaks about them disinterestedly. Nevertheless, no wonder that that opinion must have many partisans; it is equally proper to smear witty people and console those whom nature refused talents. It is therefore permissible to remark here that our Bernoulli, although being a man of innate talent, had mind enough even for those who were unable to grasp all that shone in his writings.

As all people born with a gift for observation, he was able to recognize ruses, penetrate little secrets or vices, but he only applied that art against the malicious but thought that out of duty to humanity and justice stupid people ought to be spared if only they did not wish harm. If he sometimes abandoned himself too easily to his natural vivacity, he redeemed himself by deep gentleness and pleasantness which never left him and especially by the agreeable or witty ways with which he manifested his vivacity or made up for it.

People who try to find fault in those whose brilliant qualities humiliate them, accused him of a vice very undignified considering that greatness of mind and character, almost inseparable companions of a genius. They claimed that Bernoulli was a miser. It is true that useless expenses such as occasioned by splendour and vanity, involving waste of much time and providing little pleasure, were unknown to him. But his house, his table and clothes were the very refinement still compatible with simplicity. He was charitable without being pompous and did not attempt to show it. Even during his lifetime he established a foundation in favour of poor students passing Basel. Finally, in many circumstances, when he was compelled to choose between happiness and fortune on the one hand and freedom, his peace or tastes on the other hand, it was always the former that he sacrificed.

[17] He enjoyed peace, and his life was not at all troubled by scientific quarrels[33]. He rarely asserted himself among geometers; they do not have many judges, and those judges cannot be either dazzled or captivated, and, what is even more important, they cannot be unjust. They were often shown to have been wrong in their judgement; they sided with those whose claims they attempted to favour but their pride compelled them to be just. And there were no long disputes except those about issues on the boundaries of metaphysics and geometry, and, when the former science entered to some extent, doubts, subtlety, mist and uncertainty accompanied it, perhaps owing less to the essence of the pertinent objects than to the fault of those who cultivated them. In Bernoulli's memoirs concerned with those discussions we see some features of indignation slipping out as though in spite of himself, but very rarely for suspecting that they could have disturbed his peace and sufficiently often for proving that if he enjoyed peace, it was less because of his disposition or insensibility, than owing to reason and philosophy.

[18] The staff of Basel University was excluded from positions in the government. Not that (as could have been thought, just like in some places where the Gothic prejudices were not yet rooted out) those wise republicans could have regarded the noble function of teaching as a low and despicable profession. Neither was it because, according to no less wrong ideas still spread by ignorance and dread of useful reforms, that they thought a gift for sciences incompatible with a talent for governing, as though the art of governing does not consist of revealing or discerning the truth; as though the method of discovering, recognizing and justifying it is not the same in each profession; as though, finally, the possibilities provided by the inclination to sciences do not inspire a certain independence in those who cultivate them, – yes, with independence that we cannot expect in people who only represent their own position and lose everything when forced to leave it.

Other causes had undoubtedly dictated that disposition: it was feared that in a small republic a part of a corps of educated men, of those accustomed to reflection, work and power over their students, and provided with the authority of the magistrate, will possess excessive influence dangerous for republican equality[34].

And although Bernoulli was unable to be a member of the government of his country, he knew how to be a useful citizen. The most learned, wisest and virtuous of his compatriots were honoured by being befriended by him and thought it necessary to consult him. When his advice became known to the public, it provided those who accepted it an authority of a revered name. Those who had improper intentions knew that he was able to reveal them and feared the judgement of a great man, a honour to his country, more than public opinion which, as they boasted, it was always possible to seduce, misdirect or silence.

Our Bernoulli enjoyed in Basel the respect that a man of genius only gets after surviving the jealousy of his contemporaries, taming or humbling the pride of those on the top and triumphing over the ignorance or insensitivity of common people. When he walked the streets of the city, citizens of every rank saluted him respectfully, and one of the first lessons that fathers gave their children was to explain that duty.

His life, monotonous and regular, exempted from passions and even grief except for those necessarily occurring in mortals, ensured him invariable health. In spite of his delicate constitution he preserved all his intellectual capacity almost until the age of 80. His last memoirs were still worthy of him; and what he achieved at the age when so many people are condemned to remain useless was sufficient for another geometer to earn a reputation.

Several years before his death he quit socializing which only became burdensome for him, but in the evenings he allowed to be carried to his usual place of rest to meet with five or six persons with whom he had been acquainted for a long time. He did not anymore receive strangers attracted to him by vain curiosity and made exceptions only in favour of those, famous in Europe, who excited in him the same feelings as he inspired in them.

During those last years some kind of very exhausting asthma deprived him of sleep and strength. At the beginning of March 1782 his infirmities redoubled and he only dragged out a painful existence. He was hardly able to use his intelligence a few hours daily and on 17 March, in the morning, his manservant who had left him for a short time, entered his bedroom and found him dead in his bed. He slept peacefully through the last few hours of his life and saved himself from all that could have given him a feeling of regret and suffering.

He was mourned by his family and fellow citizens, who honoured his genius and virtues. To sciences he left monuments to be recorded forever in their annals; to scientists, useful lessons in the art of enjoying glory coupled with rest and consideration; to all the people without exception, an example of happiness in the flavour of retreat, love of study and wisdom.

Notes

1. It seems that Daniel II excessively esteemed Condorcet. This is also evident in some of his comments below. O. S.

2. Wolf [v, § 1] reports that Daniel's second teacher was his elder brother, Nikolaus II. O. S.

3. If Nikolaus I, the nephew of Jakob I and Johann I and Daniel's cousin, not less famous in the scientific world as a mathematician and lawyer, were added to the three great men, Jakob, Johann and Daniel, the reputation of the Bernoulli family would have been still higher. Nikolaus is especially known because of his dissertation (1709); its worth is seen already since before long it was translated into French, supplemented by comments written by the learned Condorcet and published in Paris.

It is not for me, his son, to say to what extent Johann II, Daniel's younger and still living brother and successor at the Royal Academy in Paris, also deserves to be mentioned here. I therefore refer lovers of the history of sciences to Meister [1782 – 1793] who provided short biographies of Johann I (in pt. 1), Jakob I, Daniel and Johann II (in pt. 2) as well as their not really good portraits by H. Pfenninger.

Jakob died in 1705 and therefore, in spite of what some authors stated, could not have been the teacher of the great Leonh. Euler who was only born in 1707. That honour fell on Jak. Bernoulli's brother, Johann I. Dan. II B.

4. In § 4 Condorcet states that ranking is inadmissible. O. S.

5. That elder brother of D. Bernoulli, before he was invited to Petersburg, was already for a few years a teacher of law in Bern [v, § 3]. He died in Petersburg on 9 Aug. 1726, old style, and was buried with honours at the Empress' expense. His obituary is Anonymous (1729). Dan. II B.

6. That work was the learned *Hydrodynamica* that he extended and gave for publication in Strasbourg only some years later. Dan. II B.

7. Bernoulli had the pleasure of returning to Switzerland with his ten years younger brother Johann II who had come to Petersburg a year earlier to stay with Daniel and undertake the scientific journey back home together with him.

As soon as the President of the Imperial Academy of Sciences heard about Johann' arrival, he honoured that young newcomer with a very courteous letter written in his own hand in which he invited Johann, although unconnected with the Academy, to attend their conferences as often as he wished and from time to time submit to the *Commentarii* some pieces of his work. My father had with proper gratitude accepted and made use of that honourable offer. And although the Academy occasionally sent him invitations to enter, to which he was not indeed indifferent, it seems that the attitude on either side was not really earnest since in 1733, on Midsummer's Day, old style [24 June, new style], both brothers went back to their fatherland. Dan. II B.

8. Daniel had been honoured by various noble and advantageous invitations from abroad. He had, however, declined all of them because of scientific necessity and since he thought it his duty to dedicate his talent to his fatherland rather than to foreign princes. Dan. II B.

9. Bernoulli was appointed to the medical position while still travelling back home. He held it until 1750 when the chair of physics became vacant. The magistrate transferred him waiving the regrettably! usual decision by lot, which was an exception much honouring him, and leaving him the title of extraordinary professor of medicine and a seat and vote at the Faculty of Medicine. In addition to the professorship in physics Condorcet mistakenly named the chair of speculative philosophy. Dan. II B.

10. The Kingdom of Naples. That city was the capital of the Two Sicilies. O. S.

11. It was De Moivre who introduced recurrent series in 1720. O. S.

12. Here is Leibniz' statement (1765/1996, Book 4, chapter 16): *It is an axiom*, "*aequalibus aequlia*", *equal* [equally valid] *preconditions must be equally considered*. The explanation of Bernoulli's method above is inadequate, but my attempts to find its better description have failed. O. S.

13. That law is now considered as an axiom. O. S.

14. See [i, Note 14]. O. S.

15. Antropova (1972, p. 413) mentions the date of publication of D'Alembert's memoir (1749 for 1747) and writes out his equation, – not an integral but a partial differential equation. Condorcet apparently had in mind the integral of that equation. O. S.

16. How does that statement agree with the law of conservation of energy? O. S.

17. It seems to me that the repulsive force can also be advantageously used to solve the long studied mystery of the horizontal movement of balloons since it is as good as proved that nothing or almost nothing is achieved by sails, helms, wings etc.

The elder of the brothers Montholfier had thought about that and had a balloon with various large holes all around it and valves to open and close them. Through these holes the air can escape from within and make it possible for the repulsive force to push the balloon in the opposite direction. Dan. II B.

18. The term itself, *hydrodynamics*, was new and perhaps Bernoulli adopted it not wishing to entitle his contribution the same way as his father did concerning the theory of fluids. Later the Abbot Bossut published a writing [1771] of a wider scope treating many issues on which Bernoulli did not dwell at all, and resolved many others simpler and more precisely. M. J. A. N. C.

Johann Bernoulli published his *Hydraulics* in 1742, but indicated there: *Hydraulicam now in 1732 discovered for the first time* [v, end of § 8]. O. S.

19. When describing Condorcet's contribution of 1785 Todhunter (1865, p. 352) noted:

It is in many cases almost impossible to discover what Condorcet means to say. The obscurity and self contradiction are without any parallel. […] *No amount of examples can convey an adequate impression of the extent of the evils.*

In particular, Condorcet's expression above is extremely unfortunate as is his description of Bernoulli's mentioned memoir on the *moral* (rather than *relative*) expectation. Crépel (1987) published the summary of his apparently first manuscript on the theory of probability dated 1772. The same year, 1772, in a letter to Turgot, Condorcet (Henry 1883/1970, pp. 97 – 98) stated that he "is amusing himself" at calculating probabilities and had "compiled a small book on that subject".

In essence, Condorcet continued, he was keeping to D'Alembert's opinion and differed with him "in a few details". That book remains unknown. Then, no one had ever levelled any objections to "the rule proposed by Fermat …"; really confusing was nevertheless the paradox of the Petersburg game (involving an infinite expectation) whereas D'Alembert had proved nothing at all and is notorious for his mistakes in treating probability. O. S.

20. Bernoulli had adduced quite a few examples concerning France so that *or monarchist* should have been added to *republican*. In essence, however, the description above is superficial and wrong. Bernoulli did consider the not quite safe inoculation of smallpox (from a sick person, made under definite conditions) from the point of view of the individual as well, although not quite comprehensively. For a modern discussion of that memoir see Dietz & Heesterbeek (2000). O. S.

21. The exposition is feeble; suffice it to recall Simpson's memoir of 1756 in which he considered a series of observations as a sample from some general population and justified the arithmetic mean in an important case. Then, Condorcet did not mention Euler's commentary on Bernoulli's memoir and the same is as good as true concerning Condorcet's Eulogy on Euler [ii].

Finally, Bernoulli's recommendation actually led to the choice of a general (weighted) arithmetic mean, i. e., to the ordinary mean corrected for asymmetry of the (unknown) distribution of errors, see Sheynin (1972 and 2007, §§ 3.1 – 3.3); the former source also discusses Bernoulli's memoir on pendulums, see Note 23. Bernoulli's *Werke*, Bd. 2, include a list of papers discussing his work on probability and statistics compiled, as mentioned in the Acknowledgement, with the help of a certain mathematician (now professor), Ivo Schneider. My paper (1972) is lacking in that list, and several years later, when corresponding with the Editor of that volume of the *Werke*, I found out that he did not know about my paper although it was that same Schneider who edited it! O. S.

22. For example, Lagrange whose memoir was very interesting, although only in the general mathematical sense, and again Euler. O. S.

23. Condorcet apparently did not know that pendulums were being used for determining the acceleration of gravity. And his description is moreover very superficial. O. S.

24. Condorcet discussed only eight out of the ten prizes. One of those ignored by him concerned anchors and Bernoulli won it in 1737 together with the Marquis Poleni, the then professor of mathematics in Padua. That problem was already proposed two years previously but no one had solved it completely enough, so the Academy repeated it for 1737, separated it in three different parts about the shape; the manufacturing; and the testing of anchors and established a prize for each of these.

Johann II, the younger brother of our Dan. Bernoulli, won the first of these prizes, the second went to Tresaguet (*ancient ingénieur des ponts & chausées*). Concerning the third problem, since no piece sent to the Academy was complete enough, it was decided to share the prize between Dan. Bernoulli and the Marquis Poleni. Their writings, as the Academy itself stated in its preliminary report, contained very meaningful investigations about the shape of anchors as well as concerning the other aspects of the problem and very useful practical remarks.

The other prize carried away by our Bernoulli, although actually together with my father, and left out by Condorcet, was proposed for 1746 and concerned the theory of magnets. That awkward and difficult problem was already proposed in 1742 and 1744, both times vainly, and so it was proposed for the third time for a triple prize.

Maupertuis, who had been staying in Basel from 1744 to 1746, encouraged the brothers Bernoulli who thought that that subject was too difficult and did not dare deal with it the first two times. He vigorously persuaded them to compete for that considerable prize which, according to the Academy's bylaws, could not be proposed anymore. If, he argued, the work is difficult for you, it is also difficult for others, etc.

And Dan. Bernoulli acknowledged that he had actually considered that problem and put some thoughts on paper; however, he was only slightly satisfied with his efforts and abandoned that work. Nevertheless, at the same time he told Johann, as though jokingly, that he will turn over those thoughts to Johann provided that he will go ahead with them, and, should the outcome be happy, they will share the meal. The younger brother accepted that proposal out of which emerged the contribution that they sent to the Academy under a modest motto *In sententia permaneto, eninvero nisi alia vicerit melior*. The happy success considerably surpassed their expectations since they shared that triple prize with two other authors, Euler and Du Tour, *Ecuyer* [Cavalier] and correspondent of the Royal Academy in Paris. The contribution of both brothers [1748/41] was published under their both names (Daniel and Jean Bernoulli) and it is in order to note that their theory of magnets was very near to the Eulerian. The name *Jean* was not specified since that contribution appeared when Johann I was already dead. Dan. II B. [Not convincing. O. S.]

25. Euler "collected" 12 prizes. O. S.

26. Bernoulli's pertinent memoirs (and those concerning magnetic declination) are reprinted in vol. 7 of his *Werke*. O. S.

27. That prize, as also the prize for 1747, see below, was doubled since both were proposed previously, in 1733 and 1745 respectively. Their distribution was however postponed. Dan. II B.

28. I will never disclose here the weak side of my late grandfather. He had it, just as the greatest and wisest men have their own weaknesses, and he acted somewhat unjustly towards his son. I am a translator and there are many reliable translators who could have saved me from accomplishing that work. Basically, during his later years, which are here discussed, he was not as resolute a Cartesian as Condorcet pictures him. He knew, however, that the Academy itself was still almost completely disposed to Cartesianism and thought to offer it an agreeable sacrifice by submissively lending them their beloved system and thus solving one of their proposed problems that certainly could have been better explained according to the Newton's theory.

I will not investigate whether, or rather to what extent that long-winded anecdote could be justified and still less attempt to find out who could have told it to Condorcet. Dan. II B.

29. In 1699, when the Academy was established, both brothers, Jakob and Johann Bernoulli, were admitted as foreign members. As stated above, Jakob died in 1705 and Johann died on New Year's Day 1748. Dan. II B.

30. I am faithfully translating that anecdote but am unable to refrain from the following remark. How was it possible that Condorcet, so acutely witty, a quality that certainly no one can deny him, when describing Bernoulli's answer and putting words in his mouth, called it *plain and simple*? To my mind, these words rather betray some pride and conceit as though he wished to say "I am that Daniel Bernoulli, fama super aethera notus".

And the actual, and really quite simple answer was *My name is Bernoulli*. His companion, who as it seems cherished an enthusiastic deep respect for that name wished to answer that intended joke in Bernoulli's own coin: *And my name is Isaac Newton*. Bernoulli, however, proved that he had not at all joked by showing the addresses on various letters which he was keeping, and then came to know that his companion was Trant, also a scientist, an *adjunct* [junior scientific assistant] at the Royal Academy of Sciences in Paris. Dan. II B.

31. The event described here occurred when the brothers Bernoulli, during their journey from Petersburg to Switzerland, left Paris. Our D. Bernoulli was then 34 years old. Dan. II B.

32. Condorcet could have added that König, upon hearing the solution of that problem from Bernoulli, politely (höflich) complimented him: "Your solution can be hardly (schwerlich) good enough (perhaps he even said *impossible that it is good enough* – Dan. II B.) since I had so many days worked on it". Dan. II B.

Concerning König see also [ii, Note 7]. O. S.

33. This is simply wrong. For example, his own father made him for a long time utterly miserable, see Wolf [v, § 8] who, in particular, quoted Condorcet (§ 12)! Condorcet made the same mistake below (§ 18)and he was also completely wrong when stating that Bernoulli had been enjoying good health and was quite satisfied to live in his fatherland, again see Wolf [v]. O. S.

34. The staff of Basel University had been excluded from positions in government because of the various privileges enjoyed by the corps whose members they were; the most important of those was to be under their own jurisdiction. This already proves that the reason for that decision was not that *the noble function of teaching* was regarded as *a low and despicable profession*. Furthermore, according to the constitution of our republic no rank is actually low and despicable enough for being incapable to participate in the government.

And that decision was just as less founded on the opinion that *a gift for sciences was incompatible with a talent for governing*. This is evident because the members of a university are at liberty both to become ordinary citizens of the country once they forgo their privileges and to come back to the corps and enjoy once more the lost privileges.

Examples of academic citizens who made use of that liberty are not at all rare. We see even now two previous professors, one of them being in the lesser, the other in the great council and also holding a position at the government chancellery for which he had foregone his professorship; they may also secure a scientific position anew. Dan. II B.

The cantons had *great* and *cantonal* councils; the communes in the French speaking cantons had *lesser* councils. O. S.

V

R. Wolf

Daniel Bernoulli from Basel, 1700 – 1782

R. Wolf, Daniel Bernoulli von Basel.
Biographien zur Kulturgeschichte der Schweiz, 3. Cyclus.
Zürich, 1860, pp. 151 – 202

[1] Daniel Bernoulli, the third who earned everlasting glory for that family, was born 29 Jan. (old style) 1700 in Groningen where his father, Johann I Bernoulli, was then professor of mathematics; his mother was Dorothea Falkner[1]. On his sixth year Daniel returned to his home town, Basel, with his parents and siblings[2]. There, he attended schools soon becoming successful in his studies, so that already in 1713 he was able to obtain admittance to the Academy [to the university].

Seven teachers [Wolf mentions and briefly describes all of them] taught him philosophical and philological sciences. It was his father, however, from whom he received some instruction in mathematics, but he received likely even more instruction from Nikolaus II, his five years older brother (Wolf 1859, pp. 78) with whom he kept up most intimate friendship.

On 9 Nov. 1728 Daniel Bernoulli informed his friend Goldbach:

Nikolaus became a mathematician imperceptibly and almost against his wishes. Not that he did not like it or had not understood it quite easily, but applications each time impeded him. Perhaps he would not have at all noticed own advance since being aware of how little did it cost him, but brotherly friendship naturally compelled him to teach me mathematics although I was only eleven years old then. At first he thought to make me understand in a very short time all that he learned from his [from our] *father and managed by himself. He applied everything to make it possible for us later to study jointly, but in spite of all his efforts I remained his student to this time. Seeing himself so much higher than his student, he thus formed a sufficiently high opinion about himself for believing to be an accomplished mathematician.*

It was not yet possible to hope that during his young years, though he solved problems "demanding perfect knowledge of the new differential, integral and exponential calculus"[3], he fully understood the difficulties caused by higher mathematics even to a talented boy. On the other hand, it could have been expected that Daniel's father, already an old man and experienced teacher, will appreciate with an allowed paternal satisfaction his remarkable advance rather too high than too low and in any case try to encourage him. Nothing of the sort! The father, as it seems, conducted himself in that very way concerning Nikolaus, but, on the contrary, depressed Daniel which is clearly seen in the following example described by Condorcet [iv, § 1]:

To test the capabilities of his son, he once put him a simple problem. Young Daniel took off to his room, examined and solved it, returned thrilled with joy to report to his father and get the expected approval. The only response was, however, "Shouldn't you have solved it at once?" That answer, the tone, the accompanying gesture distressed the young man, and the memory of that first grief did not erase from his memory.

It was good that Daniel was strong enough to follow independently along his scientific route so that, as Condorcet said (Ibidem),

In spite of itself, his family obtained the honour, unique until now, we will not at all say in the history of science [why not?], *but in the annals of the world of producing three great men in only two generations*.

Indeed, in his later years he seldom received deserved recognition, and even, as we will say below, had to experience from his father such grief that could have been least of all expected to be caused by a father.

[2] After completing his philosophical studies Daniel Bernoulli was sent to stay at the house of a priest in Courtlary [in the French part of the present Bern canton] for learning French better and in 1716 he obtained the Master degree so that it was time to choose a profession. According to the opinion of his parents, he should have become a merchant, Daniel however most energetically objected. They finally agreed that he will take up medicine; Heinrich Stähelin, Emanuel König[4] and Theodor Zwinger then superbly taught it in Basel (Wolf 1859, p. 111). For two years Daniel diligently studied it in Basel without losing sight of mathematics and physics, then, in 1718, moved for further development to Heidelberg and in 1719 to Strasbourg, and only returned to Basel in 1721 to earn the doctor degree. To achieve that, he wrote a dissertation [1721/1] on the amount of air entering the lungs when a breadth is taken.

Next year, after submitting theses on logic [1722/3], Daniel unsuccessfully competed for the vacant chair of logic, then, in 1723, moved to Venice to perfect himself even more by being in contact with the famous Michelotti. Bernoulli soon won his affection to such an extent that was allowed to accompany Michelotti not only in hospitals, but also during his visits to private patients and was satisfied to be able to thank actually his teacher for such preferential treatment.

Michelotti was also a skilful mathematician and entered into scientific disputes with some of his compatriots committed to that science, especially with Riccati. Bernoulli so ably defended him that at the same time established his own scientific reputation and was prompted to publish his polemic writing [1724/4][5] and even asked to head a planned academy in Genoi. He was however unable to come to a definite decision[6] and, on the contrary, soon afterwards, when frequenting the famous anatomist Morgagni in Padua, began to negotiate with the Petersburg Academy of Sciences.

[3] Indeed, on 25 Nov. 1724 Johann wrote Joh. Jakob Scheuchzer:

My second son is offered an occupation as a mathematician in Petersburg with a yearly salary of 600 roubles, a house free of charge and a sufficient amount of firewood and enough candles. Although this is not the 2000 roubles offered to Herman [Hermann], *as he boasts, it is all the same quite honourable for a young man not more than 25 years old.*

I am sending this news to Padua, to my son where he is staying now, and I will see what he decides to do. If he follows my advice, he will accept the invitation at least if he does not find anything more satisfactory. Concerning his ability as a mathematician, I believe that he does not at all yield to Herman and that in due time he will overcome that scientist in spite of any grand idea that Herman has about his knowledge.

The "news" mentioned above caught Daniel rooted in bed taken ill with a violent fever. Considerable time had to pass before he was out of danger and able to resume his correspondence. On 25 Jan. 1725 he wrote Goldbach:

I am writing to you lying in bed, still enfeebled by high fever that failed to carry me off to the next world. Owing to my youth, I happily escaped, and, although being extremely weak, am out of danger. If everything goes in the same way, a couple of months will have to pass until I am able to get out of bed.

At about the same time he sent a letter to Basel. Anyway, while sending a copy of Daniel's book (1724/4) to Scheuchzer, Johann Bernoulli wrote him on 13 Febr. 1725:

I ought to thank you for your wishes for the recovery of my son. He really needs to be on his feet since five or six days he had been at death's door. However, thank God! He is out of danger. Last week I received a letter from him, the first since he feels himself a little better. He complains of being exhausted and does not expect to recover sufficiently to return here until May. I am sending you a copy of a small book that he published shortly before being taken ill. The book is a miscellanea [medley] *of all kinds of subjects and although he did not treat them thoroughly, you will assess what he will become capable of provided that God extends his life.*

For a long time Daniel Bernoulli was unable to decide whether he ought to accept the invitation to Petersburg and consulted in writing his brother Nikolaus, then professor of law in Bern about it. And this is what he added in the letter to Goldbach quoted above:

I have just received a letter from my brother who out of really brotherly friendship tells me that he cannot allow me to move to Muscovy and that if I absolutely wish to go there, he is prepared to sacrifice his interests (he has a chair that brings him in not less than

150 louis-d'ors) and accompany me. I think that we both can be easily found positions in Petersburg the more so since there is no field wider than the study of mathematics. If you can agree with that idea, you will acquire the merit of preventing the separation of brothers whose most intimate friendship that came into the world links them together so strongly.

[**4**] Daniel Bernoulli recovered sooner than it was hoped and to such an extent that was able to go back to Basel. And about the same time there occurred two other happy events. The Paris Academy of Sciences considered his competitive writing [1725/8] worthy of a large prize of 2500 livres; and his wish that his brother Nikolaus also receives an invitation to Petersburg came through. Soon after that Daniel decided to accept the honorary invitation, and on 11 May 1725 his father wrote to Scheuchzer:

I thank you for your triple congratulations with the return of my son, with the prize that he won in Paris, and his voyage to Petersburg.

And here is Daniel himself writing to Goldbach on 13 June 1725:

Since I know that you are involved with everything concerning me, I ought to tell you that I have accepted the chair of mechanics which they were kind enough to offer me with a yearly salary of 800 roubles and 300 German écus[7] *or 450 florins for the voyage expenses. I confess, however, that I did not decide to agree without plunging into terrible mental strife. Sometimes I vividly imagined the advantages to be expected from such a honourable invitation, and sometimes the sacrifices I will be making by accepting it. Finally ambition won the battle.*

I am mortified that the business dragged on for so long and hope that the reason for the delay will not at all be attributed to me. I never caused any difficulties about the conditions accompanying the offered chair; at first I even thought that the initial offer of 600 roubles was too generous and far above my little worth which I am still lacking. Even if I would have been rather presumptuous to ascribe it to myself, my soul is not mercenary enough to justify such pretensions. My first motive is always ambition rather than interest.

I (Wolf 1859, pp. 101 – 102) have already noted that the brothers safely arrived in Petersburg in the first days of September 1725. They were warmly received and began their scientific life spiced by being together. Regrettably, however, their happiness was short-lived: already on 20 July 1726 Nikolaus died (Wolf 1859, pp. 102 – 103) and the emerged void coupled with the influence of the inclement weather on his own delicate body made Daniel's life in Petersburg unhappy and he often cursed his departure from Basel. He wrote Goldbach about it even in 1729:

My fatal departure from Basel had cost me a brother; I would have wished to expatiate that by all my possessions and my blood.

Although in 1727 the lot for professorship of physics in Basel was unfavourable for him, he hardly wished to await the passing of the five years, for which he had committed himself, before submitting his resignation. Then, however, all possible was certainly done for persuading him to stay; not only was it agreed to raise his salary, but he was permitted to withdraw at any time retaining half of his salary as a pension. He promised to stay for a while but continued without interruption to look for a position in Basel.

[5] And so, in 1731, Bernoulli competed for the chair of logic but without success. On the contrary, in 1732 things went better for him when the young Emanuel König (see Note 4) was promoted to professorship of medicine thus vacating his chair of anatomy and botany. The Regent selected Joh. Jakob Huber, Benedict Stähelin and him for that position and now, finally, the lot favoured Bernoulli.

Daniel's younger brother Johann[8] II decided to pick him up and departed to Petersburg in autumn (Spätjahr) of the same year[9]. […] A stormy sea voyage to Danzig apparently influenced many later contributions of Daniel. […]

The mathematical prize problem for 1733 was [see the title of Bernoulli's contribution [1735/24], but no competitive writing had then won the prize. The same problem was announced for the next year with a double prize being set and Johann I and Daniel Bernoulli shared it, see Wolf (1859, p. 93) and the main text below. I do not know whether Daniel had also submitted a solution in 1733. On the contrary, it is said that during a meeting of the [Paris] Academy, when the several competitive pieces were being presented, many academicians fixed their eyes at the famous brothers Bernoulli so as to decide judging by their expression whether they had participated, but did not notice even slightest changes.

In Paris, a lot of time and even most of it was spent on visiting Maupertuis, Clairaut, Mairan, Fontenelle et al and receiving them; I can only add two following notes about that stay. It is said about 8 October: *We visited* […]. *He is a good friend of* [Chr.] *Wolff. He told us that the King of Prussia earnestly asked him to return to Halle*[10]. […]

We saw Fahrenheyd [Fahrenheit][11]. *My brother presented him a thermometer made by Delisle*[12]. […] *He supposed that the temperature of boiling water was always the same rather than changed with the air density*. […]

[In The Hague they were shown the place where De Witt was murdered[13].]

[Eight years ago the Russian envoy of that time in Prussia told them that Delisle was presented to the King of Prussia.] *The King asked him "Well, Mr. Delisle, what news are there about the Moon? You have empires and kingdoms there, isn't it true?" "Yes, Your Majesty". "And to whom do you give these kingdoms?" "Your Majesty, to those who are witty enough and know it since not being too ignorant"*. […]

[For three weeks the brothers stayed in Paris and visited Maupertuis[13].] *We began talking about Delisle and Maupertuis told us that he is not too pleased that Delisle is staying so long in Petersburg;*

that for a year now, he is keeping a place for Delisle instead of the deceased Louville and that it is desirable that Delisle returns back as soon as possible to occupy that place since he knows well enough that no one is more worthy of it.

We visited the Academy; there were about 25 members present at the meeting: Maupertuis, Mairan, Réaumur, Camus, de Fontaine, de la Condamine, Godin et al. Since it was the last sitting before holidays, the 28 competing pieces were distributed for examination among five commissioners. During the meeting there arrived one more piece and a question about whether it should be received had arisen. After some debate it was resolved to receive it, and to receive in future all those that are presented before the dissolution of the last sitting. […]

Together with Montmort we went to Miss Ferrant who in the old days had known my late brother [Nikolaus II whom Nikolaus I had recommended to Montmort – R. W.]. *That young lady understood mathematics and showed us physical experiments* […]. […]

Together with Maupertuis we saw the observatory, then dined at his place with de la Condamine who asked Maupertuis to acquaint him with us.

Before arriving in Metz we found out that one of those accompanying us during the journey was the botanist Trant, a member of the Academy from Paris.

While travelling in a carriage, Trant entered into a merry dialogue with Daniel Bernoulli and in the course of a scientific conversation asked the name of his companion. The answer was *Daniel Bernoulli*. Trant thought himself deceived and countered: *And I am Isaac Newton*. Bernoulli, however, proved that he was not jesting by showing the addresses on letters he kept by himself, and then Trant acknowledged that he was Trant. […]

[**6**] On 12 October 1733 the brothers finally arrived in their home town and Daniel never left it afterwards for a long time. On 26 October 1735 he wrote his friend Euler: "For my part, concerning my health I became, so to say, a different man since savouring our good Swiss air".

For the sciences, this revival occurred for the best. Not that Bernoulli had given much over by being engaged on both sciences entrusted to him[15]; almost all efforts he devoted to the great work with which he as though opened up a new field of application of mathematics where his name will shine forever, the *Hydrodynamica* [1738/31]. Already on 17 July 1730 he wrote to Goldbach

I have completely plunged into water which is my sole occupation and for some time now renounced all not belonging to hydrostatics or hydraulic.

From that time onward Bernoulli almost exclusively abandoned himself partly to his investigations and experiments made in Petersburg, and partly to the final preparation and editing of that great work accomplished in Basel. By the summer of 1734 he advanced so much that could allow the printing to begin. On 25 August of that year, in a letter to Professor Schöpflin then published [1734/2002, pp. 87 – 90, in original French; pp. 20 – 24, in an English translation], he provided the first public and very interesting report about his enterprise which, as I believe ought to be inserted here in full the more so since even here, in Switzerland, the journal containing it became rather rare.

[The original French text of that letter was indeed reproduced here.]

[**7**] It can be imagined how curious became the scientific world about that announcement of the [forthcoming] appearance of that work. However, although Bernoulli wrote Euler on 18 Dec. 1734 that his *Hydrodynamica* "is indeed being published by Mr. Dulsecker who will give me 30 copies and 100 thalers in recompense" [give as royalties], that process encountered various difficulties which were only surmounted by 1738 when the book was given over to the bookshops.

Thus the proverb "What lasts long finally comes out good" was confirmed and Daniel had the pleasure of seeing that his work was nearly everywhere received so warmly that he almost decided that it was too warm. On 9 Aug. 1738 he wrote Euler about it:

That work of mine must have been very favourably received provided that I ought to trust even most slightly the letters I have from everywhere. However, since flattery nowadays passes for politeness, I do not know how much delighted I should be by that approval. Your testimony would have certainly replaced for me all the others since I recognize both your friendship and naturalism.

I have not found Euler's answer, and neither do I have the mentioned letters[16]. On the contrary, it is easy to compile numerous later testimonies which attest to complete recognition of Bernoulli's work. Here are only a few examples. Condorcet [iv, 8]:

Bernoulli published only one great separate contribution, his celebrated treatise Hydrodynamica. The theory of the movement of fluids had occupied the most illustrious geometers of the 17th century, but their efforts were barely useful for something more than a better understanding of the phenomena that needed to be explained, of the questions that ought to have been answered, and in the first place of the difficulties encountered.

Daniel Bernoulli earned the glory of being the first to provide that theory in a general manner and according to the principles which if not rigorous at least seemed only to deviate slightly from the truth. One of them is the principle of conservation of live forces only subjected to exceptions when the law of continuity ceases to take place in the phenomena. The second principle consists in separating the flowing fluid into parallel channels and supposing that a common movement with the same velocity and direction takes place for all the particles in each channel.

Then, after explaining the contents of the *Hydrodynamica* in more detail, […] Condorcet goes on:

Some questions treated by Bernoulli had apparently eluded the principles he applied; however, he was able to return to them by equally ingenious and plausible physical considerations and so skilfully that it seems to stand up to a miracle.

And to conclude he adds:

This contribution will always be regarded as one of those monuments that constitute an epoch in the history of sciences.

The Abbot Bossut who himself later wrote an excellent work on hydrodynamics, and whose verdict weighs therefore twice more, expressed himself in the following way:

The theory of the outflow from openings of any size always remained imperfect until Daniel Bernoulli in some fortunate essays subjected it to a general and rigorous calculus admitting some hypotheses sufficiently conforming to experience. […] *He arrived in a very simple and elegant manner at the appropriate equations and applied general formulas to many particular and practically useful cases.* […]
Daniel Bernoulli shows sagacity of a physical geometer, attentive and accustomed to follow the course of nature. <u>For him, calculus was never anything more than a necessary tool rather than a vain display of purely theoretical formulas</u>. However the science of the movement of water has advanced from the time when Daniel Bernoulli's <u>Hydrodynamica</u> had appeared, fair-minded posterity will always reckon that work among the most elegant and wise products of human genius.

On 21 Nov. 1778 Jeanneret (Wolf 1859, pp. 213 – 214) wrote his friend Jetzler:

I have read the <u>Hydrodynamique</u> of the Abbot Bossut [1771]. *What pleases me much is to see that he justly estimated Dan. Bernoulli's merits, see his Preface, pp. 11 – 13. He* [D. B.] *is wiser than Euler, D'Alembert et al, he never plunges into analysis like those scientists so as to arrive at the horrible formulas whose appearance makes you tremble, as D. Bernoulli told me, and which in addition are useless. And the Abbot Bossut also indicates in his Preface: "Bernoulli often told me to defy all complicated formulas; he thought that nature is very simple and should not be led somewhere, and if that is done, it is because the calculation was based on wrong hypotheses*".

Finally, the great Lagrange (1811 – 1815, [t. (?)], p. 242), after introducing and applying the Johann Bernoulli's principle of conservation of the live force, says:

Daniel Bernoulli then extended that principle further and deduced the laws of the movement of liquids in vessels, a subject that was *previously only treated vaguely and arbitrarily. Finally, he* [1750/43] *showed in a very general way how was it possible to apply it to the movement of bodies being under the influence of some mutual attractions or attracted to fixed centres by forces proportional to some functions of distances*.

And then, again bearing in mind Bernoulli's *Hydrodynamica*, he (t. 2, p. 282) calls it "A writing that sparkles with an analysis as elegant in its course as simple in its results".

[**8**] Whereas Daniel Bernoulli's scientific reputation spread ever wider, he had, on the contrary, not only received no clear recognition from his father, but, as we intimated above, even had to encounter from Johann much that painfully affected him, and still more painfully because reverence for his father did not allow him to defend publicly his just rights and at most he could have only unburden himself in letters to trusted friends.

During his last years Johann Bernoulli had overcome himself and recognized Euler as a mathematician of equal worth and greeted him [greeted his works?], but he was unable to excuse his son for daring to surpass him in some instances. That in 1734 he had to share the prize of the Paris Academy with Daniel especially wounded his pride (Wolf 1859, p. 93)[17]. Condorcet [iv, § 12] noted:

Johann Bernoulli sorrowfully saw that in a sense his son became his equal as judged by a society whose favourable decision he himself had so many times aspired to and deserved. Paternal love, the strongest and perhaps the least personal of all that people can experience, yielded in his heart to his indignant glory. Little touched by seeing his family obtaining by that sharing a still unparalleled honour, insensible to the pleasure so sweet for a father to feel that his son is worthy of him, he only saw that son as a rival, and his success only as lack of respect with which he for a long time bitterly reproached Daniel[18].

There were perhaps other causes as well for that mood because his son's piece was better than his own and Daniel had imprudently hinted that he indeed thought so and his father was unable to conceal from himself that that opinion was justified. Finally, the son dared show himself as a Newtonian and abandoned Cartesianism still only supported by the name of Bernoulli. Daniel Bernoulli's admission was the last triumph previously lacking in Newton's glory that Daniel's father had the misfortune of struggling with all his life.

P. N. Fuss (1843) expressed himself still sharper:

The exorbitant jealousy of Johann Bernoulli, that formerly led him to the famous dispute with his elder brother, manifested itself concerning his son Daniel in quite a staggering manner, and, as it is even possible to say, in a manner contrary to nature, and to such an extent, that, being unable to struggle against an enemy so young and so powerful, he finally became guilty of plagiarising Daniel.

And this statement brings us to that which pained Daniel most of all. On the occasion of the publication of his *Opera omnia* [1742] (Wolf 1859, pp. 94 – 95), his father not only borrowed without acknowledgement many findings of his sons, and especially of Daniel, but appropriated much from the *Hydrodynamica*. On 4 Sept. 1743 Daniel voiced his complaint to his friend Euler:

My father had at one stroke robbed me of my entire Hydrodynamica even a jot of which, to say the truth, I do not owe him and I am thus all at once losing the fruits of ten years of work. All the propositions are taken from my Hydrodynamica but my father nevertheless called his contribution <u>*Hydraulicam now in 1732 discovered for the first time*</u> *whereas my Hydrodynamik*[19] *was only published in 1738.*

At first that was almost impossible to bear; finally, however, I accepted it all resignedly, but began feeling disgust and contempt for my previous studies. It would have been better for me to have learned the trade of a cobbler than to become a mathematician. And since then I was unable to resolve to prepare something mathematical.

[**9**] Apart from the troubled relations with his father, Daniel Bernoulli was also depressed by his position that prevented him from devoting all his time and strength to studies for which he was born and in addition offered him little comfort.

On 12 Dec. 1742 he morosely wrote to Euler:

My duties only allow me to consider mathematics in a supplementary way; and besides my weak aptitude for mathematics is so terrible that it is just exhausted and in spite of myself I ought to abstain from any meditations.

In subsequent letters he repeatedly stated that he could have decided to move to Berlin or once more to Petersburg. Concerning Berlin, formal negotiations with him indeed took place many years previously. On 24 Dec. 1740 Maupertuis wrote Friedrich II:

Brothers Bernoulli, geometers from Basel, are the two provinces that Your Majesty were unable to conquer. It will not cost you more than two thousand German écus for one of them and one and a half thousand for the other. Being more charmed by the pleasure of serving Your Majesty than flattered by the attached recompense, they are much disposed to be established in Berlin.

With them, whom we will soon get hold of, with Euler, whom we already have, Monnier, whom I have in view for astronomy, and me, with my zeal for serving you more than with my talent, – taken along with those illustrious men I already see Your Majesty's Academy more powerful than any other in Europe.

However, that confidence expressed by Maupertuis notwithstanding, the negotiations with Berlin broke down as did later talks with Petersburg although Bernoulli would have been really glad to live in a more scientific atmosphere than he found in Basel. On 4 Jan. 1746 he wrote Euler, who had once more urged him:

Gratitude for the token of your so real friendship with me does not allow me to put of my answer. My sensitivity to that matter is so much greater because I am living in a country where neither friendship nor science are known. Without my old parents holding me back I would have striven at any price to spend and end my life by such a good friend. Here, I have neither spare time nor the slightest opportunity to contribute something fostering real science[20].

And on 22 Sept. 1747, again to Euler:

Since for me the present situation in Basel is revolting beyond all measure, your latest letter prompted me to deliberate with my father about my vocation in Petersburg. It occurred, however, contrary to all assumptions that he had most strongly cautioned me against it and as though implored me not to carry out such changes during his lifetime that will soon end. He also added that at my age I should not go at all or go forever with bag, baggage and belongings, and that it is indeed better to wait beforehand for my future inheritance which cannot be for long anymore.

And when, on the next New Year's day, the father had indeed departed this life, the attempts to persuade Bernoulli unreservedly to move to Petersburg, reinforced by the above, have resumed[21]. Nevertheless, on 9 March 1748 he wrote to Euler:

Concerning the question whether now, after my father's death, to accept the invitation to Petersburg, I swear by my honour that, even if I still had a strong desire to move, I would not be able to do it. For some time now, I am very sickly and unable to carry out my present duties not to mention enduring such a long journey and living in such an inclement weather[22].

[**10**] It is also possible that at that time Daniel secretly hoped that he will be invited to succeed his father, but on that point he was greatly mistaken. He could have been elected to the Paris Academy [and was indeed elected in 1748] although Euler was still not, and [the friends of] Gabriel Cramer (see the next biography in this my collection) who was then staying in Paris actively campaigned for his election to one of their eight seats for foreign members, but the academic senate in Basel kept to the usual rut. The chair of mathematics was put out for public competition although it could have been indeed thought that Bernoulli will not participate there.

For excusing the senate it can be stated that it was required to do so by law, but, nevertheless, professorships had already been often offered without invoking competition[23].

Daniel had possibly thought about that because in the spring of 1748 he wrote Euler:

I was prevailed upon to enter the board (collegium) *of experimental physics instead of Dr. Stähelin who is very ill and helpless. The change brought about huge popularity; I always have more than a hundred students, and the duties take very much time so that I intend to continue not later than until the end of this summer*.

By offering Bernoulli that position [the senate] had compensated the injustice done him in 1727 by the unreasonable lot [§ 4]. However, if that indeed were the case, the senate should have also offered the chair of mathematics to Johann II Bernoulli whose achievements had by that time proved him an unusually excellent mathematician[24] but who just the same was out of place as professor of eloquence.

That did not happen either and out of politeness Johann certainly could not have participated. Only when the lot for the chair of mathematics fell to Jakob Christoph Ramspeck, quite worthy (wackern) in other fields but unsuitable for that position, a change was arranged so as to prevent a scandal in the scientific world. Ramspeck took over the professorship of eloquence, and Johann II got the chair of his father.

Daniel saw [had to watch] all that until 1750 when Stähelin died and without elections being held he was finally transferred to the chair of physics[25] improved by appointment to the college of ecclesiastics at St. Peter's[26].

[**11]** Partly prompted by his contributions on the prize problems of the Paris Academy (see below), Daniel gradually prepared a series of extremely valuable writings on various parts of mechanics and physics related to the *Hydrodynamica* and was able to apply higher mathematics so skilfully without departing from nature[27] or abandoning himself by playing about with analytical methods. He is therefore rightfully considered to be one of the first founders of real mathematical physics, and his works are among the finest adornments among the Petersburg, Paris and Berlin academic contributions.

It would have nevertheless led us too far afield to touch on them separately although they deserve it; Condorcet [iv, § 4] was quite right about them:

However, had any one of them been the only contribution of an author, it would not be sufficient for considering him a man of genius.[28].

We ought to restrict the description to some general opinions and a few details. At first it is desirable to follow Jeanneret's letter to Jetzler dated 26 April 1774:

Considering the Petersburg Commentarii, I also wish very much to get them, they contain excellent pieces. Among others, you will read there Daniel Bernoulli's memoirs. It seems to me that no one of our scholars treats physico-mathematical sciences more sensibly, more exactly and better conforming to nature which he always carefully examines whereas others mostly do as they please, calculate accordingly and nature ought to conform to them. And they are not *worried at all since their meditations do not leave paper. Have a look: does any of them except Dan. Bern. make experiments to check whether nature agrees with the theory.*

Each section of his *Hydrodynamica* *is followed by experiments that he made for confirming the results of his calculations. It ought to be admitted that anything not treated in such a manner and often only based on rather arbitrary principles should be rather useless. After considering all the available books it must be recognized that most of them are very little beneficial for practice. The best of them do not demean themselves by descending into details. Actually, that would not be dishonourable because applications are often more difficult than those infinitely general formulas.*

Bern. rather often mocked those people when he saw them generalizing, and among them we may name D'Alembert, because the envy tormenting them and wishing them to accomplish things better than any other geometer, often committed them to modify the subjects they treated only to make a show of providing an advice by saying that here is something not general enough or that there is a mistake since a certain magnitude was neglected[29].

[**12]** Concerning Jeanneret's occasional remark about D'Alembert it seems to be in order to hear Bernoulli himself about him especially since his opinion about that man is very typical and his correspondence with Euler offers many appropriate opportunities. On 7 July 1745 he wrote:

During Maupertuis' latest visit to Basel, he repeatedly brought up the young D'Alembert as a miraculum miraculorum who published [contributions on] *mechanics and Hydrodynamicam. I had finally told him that at the age of twenty it was impossible to comprehend all the principles of those sciences and even to attain wonderful findings. Meanwhile, all this prompted me to get hold of those writings and I was surprised to see that apart from a few details his Hydrodynamica was nothing but impertinent complacency. His criteria are sometimes simply infantile and indicate that he is not and even never will become extraordinary since his presumptions are much too great for learning something from others and his own insight much too weak for attaining the same by himself*[30].

That certainly somewhat curt opinion, especially coming from the otherwise peaceful Bernoulli, had gradually more and more modified as he became better acquainted with D'Alembert's writings and saw that Euler stood up for the young scientist. Thus, on 4 Jan. 1746 Bernoulli wrote Euler:

In pure mechanics he showed himself as a highly educated man, but, once there occur some physical or metaphysical reflections, everything becomes simply infantile.

And on 29 June 1746:

I understand from Mr D'Alembert's Hydrodynamic that in applied mathematics as far as quantities are concerned he is weak. He claims to have deduced difficult integral formulas for the force and direction of winds in all climates and seasons. I can only say that those are words that will bring about more shame than honour to mathematics. I believe that because of the Hydrodynamica everyone will ridicule him. I would treat him like you treated Robins and much more admire his real merits taken by themselves rather than stress his absurd complacency that I attribute to his youth especially since I foresee that even in which he now lacks he will become a greater man.

On 9 July 1746: "Perhaps since the publication of his *Hydrodynamica* Mr D'Alembert had some more perfected himself in physics".

On 3 Nov. 1746: "If Mr D'Alembert decides to come to Berlin, it will be a great acquisition for your Academy".

On 16 August 1749:

For me, Mr D'Alembert carries little weight in physics and mechanics and in physical hydraulics he childishly reasons against all experiments. In spite of all that, I deeply and sincerely respect him and foresee that with age he will in essence replace [abandon] *the blunders of his youth.*

Nevertheless, over the years Daniel Bernoulli's main opinion about D'Alembert remained essentially unchanged, and even on 26 January 1750 he summarized it in a letter to Euler as follows:

I consider Mr D'Alembert a great pure mathematician. However, when he invades applied mathematics, all my esteem comes to an end. His Hydrodynamica is much too infantile for me to esteem him somewhat concerning such subjects. […] *His piece about the winds says nothing and who reads it through, knows not more about them than previously. I think that what is needed is physical demonstration rather than abstract integration.*

A pernicious taste begins to creep in so that real sciences suffer much more than advance, and for real physics it often would have been better had there been no mathematics in the world[31].

Daniel Bernoulli's common sense rightfully opposes mathematical treatment of physical problems with an urge towards ensuring almost nothing except great development of analysis, as it also occasionally happened when Euler entered applied mathematics. Bernoulli wished that mathematics be only introduced into physics as an auxiliary tool and only for the sake of that science, and in such applications of mathematics none of his contemporaries surpassed him. As Condorcet [iv, § 15] put it with his inherent talent of presentation:

In the essence of the problem itself, Bernoulli attempted to find the means for simplifying it, for reducing it to its simplest form only leaving for analysis that which cannot be taken away from it. It is seen *that above all he wished to apply theory for penetrating nature; to use mathematics not only in speculative mechanics, in studying the laws of abstract movement of bodies, but in physics, when examining phenomena in the universe in their real states, and according to the manner in which observations presented them.*

No one discovered more analytical means for subjecting to calculation all the circumstances of a phenomenon, no one was able to arrange better an experiment to apply it either for confirming the results of a theory or for serving as a basis for calculation. He was invariably a philosopher and physicist as well as a mathematician.

Perceptiveness seems to have been the dominant quality of his mind and he possessed it to such a large extent, and applied it so fortunately, and it served him so well, that in essence it became majestic and led to admiration and surprise that seem to be reserved for miracles caused by the force and depth of genius.

[**13]** As stated above, the prize problems set by the Paris Academy led Daniel Bernoulli to some of his best investigations, and it is therefore opportune to discuss separately his crowned papers if not done above. He earned his first prize in 1725 and the second one in 1734, see §§ 4 and 5 above. It seems that Euler had commented not really favourably on the contribution based on that second problem. At least on 25 Jan. 1737 Bernoulli wrote him concerning that issue:

Your opinion about my crowned piece would have strongly mortified me had I not realized that you have only read it superficially and in great haste. I never thought about moving the plane of the solar equator for putting in order the [planetary] *inclinations and* [their] *eccentricities.*

I only remarked that, since the [position] *of that plane is still uncertain, it is not unreasonable to find out where should it be placed for the arithmetic mean of all the inclinations to become minimal. And that I have investigated, and do not regret it. I can assure you that, after studying all my correspondence, I have concluded that my piece ought to be almost the best of my contributions*.

And after Euler had again remonstrated, Bernoulli wrote him on 16 March of the same year:

You are telling me that it is clearly seen that I had compiled my letter in a hurry; however, I also see that you had glanced at it hastily.

It is highly probable that Euler was unable to appreciate properly Bernoulli's contribution because, in particular, it was written in Newton's spirit[32] whereas at that time and even many years later, although beginning to base himself gradually on particular occasions on Newton, in general he still kept to the Cartesian system[33]. This follows from Bernoulli's letter to him dated 4 Febr. 1744:

I believe that the ether gravitates towards the Sun like the air does towards the Earth and cannot keep back from you that concerning that *issue I am a pure Newtonian and am amazed that you are adhering to the Cartesian principles for so long. Here, some kind of a passion likely creeps in.*

Since God was able to create a soul whose nature is incomprehensible to us, He could have also established universal attraction of matter although such attraction is above our understanding. On the other hand, Cartesian principles always involve something over our heads.

And in his later letters to Euler Bernoulli had again been discussing this subject until finally Euler completely reconciled himself with the gravitational theory. <u>To Bernoulli undisputedly belongs the most honourable title</u> [merit] <u>of getting rid so early, already in the first half of the 18th century, of the generally circulated prejudice against the Newtonian theory, and of assisting in the gradual recognition of that theory on the Continent</u>.

[14] In 1737, Daniel Bernoulli shared his third prize [1738/28] with professor Poleni from Padua for investigating the best way of examining anchors. At the same time his brother Johann was crowned for studying the best form of anchors.

In 1740 Bernoulli shared the fourth prize for the problem about sea tides with Euler, Maclaurin and the Cartesian Cavalleri. On 30 April 1740 he wrote Euler that

The prize is divided into four parts one of which is awarded to you; another one went to Maclaurin, the third part, to an unknown Cartesian, and one granted to me. I am informed that no other competing contribution resembling those three had been ever sent to Paris. It was not wished [my correspondents did not wish] *to praise the fourth paper and its only merit could have possibly been in that it was not antiCartesian.*

Bernoulli got the fifth prize in 1743 for his contribution [1748/41] that Condorcet [iv, § 13] described as the work "in which he displayed most perceptiveness and wit". Bernoulli continued to busy himself actively with that subject and Euler had also studied it in a work sent to Paris but only awarded an accessit [honourable reference]. Bernoulli's ideas prompted the mechanician Johann Dietrich in Basel, who attained a good reputation for producing physical instruments and in particular manufactured excellent artificial magnets, to turn out a considerable number of dip needles thought to be quite superb. Indeed, on 24 June 1755 Euler wrote him:

Two days ago I received by post your dip needles and had to pay 10 reichsthalers and 10 groschen postage and in addition 2 reichsthalers and 12 groschen excise. However, I find that that instrument is so excellent and will recommend it accordingly to the Academy and hope to gain for you more than 15 louis-d'ors for each after deducting my expenses.

No local mechanician will be allowed to see your instrument even though not many of them should be afraid of. These days I carried out with greatest pleasure all kind of experiments with this instrument and reliably concluded that here in Berlin the inclination is 71°30'.

[**15]** The sixth prize for answering the question about the physical cause of the magnet Bernoulli shared in 1746 with his brother Johann, Euler and the French physicist Du Tour. That question was vainly asked in 1742 and 1744 and once more in 1746 with a triple prize promised.

Maupertuis, who just then was in Basel, insistently asked both brothers to deal with that question. Daniel explained that he had already wrote down some appropriate ideas but was not satisfied with them. Then, half-jokingly, he told Johann that he was prepared to give him that work for continuing it and to share with him the feast should the outcome by happy.

Johann agreed and the result was unexpectedly successful. On 29 June 1746 Daniel informed Euler about that:

I congratulate you with getting a part of the Paris prize. Contrary to all safeguards I have also obtained its part together with my brother. Should I be flattering myself even in the slightest that people will deliberate so much on our ideas, I would have worked them out better. The whole piece is hardly 32 pages long[34] *and only considers some main phenomena, but the ideas are quite new and can indeed be somewhat useful in physics.*

In 1747 Daniel [1750/42] shared his seventh prize with someone whom he mistakenly suspected to be Euler. On 29 April 1747 he wrote the suspected author:

I have just heard from Paris that I was awarded half the double prize for this year and the other half to a writing attributed to you. Had you competed, I would not at all doubt it and would have sincerely congratulated you beforehand. I was luckier in Paris than in Berlin.

In spite of that, I hesitate to continue competing; I fear that my luck can finally lead to bad consequences since the scientific public will look for some bias although I conceal myself as much as possible [by using mottos].

Nevertheless Bernoulli competed many times more and in 1751 he got his eighth and doubled prize [1769/44]. The ninth prize was awarded him in 1753 [1769/47] and finally, in 1757, he earned his tenth prize [1771/48]. Had Euler, who earned twelve prizes completely or partially, lived in his home town, the Paris Academy, which had been making almost superfluous efforts over many decades for studying the competing mathematical contributions, could have without committing any great mistakes transferred all its prizes to Basel.

[**16**] Already the mentioned solutions of the prize questions of 1725, 1734, 1740 and 1747, and above all his attitude towards the gravitational theory (§ 13) would have put him among the most deserving men in astronomy, but there is something else to be said not to be found in J. A. Mallet (Wolf 1959, pp. 249 – 268).

Thus, in 1728, as a member of the Petersburg Academy, he discussed there Delisle's report about whether the real system of the world and the rotation of the Earth can be established solely by astronomical facts (Delisle 1728)[35], and he published many valuable astronomical memoirs[36]. Neither was he completely alien to practical astronomy; we know from von Zach (year?) that among other mathematical and astronomical books from Daniel Bernoulli's literary estate he bought a rare complete collection of the periodical *Connaissance des temps* from 1679 onward and found there, in the volume for 1679, a small piece of paper in Bernoulli's handwriting providing a complete observation of the total lunar eclipse of 26 March 1736.

However, he had no means for regular observations, and, being "confined to a country where you hardly hear astronomy mentioned", as he had expressed himself in a letter of 1769 that I have published in 1853 in the *Berner-Mitteilungen*, and having no external stimuli. It is also worthwhile to mention that in 1779 his repeated representations that the city [tower?] clock, which for a long time (it was thought since the Basler-Concil [of 1499]) at 12 noon showed 1 p. m., was brought in accord with the rest of the world.

That was a change not at all easy to be introduced which in particular caused the appearance of a cartoon still kept in the collection of pictures at Basler-Antistitium and showing Bernoulli pushing back the [hour] hand of the clock from 1 to 12. Merchants had been supporting his efforts but, on the contrary, tailors, cobblers and others attempted to prevent the unpopular innovation whereas the municipal authorities were helpless.

[**17**] And finally it is also possible to include Bernoulli's memoir [1778/72] in which he proved to astronomers that in most cases the choice of the usual arithmetic mean was inadmissible but that […][37] And that contribution leads us to discuss many more applications of the theory of probability to civil life studied by Bernoulli. In 1760 he submitted his memoir [1766/51] on inoculation of smallpox and essentially strove to introduce that procedure in his home town and he forwarded memoirs on the duration of marriages [1768/56] and on similar subjects to the Petersburg Academy.

Statistical studies of various kind had also stimulated him[38]. For example, in July 1764 he wrote to Dr Hirzel in Zürich:

How useful it would be to find out the proportion of the diseases that carry men off in various climates and thus to determine properly those that are endemic, i. e., those that in a given place attack and kill more often than elsewhere[39]*. It seems to me, for example, that in our Basel more people come down with stroke. Should that conjecture be justified, we ought to think about the relevant cause and really investigate everything.*

Here, however, it is necessary to note that all cities whose inhabitants are not too numerous and in addition belong to unchanging families, will after a long time be afflicted with a hereditary and predominant situation. In our city we have worthy families suffering from strokes more commonly than others. Here also there are many bent and hunch-backed inhabitants whereas I do not remember noticing anyone of such people in the populous Petersburg.

It had not probably ever happened that a foreign member of the Paris Academy did not at the same time belong to other scientific societies as well, and Bernoulli added to that membership many others. He was on the list of the Berlin Academy since 1747; of the Royal Society, from 1750; of the Economic Society in Bern, since 1762; of the Physical Society in Zürich, from 1763; and of the Mannheim Society, since 1767[40].

Accordingly, his correspondence was rather extended, and the names such as La Condamine, Bouguer, Clairaut, Maupertuis, Lalande, Buffon, Lambert, Leonhard and Joh. Albrecht Euler, Lagrange, Mallet (Wolf 1859, pp. 249 – 268), Jallabert testify that it was scientifically highly interesting. Many of the letters are published; P. N. Fuss (1843) included 37 of them to Goldbach (1723 – 1730), 58 to Leonhard Euler (1726 – 1755), 5 to Nic. Fuss (1773 – 1778). I published passages from various letters to Mallet, etc.

[18] Nevertheless, a large part of the correspondence was not yet studied and at best being kept by someone unknown. After Bernoulli's death the whole correspondence certainly passed to his nephew Johann III. Indeed, on 1 Nov. 1796 Scheibel wrote from Breslau to Kästner in Göttingen:

In August, Dir.[ector of the mathematical class of the Berlin Academy] *Bernoulli from Berlin suddenly visited me. An exceptionally good-natured man, about whom it is only to be greatly regretted that in his youth, when diligently making* [astronomical] *observations during a cold winter, he so seriously weakened his hearing that has to use all the time an ear-trumpet. <u>He has Daniel Bernoulli's correspondence</u>.*

I suggested to him to have it published in the format of the <u>Commerc. Epist. Leibn. Bern</u> with explanatory comments, [but only] *by a foreign publisher (Lausanne, Geneva etc.) and not by subscription. He went from here to Oels, to the Duke* [Ernst] *with whom he was well acquainted in Berlin.*

And indeed Johann III himself had already previously thought of separately publishing at least some of the letters since in vol. 2 of Lambert's German correspondence that he edited (Lambert 1781 – 1784), he wrote quite clearly:

Lambert's important correspondence with my uncle, Mr D. Bernoulli, will appear in the first volume of the French scientific correspondence.

Regrettably, however, that volume had never been published and my inquiries by letters and travels made in 1847 and 1848 about the entire correspondence were unsuccessful (Wolf 1859, pp. 87 – 88). Only in November 1858, when Johann Bernoulli's biography was already published, the Pulkovo astronomer Wagner notified me that among the enumeration of the purchases for the Duke's library in Friedenstein [a castle in Gotha], was a note: "1793, 7 Dec., 860 thalers to J. Bernoulli in Friedrichsfelde near Berlin", and "1799, 26 July, to Director Bernoulli in Köpenik[41] 300 luis-d'or for his father's manuscripts, about 100 volumes and convolutes" (Beck 1851).

And there perhaps are also some of the lost letters. After that I turned to Professor Habicht in Gotha who on 22 Dec. 1858 was kind enough to offer me more detailed information and even a survey of the available letters. It indicates that apparently the Gotha collection mostly includes the brothers' letters to their correspondents, perhaps mostly in copies and extracts.

Johann I Bernoulli is the best represented but most of his main correspondents (Wolf 1859, p. 87) are either not included at all or only a few of their letters are present. Nikolaus II: a few letters to Daniel; Daniel: letters to Johann II, Gabriel Cramer, Euler, Lambert, Fontenelle and some other correspondents, although again not many letters; Johann II: a few letters to Maupertuis, La Condamine, Lambert, Kästner, Formey et al; and finally Jakob II: a few letters to Johann III.

It is highly desirable that that collection which must certainly offer many interesting exclusions from the history of mathematical sciences [many changes in …] would be published in expedient extracts. Still more desirable however would be the discovery of the rest of the Bernoullis' correspondence since the Gotha collection rather clearly proves that Johann III, who is known to have been always lacking money, after failing to sell the family superb collection of letters in full, separated it and that Duke Ernst could have only acquired the rest because the main part of the collection was already sold [to various buyers].

[**19]** To conclude, some details of Daniel Bernoulli's character, of his last years and death. Daniel, as his appended [to Wolf (1860)] very successful portrait already indicates, was a charming, gentle and friendly man, and, besides, a good companion who not only possessed the rather often gift of narrating, but also a talent of prompting others to speak out. And it should not be doubted that he would have become a good husband and father had he been able to decide, as he put it, to "expose [myself] to the danger of at once losing freedom and peace".

He kept to simple and pure moral values without avoiding the pleasures of life. He was charitable, but mostly secretly[42], religious but not overly pious. Especially in his later years he did not at all receive, or received very reluctantly, those strangers who considered him remarkable and wished to see him. On the contrary, he liked very much the visits of his friends and colleagues and never allowed them to feel his high scientific position. Thus, on 27 July1772 Jetzler recounted an episode in a letter to his friend Jeanneret:

When passing through Basel, I had the pleasure of seeing the brothers Bernoulli whose politeness, especially Daniel's, had quite charmed me. Actually, I find that that great geometer is the most friendly man about town. I could have told you much since you know him enough. Dan. Bern. went to see me where I was living. My God! I was surprised since it was too much for a math. of the first rank to visit a man who began to deserve a place for himself in the last rank. That honour would have much flattered me had I deserved it, but shouldn't I blush?

He willingly assisted beginners in science and did not regard it beneath his dignity to visit their lectures and thus to encourage them and their listeners. Abel Socin[43] wrote in his diary that

In 1760 I received from Paris Franklin's letters about electricity and studied them from 9 p. m. to 1 a. m. one after another for six weeks with my friend Fürstenberger[44]. *We got ourselves most of the instruments needed and I also delivered two relevant courses. Once Professor Bernoulli came to a lecture after which I told my listeners that if they wished to know something better they ought to turn to that genuine professor.*

Bernoulli was not jealous as old men often are, he was glad when younger men caring about science joined him, and on 13 March 1778 he wrote Fuss in Petersburg quite cheerfully in connection with that same Socin:

Actually, there are many people in our city who have physical studies quite well equipped, especially as far as electricity is concerned, and Professor Socin will soon join them. He was distinguishably employed in Hanau[45], *declared his wish to return to his fatherland and the lot won for him a vacant seat in our cantonal parliament. He is the author of a treatise* [1777, 1778] *published not long ago on the true principles and mechanism of electricity*[46].

His own lectures on physics were indeed excellent owing to his talent for simplification and performing deductions and experiments and achieving important results by least efforts. He continued lecturing until 1776, then gave them over to his nephew Daniel[47], and in 1780, when the latter was promoted to professor of eloquence, asked his younger brother Jakob[48] to substitute.

Not that he was compelled to step down because of decreased mental power, as Holzhalb mistakenly reported; indeed, his last contributions testify to the contrary and Condorcet [iv, § 18] quite rightfully said that

What he achieved at the age when so many people are condemned to remain useless, is sufficient for another geometer to earn a reputation.

No, but his physical fitness, never good enough, did not suffice anymore, especially so since a very burdensome breathlessness became ever more pronounced. And in a letter to Euler[49], that Fuss got between 1754 and 1766, since Euler again wished Bernoulli to accept an invitation to Berlin, Bernoulli complained that

My age and my health prevent me to accept it. A least effort exhausts me, and I am just a depontain [?]. *In Prussia, I would have only passed a feeble and useless remainder of my life almost burnt out in Russia and Switzerland.*

And we ought not be surprised that in a letter of 7 June 1777 to Fuss in Petersburg Bernoulli moaned about "infirmities inseparably linked to old age" and all the less be astonished that he added

A catarral fever sufficiently severe for carrying off a sturdier person has joined the natural state of exhaustion and suffering.

That seems to be one of his last letters. In the following years his health essentially worsened and by the beginning of March 1782 his troubles became so serious that he had hardly been keeping his mind and feelings under control for longer than a few hours daily.

In the morning of 17 March 1782 he slept unusually peacefully and calmly so that his manservant thought that Daniel will soon be able to leave his bed, but, upon entering the bedroom once more, found his master dead.

In spite of the advanced age of the deceased, it was a loss for the family, for his home town and the whole scientific world because he was loved and respected everywhere and had no enemies. This explains the general mourning, the various attempts to mark his memory and the honourable response seen in the letters of his contemporaries. Thus, 3 April 1782 Jeanneret wrote Jetzler:

That same day that you wrote me (on 17 March) Daniel Bernoulli quit this world and fell asleep perhaps not to wake until the Last Judgement; you undoubtedly know this already. Well, here is a new loss for the science: he was a great man at least in this world, and in him I have lost a good friend. His nephew pointed out to me that until the end he had a lucid mind. Not that he was able to meditate about and occupy himself with subjects as difficult as previously, but he always had the same presence of mind in matters of everyday life or concerning advice about an event.

It is desirable that such clever men could leave their intellect to those whom they leave here, just as Eli'jah had left his spirit to his disciple by leaving him his mantle [2 Kings 2: 8 – 11]. *I would have at once got hold of the mantle left by Daniel Bernoulli had I known that it can have the same property. He loved glory but did not seek it by making himself difficult to understand; on the contrary, he liked clarifying and simplifying the most difficult subjects.*

And then on 30 April 1782 Jetzler wrote Johann III in Berlin:

You have lost your uncle and mathematical physics lost its greatest man. I, if only I could have also inherited some of the spirit of that great man! He surely was the greatest favourite of nature. To him it revealed its secrets whereas other great men became absorbed in the field of the just possible and often rested content with hypotheses while he established the truth.

We conclude finally with a quotation of the last lines of Condorcet [iv, 18]:

To sciences he left monuments to be forever recorded in their annals; to scientists, useful lessons in the art of enjoying glory coupled with rest and consideration; to all the people, an example of happiness in the flavour of retreat, love of study and wisdom[50].

Notes

1. My sources were: Condorcet, as translated into German and commented on by Daniel II Bernoulli [iv]; Daniel II Bernoulli (1783); Lacroix (1811); P. N. Fuss (1843); the correspondence of Scheuchzer, Mallet, Jetzler etc, etc. Concerning Johann I Bernoulli I refer to my paper (1859, pp. 71 – 104) and I also quote Ritter's autobiography (*Börner's Nachr.*, Bd. 2):

To compile a short sketch of the great Jo. Bernoulli, it is necessary to know that who in those times wished to benefit by his lectures should have already mastered algebra. For J. B. that subject was too narrow for intelligibly, as was his wont, dwelling on it. He therefore, also in these latest years, read lectures on geometry and algebra with extreme displeasure. For him, the subject should have been pure transcendentalia into which he was completely absorbed and on which he dwelt long enough for his listeners to grasp it clearly.

He could have indeed allowed his students to turn to him with their doubts, was pleased when that happened and resolved their problems willingly. His disposition was cheerful and he was able to entertain a whole company with his meaningful ideas; he read his lectures diligently and his usually distressing gout did not prevent him from delivering them. He wrote great Latin satirical verse and deserved a place between Martialis and Owen. He had quite a good command of French and was really exceptionally honoured by the Paris Academy of Sciences which declared that, after he had earned three prizes, it will not crown him anymore so that other foreigners will not become jealous and discouraged. [But how about Euler and Daniel Bernoulli?] *He was very generous and often presented his Sostium* [?] *to his poor students.* R. W.

2. In Basel, his father succeeded his deceased brother, Jakob Bernoulli, as professor of mathematics. O. S.

3. In other words, the calculus of exponential functions, their algebra. O. S.

4. Emanuel König (1658 – 1731) from Basel, at first professor of Greek language, then physics, and in those days, of medicine. Later he became the father-in-law of Johann II, brother of Daniel. R. W.

5. That book includes studies of series, probability theory, outflow of water etc., solution of problems posed by Riccati, Goldbach, etc. Problems of pure mathematics, namely pertaining to the theory of series, also constituted the subject of later works published in Petersburg, in the *Leipziger-Acten* etc.

We stress as typical that Bernoulli applied the theory of probability for determining the sum of an infinite series whose terms periodically made up the same definite sum. Then, he [1747/77] very favourably referred to Clairaut's writing [1746] mentioned in the title of that contribution. R. W.

6. Was not sure of his powers, as he himself explained in his *Autobiography* [iii]. O. S.

7. I can only add that the French écu was roughly on a par with the reichsthaler. O. S.

8. Johann II Bernoulli was born in Basel on 18 May 1710. Already in 1721 he was admitted to the university, earned his first academic degree for the report (1723) and in 1724, simultaneously with Euler, became Master after making his second report (1724). After a stay at Vivis [Vevey, Switzerland] he studied the law and received a doctorate for a dissertation (1729), but in 1731, 1734 and 1746 vainly competed for professorship of law. At the same time he studied mathematics with great success under the guidance of his father. His journey to Petersburg and back home with Daniel is discussed in our main text here, his friendship with Maupertuis and his later journey are expounded in Note 14.

In 1744 he had married Susanna König, cf. Note 4, who had borne him the repeatedly mentioned Johann III, Daniel II and Jakob II, and two more sons, Emanuel, a merchant, and Nikolaus, a chemist and pharmacist. After Daniel's death he succeeded him as foreign member of the Paris Academy, and even previously he became member of the academies in Berlin, Stockholm, etc.

His spirit was powerful but his body weak so that he had to avoid great efforts. When being in companies, was shy and lacked confidence but showed much liveliness and great dexterity when using Latin and French in his correspondence. Published nothing apart from his competing writings, some occasional scientific memoirs and contribution (1740). He died on 17 July 1790; cf. Wolf (1859, pp. 67 – 68). R. W.

9. The author recounts in detail the brothers' journey back home, mainly drawing on the diary of Johann II which he received from the latter's grandson, "a renown technologist" Christoph Bernoulli living in Basel. In 1842 that same Chr. B. published a contribution on population statistics. Passages from the diary mentioned above are italicised. O. S.

10. Wolff had indeed returned (from Magdeburg), but only in 1740. O. S.

11. Gabriel Daniel Fahrenheit from Danzig was an unsuccessful merchant, then he started manufacturing thermometers in Holland. He established two fixed points on his spirit thermometer, the first one after submerging the thermometer in a mixture of water, ice and sal ammoniac, then in water and ice only. He divided the distance between these two points in 32 parts and had not used the boiling point of water. R. W.

12. Joseph Nicolas Delisle only belatedly threw himself into mathematics and astronomy but so eagerly that Peter the Great and Ekaterina I attempted to entice him from Paris to Petersburg which had finally succeeded, and he had been staying in Russia for a considerable time (1725 – 1747). In 1733 he presented a mercury thermometer to the Petersburg Academy with the boiling point of water set at zero. He did not fix any second point and assumed 1/10,000 of the volume of mercury at 0° as one degree so that the freezing point of water occurred at 150°. R. W.

Delisle published a paper describing his thermometer (read 1733). O. S.

13. In his younger years Johann De Witt (1625 – 1672) distinguished himself as a mathematician; he was one of the first to pick up and perfect the Cartesian analytical methods. Later he devoted himself to a political career, became the Grand Pensioner (leading statesman) of Holland and used all his influence for holding the House of Orange as far as possible from the executive power.

He became the mortal enemy of the Orange faction and it is likely that on 20 Aug. 1672 it either directly or at least by spreading perfidious rumours provoked a mob in The Hague that assassinated Johann and his brother Cornelius. Not being satisfied with the death of those whom it previously praised, the enraged rabble hauled the bodies to a gallows and hanged then naked upside down (which is also testified by many old copper engravings owned by the Zürich city library) and without rest maltreated the bodies.

Only in the evening it became possible to disperse the crowd by order of parliament and bury the dead. However, the demand to investigate and to punish the assassins made by the stadtholder, Prince Wilhelm III of Orange seems to be hardly successful. On the other hand, he did not dare prevent the casting of medals honouring the memory of both brothers. Later he had to give evidence that they were excellent municipal officers and true republicans. R. W.

14. In 1729 Maupertuis, already a member of the Paris Academy, came to Basel to hear [the lectures of?] Johann I Bernoulli and became very friendly with Johann II. In 1739 the latter visited Maupertuis in Paris and through him became acquainted with the Marquise Du Chatelet. After König's departure (Wolf 1959, pp. 151 – 153) he stayed with her to introduce her still further to mathematics.

Johann II invariably kept friendly to Maupertuis, received him repeatedly for a long time in Basel and tended him there during his last painful illness, and Maupertuis died despaired of life in his arms. Then Johann II took care of erecting a fine monument to him in Dornach [Switzerland]. For the sake of this true friendship Johann II certainly had to take unfounded accusations of sharing with Maupertuis wretched and entirely negative views toward religion. On 4 March 1763 the really not very tolerant Bonnet wrote Haller:

That is the dreadful secret of our modern alleged philosophers who harden their hearts and love no one except themselves. They should have ended by hating themselves and dying in despair. I am assured that the incredulous Maupertuis thus died in the arms of his friend, the just as incredulous Bernoulli. I would much like to know about the last hours of these declared enemies of human happiness.

15. Except [1721/2] and two other early memoirs [1728/10, 11] Bernoulli apparently did not publish anything concerning those branches of science. R. W.

16. Did not have the letters but described them? O. S.
17. A passage from Condorcet [iv, § 12] follows ending by a description of the problem concerning sea tides. On Johann's discussion of the prize problem about the inclinations of the planetary orbits see Speiser (2008, pp. 108 – 119). O. S.
18. Smirnov (1959, p. 435), without mentioning his source, quotes Leibniz (in Russian): "I am glad that your son bears the family stamp and retains the hereditary family lustre". O. S.
19. Note the discordant spelling of this word. O. S.
20. This contradicts the author's statement just above. O. S.
21. It is seen here how good was the memory that Bernoulli left about himself in Petersburg. And he always belonged to the Petersburg Academy that granted him a pension and from time to time sent them his memoirs. It is also desirable to note here that he was one of the seven foreign scientists whom the Empress Ekaterina II granted a gold medal commemorating the peace made with the Turks [see the end of Bernoulli's *Autobiography* [iii]]. He later presented it along with other medals which he continued to receive to the library of Basel. R. W.
22. Condorcet [iv, § 18] was apparently mistaken when stating the opposite. I am now adding the few next lines of the same passage as only quoted by Smirnov (1959, p. 446) in Russian (O. S.):

I am therefore asking you to present to the President my deep gratitude for the honour and the kind faith rendered me. Incidentally, even without the pension I have more than I need for covering my modest expenses and consider everything philosophically.

23. For example, to Johann I Bernoulli (Wolf 1859, p. 83) and several times more when thought desirable. R. W.
24. By that time he had already won four prizes of the Paris Academy three of which he already received (Wolf 1859, p. 93) and in 1746 won 1/3 of the triple prize of 7500 livres together with his brother for the nature of the magnet, see below. R. W.
25. Condorcet [iv, beginning of § 4] and other biographers mistakenly report that in addition the teaching of speculative philosophy was also entrusted him. On the contrary, a place and a vote had been left for Daniel Bernoulli at the Faculty of Medicine, and he was also granted the title of extraordinary professor of medicine. R. W.
26. In his *Autobiography* [iii], Bernoulli states that that happened in 1753 and that in 1754 he became Dean of that college. Concerning such colleges in Germany, the Russian *Brockhaus & Efron Enc. Dict.* (halfvol. 27, 1895, article *Kapitul*) notes that the position of canon "became a source of income for the imperial nobility". O. S.
27. Applying higher mathematics without departing from nature: this phrase is somewhat dubious since mathematics originated when it departed from nature and was and is departing ever farther. O. S.
28. Along with many important, and partly written in praiseworthy competition with Euler contributions about the vibration of strings and elastic membranes, it is especially worthwhile to mention his proof [see however [iv, Note 13]]of the parallelogram law of forces [1728/9] and the first proof of the principle of virtual velocities formulated by his father as well as his writing [1728/11] where he attempted to determine the position and size of the blind spot etc, etc. R. W.
29. Nevertheless, such mistakes ought to be corrected. O. S.
30. Bernoulli several times mentioned D'Alembert's mechanics and hydrodynamics, but hardly as book titles. Each time *hydrodynamics* began with a capital *h*, but perhaps his texts were in German. I can refer to D'Alembert (1743; 1744; 1752). O. S.
31. Was not this statement formulated rashly? O. S.
32. I do not see what the alleged change of the position of the solar equator had to do with the Newtonian spirit. And Condorcet [§ 12] politely but resolutely noted that Bernoulli had explained a certain fact in that contribution "only inventively". O. S.
33. For example, when solving the prize problem of 1740, see § 14. R. W.

Much more correctly (Speiser 2008, p. 110), Johann was "neither a Cartesian, nor a Newtonian". And on p. 259: "Euler's express goal was to extend Newton's foundations of mechanics to other branches of science as well". O. S.

34. The German text was *hardly two Bogen* (sheets). A *Bogen* at least usually meant 16 pages. O. S.

35. Both speakers concluded that the answer should have still been *No, it cannot*, but that the Copernican system contained the proof of its truth in itself. R. W. [Not really understandable.]

36. For example, his memoir [1735/17]. His latest memoir [1780/73] should not be forgotten either. R. W.

37. The author was mistaken, see for example Sheynin (2007, p. 293). O. S.

38. The author's subdivision of the mentioned contributions into stochastic and statistical was extremely unfortunate. O. S.

39. According to a modern definition, endemic for a given region are those diseases that persist there without external output. O. S.

40. Condorcet [iv, § 1] also mentioned the Institute [Academy] of Bologna and the Academy in Turin. R. W.

Bernoulli himself, in his *Autobiography* [iii], also mentioned the Academy of the Electoral Palatinate. The author omitted the Petersburg Academy whose member Bernoulli had become previously. O. S.

41. In Germany, there are at least five Friedrichsfelde. Köpenick is situated to the south-east of Berlin.

42. Thus, it became known that in 1762 he gave the university 50 doubloons [gold, presumably Swiss coins] so that the Rectors of the day would be able to spend the interest obtained to be giving some money for journeying to poor travelling students. R. W.

43. According to the kind information received from his grandson, councillor Peter Merian in Basel Joh. Abel Socin from Basel was born there on 16 Jan. 1729, studied mathematics under Johann I and II Bernoulli, physics under the guidance of Daniel Bernoulli, Joh. Rudolf and Friedrich Zwinger, and medicine under the younger Emanuel König. In 1758 became Doctor of Medicine under Daniel's chairmanship, then educated himself still further in Leiden [Holland] and in 1761 accepted an invitation to a gymnasium in Hanau [in Hessen, Germany] as professor of medicine and physics.

There he earned much approval as teacher and physician and was appointed court councillor of Hessen and court physician. He returned to Basel in 1778, became member of the cantonal parliament and lived for his favourite science. He had died on 24 Oct. 1808 and his grandsons, Peter and Rudolf Merian, have inherited his love for mathematics and physics. R. W.

44. Johann Fürstenberger from Basel was born in 1726 and probably died still in the 18th century abroad. He is especially known as the inventor of the electric lamp (Ehrmann 1780). Issuing from his ideas, the mechanician Brander from Augsburg [Germany] seems to have manufactured them many times. R. W.

45. Germany, Hessen, near Frankfurt/Main. O. S.

46. He devoted both editions (1777, 1778) to Daniel Bernoulli and left a manuscript of the second part translated into French in 1805. He also published a Latin memoir in the fifth volume of the *Act. Helv*. and several other contributions. R. W.

47. See Note 1 and Wolf (1858, pp. 133 – 134) where however it is mistakenly stated that he became professor of physics. Until the revolutionary years he remained professor of eloquence and died being the manager of a cathedral. R. W. [Revolutionary events in Switzerland began in 1792 and continued rather long.]

48. Jakob Bernoulli, the son of Johann II, was born on 17 Oct. 1759. He studied the law and earned a doctorate in 1778 but also quite eagerly devoted himself to the study of mathematics and physics choosing his uncle Daniel as a prime example. He was much more talented than his brothers and upon the death of his uncle applied to be his successor preparing for that occasion physical and mathematical theses (1782). Thus, due to him the lustre of the Bernoulli family would have lightened up once more, but the lot was against him and favoured the Doctor of Medicine Joh. Jakob Thurneisen who otherwise remains unknown.

Jakob then departed to Turin as secretary to the Imperial Envoy Count von Breuner and also accompanied him to Venice. Later he secured a mathematical professorship at the Petersburg Academy and married one of Euler's granddaughters. He presented many samples of mathematical competence to the Petersburg academic editions but then, on 15 Aug. 1789, drowned while swimming in the Neva. His obituary is Anonymous (1793). R. W.

49. This letter is published (P. N. Fuss 1843, t. 2, pp. 653 – 655). O. S.

50. Hutton (1795 – 1796) reports [essentially repeating Condorcet [iv, § 18]] that

He was extremely respected at Basel; and to bow to Daniel Bernoulli, when they met him in the streets, was one of the first lessons which every father gave every child.

Information about Some Scientists and Others Mentioned by Wolf

Roman numbers in square brackets indicate the appropriate number of Wolf's collection (e. g., I means 1858 etc.)

Camus C. E. L., 1699 – 1768, mathematician, mechanician, member of the Paris Academy
Cavalleri A., 1698 – 1763, physicist
Cramer G., 1704 – 1752, mathematician [III, pp. 203 – 226]
De Fontaine J. C., 1715 – 1807, philosopher
--- **A.**, 1705 – 1771, mathematician
Du Chatelet-Laumont G. E., 1706 – 1749, Marquise
Dutour de Salvert E. F., 1711 – 1789, physicist
Engelhard N., 1696 – 1765, mathematician and physicist [III, pp. 325]
Formey J. H. S., 1711 – 1797, philosopher, author, perpetual secretary of the Berlin Academy
Godin L., 1704 – 1760, astronomer
Goldbach Ch., 1690 – 1764, mathematician, member of the Petersburg Academy
Hirzel H. K., 1725 – 1803, physician and botanist
Holzhalb H. J., 1723 – 1807, historian of sciences in Switzerland
Huber J. J., astronomer [I, pp. 442]
Jallabert J., 1712 – 1768, physicist
Jeanneret S. R., member of the Berlin Academy [III, pp. 213 – 214]
Jetzeler C., 1734 – 1791, physicist [II, pp. 207 – 230]
Kästner A. G., 1775 – 1806, mathematician. Both Wolf, in three of his first collections, and Youshkevich (1968) mention him many times
König S., 1712 – 1757, see [ii, Note 7]
Mairan J. J. d'Ortous, 1678 – 1771, physicist, member of the Paris Academy
Mallet J. A., 1740 – 1790, astronomer [II, pp. 249 – 268]
Martialis, ca. 40 – ca. 104, Roman poet
Owen G., 1564 – 1622, English poet
Poleni G., 1683 – 1761, mathematician, natural scientist
Scheibel J. E., 1736 – 1809, mathematician and astronomer
Scheuchzer J. J., 1672 – 1733, physician, learned physics and mathematics, taught mathematics, was interested in observations and their treatment in natural sciences [III, pp. 181 – 228]
Schöpflein J. D., 1694 – 1771, historian, lawyer
Socin A., 1729 – 1808, physicist [I, pp. 133]
Stähelin B., 1695 – 1750, botanist, physician, and physicist [II, p. 111]
Trembley J., 1749 – 1811, lawyer and mathematician
Wagner A. F., 1828 – 1886, astronomer in Pulkovo
Zwinger T., 1658 – 1724 [III, pp. 119 – 132]

The Bernoulli Family

Johann I [II, pp. 71 – 104]
Johann II [III, pp. 67 – 68, 93]
Jakob I [I, pp. 133 – 166]
Jacob II, 1759 – 1787, member of the Petersburg Academy. See Bibliography

Joint Bibliography to Condorcet [iv] and Wolf [v]

The list of the works of Daniel Bernoulli is separated [vii]

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Anonymous (1793)**,** Histoire précis de la vie [of Jacob Bernoulli], *Nova acta petrop*., for 1789, 23 – 32. With list of publications.

Antropova V. I. (1972), Partial differential equations. In Youshkevich (1972, pp. 409 – 451).

Beck A. (1851), *Ernst der Zweite, Herzog zu Sahsen-Gotha und Altenburg*. Gotha.

Bernoulli Daniel II (1783 and 1787), see [iv]

Bernoulli Jacob (1782), *Theses physicae et physico-mathematicae quas vacante cathedra physica… 1782 defendera conabitur*, 8 pp. Report.

Bernoulli Johann I (1727), Discours sur les loix de la communication des mouvement… In author's book (1742, t. 3, pp. 1 – 107).

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--- (1802), *Histoire générale des mathématiques*, tt. 1 – 2. Paris, 1810.

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VI

Gleb K. Michajlov

The Life and Work of Daniel Bernoulli

G. K. Michajlov, Daniel Bernoullis Leben und Werk. *Gelehrte aus Basel an der St.Petersburger Akademie der Wissenschaften des 18. Jahrhunderts.* Vorträge des Symposiums während der *Schweizer Wochen* anlässlich der Feierlichkeiten *300 Jahre St. Petersburg* (St. Petersburg, 10. Juli 2003). Aachen, Shaker, 2005, pp. 77 – 87

1. Daniel Bernoulli[1] belongs to the indeed most famous mathematical dynasty of the world that produced three greatest mathematicians and many more scholars of the first rank as well. The founders of that dynasty were the brothers Jacob and Johann Bernoulli from Basel whose contributions to the construction of analysis, probability theory and almost all the other disciplines of pure and applied mathematics can hardly be overestimated.

Although Johann had made his first steps in science under the guidance of his elder brother, they soon became not only rivals, but even enemies. The jealous Johann did not want to be inferior in any subject to his brother; later, that same feeling, jealousy, even estranged him from his congenial son Daniel.

Daniel, the second son of Johann Bernoulli, was born on 8 February 1700[2] in Groningen where his father held the chair of mathematics; at that time, he was unable to secure for himself any suitable position in Basel. A few days after Daniel's birth, his father excused himself for having delayed an answer to one of his correspondents by explaining that that birth had disturbed his work. This episode can be considered as an omen for the future relations between him and Daniel.

Because of the urgent desire of his father-in-law, Johann Bernoulli was compelled to return to Basel. He, and his entire family, wife and four little children, two daughters and two sons, had left Groningen on 18 August 1705 and, after a journey of many weeks, arrived in his home town on 20 September. His brother Jacob had died shortly before that, on 16 August 1705, so that the chair of mathematics at Basel University became vacant. Johann was immediately elected to hold that position, and hold it he did until his death on 1 January 1748.

In 1712, after attending a gymnasium [a secondary school] in Basel, Daniel Bernoulli went for a year to a parish priest in Courtelary in the Berner Jura to master the French language, then, on 21 March 1713, registered as a student at the philosophical faculty of Basel University. On 4 April 1715 he read a report [1715] on the benefits of leading a virtuous and erudite life thus earning for himself the degree of Bachelor of Arts.

Then, on 26 November 1716, he delivered a speech on the banality of those who only respect mathematical studies because in the olden days mathematicians had been considered magicians [1716]. This time, he received the Master's degree. Before that, on 10 October, Daniel Bernoulli registered as a student at the medical faculty, but then, in the spring of 1718, he registered as a student of an analogous faculty in Heidelberg. A year later he studied for some time in Strasbourg, and, after returning home, became, on 12 May 1721, *Candidatus Medicinae*. Finally, also in Basel, on 2 September, Daniel defended his dissertation on the physiology of breathing [1721/1]. It goes without saying that Johann Bernoulli had been strongly influencing the education of his son, who already then displayed his outstanding talent.

In those times, appointments to a university chair had been made by lot, and many, even quite young men often stood for those positions. Each of three groups of university professors selected a candidate by secret ballot, and then the impartial lot finally determined the lucky beggar. The young Daniel Bernoulli also, and even twice in succession, ineffectively attempted to obtain a professorship; in 1721, he was a candidate for the chair of anatomy and botany, and next year, for the chair of logic.

The portrait of Daniel, then twenty years old, shows a delightful young man with almost feminine features. He probably could have won favour from women as well as from men; however, we know no further details about this more intimate side of his life.

During 1723 – 1725 Daniel had been staying in Italy where, according to the plans of his father, he should have studied medicine. At the beginning of June 1723 he arrived in Venice for studying practical medicine under the direction of an experienced physician, Pietro Antonio Michelotti. Once there, he nevertheless immediately struck up a close relationship with Count Giovanni Vezzi preferring to tarry in his estate Nervesa and enjoying himself with him both there and in Venice.

After a lapse of three months, Michelotti felt himself compelled to complain to Johann Bernoulli about Daniel's frivolous behaviour. On 20 August 1723 he wrote[3]:

The real cause of his missing two months of practice and of his failing to remain with me long enough to see my patients, is that he is obliged to stay with Count Vezzi. […] *At the end of September he is going to Nervesa situated in the countryside and will lose two more months of practice.* […] *Because, day in and day out, instead of compiling annotations and remarks, he will go to fancy-dress parties, to the Opera or the Comedy and waste his time in vain.*

To save the situation Johann had to intervene actively. Next year, 1724, Daniel moved to Padua and practiced under Giovanni Battista Morgagni, but at the end of that year he was for a long time taken seriously ill. Already then Daniel was interested in mathematics more than in medicine. In July 1724 he published in Venice a small book [1724/4] divided into four independent parts[4]. The two larger of them were devoted to problems in the theory of probability and the outflow of water from vessels.

There, Daniel showed himself as an irreconcilable polemicist sharply criticizing his opponents. On the whole, the *Exercitationes* was directed against the famous mathematician, the then twice older Count Jacopo Riccati. It should be noted that in those days Bernoulli still remained under complete scientific influence of his father, and, for example, in the part of the book devoted to hydraulics he came out as his alter ego with false arguments against Newton and made some scientific mistakes as well[5]. In 1724, during his stay in Italy, Daniel was elected member of the Bologna Academy and in April 1725 the Paris Academy of Sciences awarded him his second prize for his *Discours* [1725/7].

2. In 1724, Russian authorities had begun preparing the establishment of the Petersburg Academy of Sciences as planned by Peter the Great. They looked for appropriate scientists living in Western Europe so as to invite them to Petersburg. The Bernoulli family was known the world over and its members seemed suitable for bringing fame to the envisaged academy.

In December 1724 the Russian envoy in Berlin informed the Petersburg authorities that Christian Wolff had recommended the young Daniel Bernoulli. Apart from the famous Johann Bernoulli, there also was Daniel's brother Nikolaus II most actively engaged in mathematics, and at first it remained unclear for those authorities whom indeed of the Bernoullis should they invite.

In 1725 they ultimately decided to invite both brothers, Nikolaus II and Daniel. In July of that year the negotiations with them were concluded and the contracts signed. On 5 September the brothers undertook the journey from Basel and arrived in Petersburg on 7 November 1725. On 26 August those members of the new Academy who came earlier were already introduced to the new Empress Catherine I.

Daniel obtained the position of professor of physiology with 800 roubles yearly ("with free dwelling, firewood and light"), and Nikolaus became professor of mathematics with 1,000 roubles annually[6]. On 18 December, at a sitting of the not yet officially established Academy,
Daniel read out his first report on the excretion of liquids from a live body[7]. The first official meeting of the Academy only took place in the morning of 7 January 1726[8].

On 1 February Daniel delivered his second report, this time on the composition and resolution of forces and presented pertinent experiments and calculations. On 4 February the brothers gave their first talks. From seven to eight o'clock in the morning Daniel Bernoulli read about the application of mathematics in medicine and Nikolaus followed from eight to nine o'clock on mathematics and its application to physics, and especially to mechanics. Incidentally, only a few listeners attended these lectures because at the time Russian young men had not been sufficiently prepared for them.

Daniel very actively participated in the academic conferences, read many reports on various subjects of physiology, mechanics and mathematics and lively took part in discussing the reports of other academicians. His intervention in the debates was often very sharp, so that the President had been sometimes compelled to call him to order.

Daniel intensely argued with the eldest member of the Academy, the mathematician Jacob Hermann from Basel, but especially with Georg Bernhard Bilfinger, the professor of experimental physics. The debates with the latter led to wrangles which troubled the management of the Academy to such an extent that, after investigation[9], it demanded detailed written statements from both of them. In these conflicts Daniel had been mostly in the right, but the form of his argumentation showed him as a genuine son of his bellicose father.

In September 1730, in view of the forthcoming departure of Hermann and Bilfinger from Petersburg[10], the Academy concluded a new contract with Daniel according to which he obtained the professorship of mathematics with a salary of 1200 roubles. By that time he became a great scientist, already world-famous, and began trying to occupy a privileged position at the Academy. Indicating that it was difficult for him to get used to the Petersburg climate and that his health was deteriorating, he soon asked to be discharged from the Academy. Nevertheless, as he added, he was prepared to remain in the city either if appointed Dean of the Academy or have a state rank conferred upon him.

However, his efforts proved unsuccessful, and, when, in June 1732, his younger brother Johann II came to Petersburg for a private visit, Daniel recommended him for his own position provided that he himself was discharged. All this turned out to no avail and on 5 July 1733 both brothers left Petersburg[11] and travelled to Basel through Danzig, Hamburg, Amsterdam, Paris and Strasbourg. On 24 October, after a journey of 16 weeks, they safely arrived at their destination. While in Paris, they attended a meeting of the Academy of Sciences and became acquainted with many French scientists.

Upon his leaving Russia, Daniel Bernoulli was appointed foreign member of the Petersburg Academy. A yearly pension of 200 roubles was stipulated as well, but the Academy was being badly financed and had been always experiencing difficulties with its payment. As a result, for 25 years, from 1742 to 1766, Bernoulli had received no pension at all in spite of his regular requests addressed to the Presidents of the Academy and the Emperor or Empress of the day.

In the 1740s the previous friendly relations between Daniel Bernoulli and Euler had gradually worsened and their correspondence became sporadic and after 1754 completely broke off for 12 years. In many respects Daniel felt himself offended. First of all, because of Euler's neutral attitude towards the discord between him and his father Johann, see Michajlov (2005); then, on account of Euler's insufficient, as he thought, support at the Petersburg Academy, and, finally and above all, due to Euler's evasive and unfair behaviour at the awarding of prises by the Berlin Academy for 1746 the more so since Euler himself had asked Bernoulli to participate in that competition[12].

Only in 1767, after Euler had returned to Petersburg, Daniel's good terms with the Petersburg Academy and with Euler were fully restored and Bernoulli had once more begun to send his discourses for publication to that Academy.

In 1776, on the occasion of its 50th anniversary, the Petersburg Academy ordered Daniel Bernoulli's portrait for adorning its premises. Ten years later, the Director of the Academy, the Princess Dashkova, specially commanded to hang the portrait of the highly respected in Russia Michael Lomonosov as a counterpart to that of Bernoulli[13].

3. But let us return to the 1730s. Already during his journey back from Petersburg, Basel University elected him to the chair of anatomy and botany and immediately conferred upon him the degree of Doctor of Medicine. On 16 September 1743, he exchanged his chair for that of physiology, and finally on 8 December 1750 Daniel became professor of physics but retained his place and vote at the medical faculty as well. Between 1740/41 and 1760/61 he was seven times elected Dean of that faculty, and twice, in 1744/45 and 1756/57, Rector.

His lectures, especially on physics accompanied by various experiments, ensured him full auditoriums. At the age of 76, Daniel Bernoulli gave over his lectures at the University to his nephew and namesake Daniel II, and, from 1780, to another nephew, Jacob II Bernoulli.

Although Daniel had begun as professor of physiology and for almost 20 years taught at a medical faculty, his personal scientific interests had always laid in the field of mathematics and mechanics, and, more specifically, mostly of probability theory, theory of oscillations, hydraulics and acoustics. In all, he published about 80 memoirs, 49 of them in the yearly *Commentarii* of the Petersburg Academy (for 1726 – 1743 and 1766 – 1776) and ten among those *qui a remporte le prix de l'Académie Royale des Sciences* (for 1725 – 1757), and he also produced his celebrated *Hydrodynamica* [1738/31][14].

Daniel Bernoulli's successes in mathematical sciences were acknowledged by his election to the most famous academies of Europe: after the Petersburg Academy, there followed foreign memberships of the academies in Berlin (1746) and Paris (in 1748, replacing his deceased father) and of the Royal Society of London (1750) not to mention lesser local scientific societies. From 1747 onward he was repeatedly invited to Berlin, and again to Petersburg, but he never left Switzerland anymore, not even for short journeys.

4. Daniel Bernoulli lived in Basel, quite near to the Peterskirche. His house, the so-called Kleine Engelhof in the Stiftgasse am Nadelberg lane, was directly linked with the house of his brother Johann II, the Gross Engelhof. Unlike Leonhard Euler, Daniel did not sit all day long at his desk. His way of life was rather unconstrained. In 1963, Otto Spiess, the best authority on Bernoulli in the 20th century, provided a meaningful picture of life in Basel during 1760 – 1761. We find there that Daniel Bernoulli, a confirmed bachelor, spent almost each afternoon with his friends in one of the then popular tobacco parlours. In fine weather he liked to go walking for several hours around Petersplatz or the Rheinbrücke, mostly accompanied by his students younger by about 40 years and often with his brother Johann II as well, and he willingly visited the homes of his friends and students.

In those years, he should be imagined as a short, plump gentleman, mostly cheerful and lively, whose 60 years of age were not yet noticeable. So he is depicted on his portrait of the 1750s now kept in the atrium of the museum on Augustinergasse in Basel.

In this connection, it is interesting to quote the diaries of the young Hungarian Count Samuel Teleki for 1760 – 1761 where he described his studies under Daniel Bernoulli (Spiess 1936, pp. 139 – 140):

Daniel Bernoulli habitually refused to give private lessons to anyone. In the summer he read (publice) Experimentum physicum mostly once weekly at the Auditorium physicum[15] *and very nicely at that. I had attended those Collegium both summers and heard them out willingly indeed. After advancing in higher mathematics and* [infinitesimal] *calculus under his brother* [Johann II], *I felt a strong desire also somewhat to profit privately from that exceptionally knowledgeable person.*

Accordingly, one evening, during a walk around Petersplatz, and a talk about mathematics and my own studies, I made known my wish to learn privately mechanics from him since I understood the calculi sufficiently well to be able also to grasp and learn mechanics. […] *Upon hearing my statement, he at once promised me his help. This dear man had from the beginning taken a liking and was always very kind to me. He often visited me and I likewise sometimes went to him. I own that I have learned much from the discussions with him. He wished very much to promote my progress in mathematics, and his talks were always very instructive for me.*

In summertime he was accustomed to go often for a walk around Petersplatz, the best promenade in Basel, sometimes alone, and sometimes with his brother, and when we met there, we always went along together, occasionally walking until ten or eleven o'clock. In such cases he usually never spoke about anything except physics and mathematics, always expressing deep knowledge. I have never met anyone more excelled in science and conversation. He wonderfully explained the most difficult subjects orderly and clearly to the greatest degree.

On 18 May 1761 I began to hear my private lectures in mechanics given by that endearing and learned man. […] *I would have willingly continued those studies until departing from Basel, but, contrary to my expectations, I was compelled to break them off on 15 July. That day, at one o'clock in the afternoon, Bernoulli had to go on business to another region of the country, and to my bad luck he was unable to cancel his journey.*

From 10 to 12 o'clock on that day, without interruption, he explained to me very nicely the laws of motion and of the live forces since he desired to make up for what he had to leave out due to his journey. At 12 o'clock we parted warmly and kissed each other. And I was thus compelled to separate myself from my good teacher whom I will always respect and love and to whom I am very grateful for my studies. While departing, he allowed me to ask his advice by letter if I were unable to understand something in my further studies.

In the last years of his life Daniel Bernoulli had been suffering from dispositions of old age and was unable either to teach or to work actively and creatively. He died in Basel on 17 March 1782, a year and a half predeceasing the seven years younger Leonhard Euler.

Exactly a year later the nephew of the great Daniel Bernoulli, Daniel II, delivered a fine eulogy for him published the same year in Basel. In 1785, the Marquis de Condorcet published an eulogy for him and

Daniel II translated it into German, supplemented it by his comments and published it in Basel in 1787 [see[iv]].

In 1882 and 2000, Daniel Bernoulli's fellow citizens commemorated the great scientists from Basel. His complete works to be contained in eight volumes are being edited in Basel since 1982. Daniel was buried in the Peterskirche next to his father. The Latin inscription on his monument reads in translation:

God mourns for the best and greatest. Daniel Bernoulli, son of Johann, mathematician, physicist, philosopher gave his mortal remains to this small grave. Hardly anyone was equal to him, and no greater man did the world see. For considering him their own competed the most famous academies and societies of sciences and arts, namely the Imperial Petersburg Academy, the Royal academies in Paris, London and Berlin, etc.

Since adorning and glorifying the Russian academy in Petersburg for eight years, and the University of his home town, Basel, by teaching as a professor for 49 years, benefiting the entire world during all his life, and attaining 82 years, 1 month and 6 days, he felt himself repleted with work, honours and years and was recalled to a better life on the 16th day before the April calends of 1782.

A monument to his genius more durable than of bronze he himself had erected during his lifetime by his discoveries, writings and merits.

His brother Johann, sister Dorothea, and the children of his brother, Emanuel and his sister Catharina sorrowfully set up an epitaph for his body.

Notes

1. The main sources for Daniel Bernoulli's life and work are the commemorative writings published in the 18th century, Bernoulli, Daniel II (1787) and Condorcet (1785; 1787), his handwritten autobiography from the year 1776 only extant in a Russian translation (Bernoulli 1738/1959, pp. 427 – 432) [see [iii]], his correspondence with Goldbach and Euler (Fuss 1843, t. 2, pp. 171 – 655), The Minutes of the proceedings of the Petersburg Academy of Sciences (*Protokoly* 1897 – 1911, especially vol. 1) and the collection of documents of that Academy for the second quarter of the 18th century in 10 volumes (*Materialy* 1885 – 1900), as well as his *Werke* [see[vii]].

In the 1950s, I have given Prof. Otto Spiess copies of many documents concerning Daniel Bernoulli from the Petersburg Archive of the Russian Academy of Sciences, and they are now in the Bernoulli-Archiv, Basel. The best, although old secondary source for Daniel Bernoulli's life and work is Rudolf Wolf's sketch [v]. G. M.

2. I provide all dates in the new Gregorian style although in 1700 Holland still kept to the old Julian style. Also in Basel, the new style was only introduced in 1701 whereas Russia applied the old style until the 20th century (1918). In the 18th century, the old style had been 11 days behind the new one. G. M.

3. This letter is kept at the Manuscript section of the Basel University Library (Ms LIa663, No. 64*). G. M.

4. At the expense of a "noble Venetian, a friend of the author", as Daniel said much later. He most likely meant the Count Vezzi. G. M.

5. Later Daniel did not wish to recall that defective publication and only once briefly mentioned his mistakes [1738/31]. G. M.

A copy of that book from the Saltykov-Shchedrin Library in Petersburg carries an inscription "donum Auctoris". O. S.

6. Nikolaus II Bernoulli died in Petersburg on 9 August 1726 being 31 years of age from an ulcer in the abdominal cavity. The original German version of my paper mistakenly mentioned an ulcer of the lungs. G. M.

7. This unpublished report [1725] was directed against the views of the Scottish physician and physiologist Archibald Pitcairn. G. M.

8. Daniel recalled that official sitting 50 years later in a letter to Johann Albrecht Euler of 18 March 1775 (Petersburg branch of the Archive of the Russian Academy of Sciences, Fond 1, Inventory 3, No. 62, 81 – 82rev):

A splendid feast at Duke Holstein. Everything shone at his court, but the Duchess seemed to eclipse everyone. The academicians were astonished, but the friendly glances cast by the Duke and Duchess soon soothed them, and they dared abandon themselves to the pleasant merrymaking of such an amiable reception.

I am recalling all that and, pray believe me if I say, that I remember it better than the festive event itself because we ended it blind drunk.

Duke Carl Friedrich Holstein was the husband of Anna Petrovna, the elder daughter of Peter the Great, and until Empress Catherine's death he held one of the highest positions at her court. G. M.

9. The documents concerning this investigation are collected in a convolute kept at the Petersburg branch of the Archive of the Russian Academy of Sciences. It is entitled "Quarrels between Messrs Bilfinger and Bernoulli, 1729". In 1885, they were partly published (*Materialy* 1885 – 1900, vol. 1). G. M.

10. They only left Petersburg on 25 January 1731. G. M.

11. A photo of Daniel Bernoulli's passport issued by the Petersburg Academy on behalf of the Empress is in the original German version of my paper. G. M.

12. The prize finally went to Jean D'Alembert, Bernoulli's enemy and rival, above all in the interests of the Prussian Court. G. M.

13. After the October 1917 coup-d'état Bernoulli's portrait regrettably disappeared. G. M.

14. Bernoulli's *Hydrodynamica* [1738/31] was the most significant contribution of the 18th century on physical mechanics. In 1727, at a meeting of the Petersburg Academy of Sciences, he read his first essential work on the hydraulic theory of the motion of fluids [1729/12] based on the principle of conservation of live forces. His next fundamental step in the development of the theory of the motion of fluids [1735/19] was connected with introducing the concept of hydrodynamic pressure in a current and the method of its determination. These two contributions had indeed laid the foundation of his *Hydrodynamics*. Elsewhere, I have discussed Bernoulli's hydraulic researches in somewhat more detail (Michajlov 1994; 2005). G. M.

15. The *Auditorium Physicum* with the collection of instruments was at that time situated in the Stachelschützenhaus on Petersplatz, the premises of the present Institute for Medical Microbiology of Basel University. G. M.

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Condorcet Marie Jean Antoine Nicolas Caritat, Marquis de (1785), see [iv].

--- (1787), *Lobrede auf Herrn Daniel Bernoulli.* Aus dem Französischen übersetzt und mit Anmerkungen begleitet von Daniel [II] Bernoulli. Basel, bey Johann Schweighauser, see [iv].

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Spiess Otto (1936), *Basel anno 1760. Nach den Tagebüchern der ungarischen Grafen Joseph und Samuel Teleki*. Basel, Birkhäuser.

Wolf, Rudolf (1860), Daniel Bernoulli from Basel, see [v].

VII

Daniel Bernoulli

List of Contributions

as compiled by H. Straub (Bernoulli, *Werke*, Bd. 5, 2002, pp. 721 – 727) and supplemented by P. Radelet-de Grave, V. Scheuber, and M. Mattmüller.
Memoirs crowned by the Paris Academy
are indicated by stating the year of their publication in bold type

Straub apparently separated the complete list into sub-sequences according to the place of publication (to the periodicals etc.). These periodicals are only mentioned when the pertinent memoirs are not (yet?) included in the published volumes of the *Werke*; I indicate the number of volumes in square brackets. The last eight memoirs were indicated by letters rather than numbers; here, they are entered under numbers 76 – 83.

A somewhat less comprehensive list is appended to Straub (1970).

Michajlov [vi] provided the titles of three unpublished reports made by Bernoulli, and I included them at the beginning of the list below.

Abbreviation

CP, NCP = *Commentarii, Novi Commentarii Acad. Scient. Imp. Petrop.*

Mém. Berlin = *Mém. Acad. Roy. Sci. et Belles Lettres Berlin*

Mém. Paris = *Mém. math. phys. Acad. Roy. Sci. Paris*

Prix = *Pièces qui ont remporté les prix de l'Académie Royale des Sciences Paris*

(1715), Nobilitatem virtutis ac eruditionis praeferendam esse generis vel muneris nobilitati.

(1716), De insulsitate illorum, qui mathematica studia spernunt eo nomine, quod Mathematici quondam maleficis fuerint adnumerati.

(1725), De secretione humorum in corpore animali, contra Pitcarnium.

1 (1721), *Dissertatio inauguralis physico-medica de respiratione*. Basel [1, pp. 61 – 83].

2 (1721), *Positiones miscellaneae medico-anatomico-botanicae.* Basel [1, pp. 85 – 91]

3 (1722), *Theses logicae sistentes methodum examinandi syllogismorum validitatem*… [1, pp. 257 – 264].

4 (1724), *Exercitationes quaedam mathematicae* [1, pp. 297 – 362].

5 (1724), Notata in praecedens schediasma Illustr. Co. Jacobi Riccati [1, pp. 272 – 274].

6 (1725), Explanatio notationum suarum, quae exstant suppl. t. 8, sect. 2 [1, pp. 346 – 349].

7 (1725), Solutio problematis Riccatiani propositi in *Acta Lips.* … [1, pp. 349 – 351].

8 (1725), Discours sur la manière la plus parfaite de conserver sur mer l'égalité du mouvement des clepsidres ou sablièrs [7, pp. 221 – 239].

9 (1728), Examen principiorum mechanicae, et demonstrationes geometricae de compositione et resolutione virium [3, pp. 119 – 135]. French translation 1987

10 (1728), Tentamen novae de motu musculorum theoriae [1, pp. 92 – 103].

11 (1728), Experimentum circa nervum opticum [1, pp. 104 – 106].

12 (1729), Theoria nova de motu aquarum per canales quoscunque

fluentium. CP, t. 2 for 1727, pp. 111 – 125.
13 (1729), De mutua relatione centri virium, centri oscillationis et centri gravitates… [3, pp. 136 – 144].
14 (1729), Dissertatio de actione fluidorum in corpora solida et motu solidorum in fluidis. CP, t. 2 for 1727, pp. 304 – 342 and CP, t. 3 for 1728, pp. 214 – 229.
15 (1732), Methodus universalis determinandae curvaturae fili... CP, t. 3 for 1728, pp. 62 – 69.
16 (1732), Observationes de seriebus quae formantur ex additione vel subtractione quacunque terminorum se mutuo conseqentium… [2, pp. 49 – 64].
17 (1735), Problema astronomicum inveniendi altitudinem poli una cum declinatione stellae ejusdemque culminatione… [1, pp. 443 – 447].
18 (1735), Theorema de motu curvilineo corporum, quae resistentiam patiuntur velocitatis suae quadrato proportionalem… CP, t. 4 for 1729, pp. 136 – 143 and (1738): CP, t. 5 for 1730/1731, pp. 126 – 142.
19 (1735), Experimenta coram societate instituta in confirmationem theoriae pressionum quas latera canalis ab aqua transfluente sustinent. CP, t. 4 for 1729, pp. 194 – 201.
20 (1738), Notationes de aequationibus, quae progrediuntur in infinitum, earumque resolutione per methodum serierum recurrentium… [2, pp. 65 – 80].
21 (1738), Dissertatio brevis de motibus corporum reciprocis seu oscillatoriis, quae ubique resistentiam patiuntur quadrato velocitatis suae proportionalem… CP, t. 5 for 1730/1731, pp. 106 – 125.
22 (1738), Specimen theoriae novae de mensura sortis [2, pp. 223 – 234]. German, English, Russian translations: 1896, 1954, 1999.
23 (1739), Theoremata de oscillationibus corporum filo flexili connexorum et catenae verticaliter suspensae. CP, t. 6 for 1732/1733, pp. 108 – 122.
24 (1735), Quelle est la cause physique de l'inclinaison des plans des orbites des planetes par rapport au plan de l'équateur de la revolution du Soleil autour de son axe… [3, pp. 303 – 326].
25 (1740), Demonstrationes theorematum suorum de oscillationibus corporum filo flexili connexorum et catenae verticaliter suspensae. CP, t. 7 for 1734/1735, pp. 162 – 173.
26 (1741), De legibus quibusdam mechanicis, quas natura constanter affectat, nondum descriptis, earumque usu hydrodynamico, pro determinanda vi venae aqueae contra planum incurrentis [5, pp. 425 – 444].
27 (1744), De variatione motuum a percussione excentrica [3, pp. 145 – 159].
28 (1737), Réflexions sur la meilleure figure à donner aux ancres... *Prix* 1737, pp. 49 – 84.
29 (1747), Commentationes de immutatione et extensione principii conservationis virium vivarum, quae pro motu corporum coelestium requiritur [3, pp. 160 – 169].
30 (1747), Commentationes de statu aequilibrii corporum humido insidentium. CP, t. 10 for 1738, pp. 147 – 163.
31 (1738), *Hydrodynamica* [5, pp. 93 – 424]. German and English translations: 1965, 1968. Russian translation by V. S. Gokhman (1959) *Gidrodinamika.* Leningrad.
32 (1750), De motibus oscillatoriis corporum humido insidentium. CP, t. 11 for 1739, pp. 100 – 115.
33 (1741), Traité sur le flux et reflux de la mer [3, pp. 327 – 438].
34 (1750), Commentationes de oscillationibus compositis praesertim iis quae fiunt in corporibus ex filo flexili suspensis. CP, t. 12 for 1740, pp. 97 – 108.
35 (1751), Excerpta ex litteris ad Leonhardum Euler [2, pp. 81 – 93].
36 (1751), De motu mixto, quo corpora sphaeroidica super plano inclinato descendunt [3, pp. 170 – 177].
37 (1751), De vibrationibus et sono laminarum elasticarum… CP, t. 13 for 1741 – 1743, pp. 105 – 120.
38 (1751), De sonis multifariis quos laminae elasticae diversimode edunt disquisitiones mechanico-geometricae experimentis acusticis illustratae et confirmatae. CP, t. 13 for 1741 – 1743, pp. 167 – 196.
39 (1748), Sur la manière de construire les boussoles d'inclinaison… [7, pp. 67 – 111].

40 (1746), Nouveau problème de mécanique [3, pp. 179 – 196]. German translation 1914.
41 (1748), Nouveaux principes de mécanique et de physique, tendans à expliquer la nature & les propriétés de l'aiman [7, pp. 113 – 135] and 1750 [7, pp. 295 – 319]. Coauthor Johann II Bernoulli.
42 (1750), La meilleure manière de trouver l'heure en mer [7, pp. 241 – 293] and 1750 [7, pp. 295 – 319].
43 (1750), Remarques sur le principe de la conservation des forces vives pris dans un sens général [3, pp. 197 – 206].
44 (1769), Sur la nature et la cause des courans... [5, pp. 535 – 611].
45 (1755), Réflexions et éclaircissemens sur les nouvelles vibrations des cordes... *Mém. Berlin* for 1753, pp. 147 – 172.
46 (1755), Sur le mélange de plusieurs espèces de vibrations simples isochrones, qui peuvent coexister dans un même système de corps. *Mém. Berlin* for 1753, pp. 173 – 195.
47 (1769), Recherches sur la manière la plus avantageuse de suppléer à l'action du vent sur les grands vaisseaux... *Prix* 1753, pp. 3 – 99.
48 (1771), Sur la meilleure manière de diminuer le roulis & le tangage d'un navire… *Prix* for 1757, pp. 3 – 96.
49 (1758), Sur les nouvelles aiguilles d'inclinaison… [7, pp. 140 – 151].
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61 (1771), Examen physico-mechanicum de motu mixto qui laminis elasticis a percussione simul imprimitur. NCP, t. 15 for 1770, pp. 361 – 380.
62 (1772), De summationibus serierum quarundam incongrue veris earumque interpretatione atque usu [2, pp. 101 – 116].
63 (1772), De vibrationibus chordarum, ex duabus partibus, tam longitudine quam crassitie, ab invicem diversis, compositarum. NCP, t. 16 for 1771, pp. 257 – 280.
64 (1773), De indole singulari serierum infinitarum quas sinus vel cosinus angulorum arithmetice progredientium formant, earumque summatione et usu [2, pp. 119 – 134].
65 (1773), Expositio theoretica singularis machinae hydraulicae tiguri Helvetiorum exstructae. NCP, t. 17 for 1772, pp. 251 – 271.
66 (1774), Theoria elementari[s] serierum, ex sinibus atque cosinibus arcuum arithmetice progredientium diversimode compositarum, dilucidata [2, pp. 138 – 151].
67 (1774), Vera determinatio centri oscillationis in corporibus qualibusscunque filo flexili suspensis eiusque ab regula communi discrepantia. NCP, t. 18 for 1773, pp. 247 – 267.

68 (1775), Commentatio physico-mechanica generalior principii de coexistentia vibrationum simplicium haud perturbatarum in systemate composito. NCP, t. 19 for 1774, pp. 239 – 259.
69 (1775), Commentatio physico-mechanica specialior de motibus reciprocis compositis multifariis nondum exploratis qui in pendulis bimembribus facilius observari possint in confirmationem principii sui de coexistentia vibrationum simpliciorum. NCP, t. 19 for 1774, pp. 260 – 284.
70 (1776), Adversaria analytica miscellanea de fractionibus continuis [2, pp. 156 – 172].
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73 (1780), Specimen philosophicum de compensationibus horologicis, et veriori mensura temporis [2, pp. 376 – 390].
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75 (written 1737/1941), Oratio physiologica de vita [1, pp. 107 – 116].
76 (1755), Remarques sur les aimans artificiels de Basle [7, pp. 137 – 139].
77 (1747), Extrait d'une lettre à Garcin sur les *Elemens d'Algèbre* de Clairaut [2, pp. 94 – 97].
78 (1751), Diverses reflexions concernant la physique générale [5, pp. 622 – 630] and (1755) [5, pp. 631 – 640].
79 (1734), Letter of 1734 [5, pp. 87 – 90 in French; pp. 20 – 24, English translation].
80 (1725), Animadversiones in solutionem problematis Lunular[um] quadrabilium [1, pp. 292 – 294].
81 (1728), Discussion of Delisle's report on the system of the world. In Delisle J. N. (1728), *Discours lû dans l'assemblée publique de l'Acad. Sci.* St. Pétersbourg, pp. 17 – 24.
82 (1734), Remarques sur les observationes méteorologiques… [5, pp. 503 – 508]. Published anonymously.
83 (1755), Abhandlung von der Höhe der in vorstehendem Stücke beschriebenen Orte [5, pp. 641 – 645].
Werke, Basel. Volumes 1, 2, 3, 5, 7 и 8 (1996, 1982, 1987, 2002, 1994 и 2004) have appeared.

An additional list containing seven unpublished writings is appended in *Werke*, vol. 5

Additional Information

1. Vol. 2 of the *Werke* (and possibly other volumes as well) contain not only memoirs, but their abstracts compiled by Bernoulli himself. No one had ever referred to them.

2. The *Gesamtverzeichnis des deutschsprachigen Schrifttums 1700 – 1910* mentions Bernoulli's

Specimen inaugurale de usu medico tabularum baptismalium, matrimonialium et emortualium. Med. Diss. Basel, 1771.

3. Süssmilch (1776, т. 3, c. 31) referred to his

Integral calculs auf die Tödtlichkeit der Kinderblattern gezeigt hat.

VIII

J. H. S. Formey

Eulogy on Lambert

Nouveaux Mémoires Acad. Roy. Sci. et Belles-Lettres Berlin avec l'Histoire pour le méme année 1778, 1780, pp. 72 – 90 of the first paging.
Reprint: J. H. Lambert, *Opera math*., t. 1. Zürich, 1946, pp. 1 – 15.

[1] When beginning this work today, agonizing and even beyond my powers, I thought seeing a Janus whose both faces are equally extraordinary and difficult to describe. One of them shows me a scholar radiantly combining all the traits, knowledge and talent which can serve not only for illustrating a man of letters and philosopher, but which, if shared among many individuals, would have made them famous. The other face presents a man, only an ordinary man almost such as nature creates without skill or art. It reminds me a marble block from which a sculptor, yet undecided, can either carve a deity or chisel a basin.

Such people are certainly rare and we ought to identify them by their specific features. Those traits which they please to allow me to acquaint myself with, together with those which we can all see and observe will perhaps distinguish this Eulogy from all the other ones only presenting ordinary and vague topics.

Jean Henri [German: Johann-Heinrich] Lambert was born in Mulhouse [German: Mühlhausen] 26 April 1728[1]. What is usually said about the first years of life and education of a scientist is a kind of commonplace, and boring at that. They were endowed with a cheerful natural disposition, managed to develop successfully and thus attained all that was in keeping with their possessed knowledge and position held. In such literature we often find only obscure names and useless dates; here, however, not a single trait is to be lost, no circumstance ought to be neglected.

The father of our academician was an honest citizen, a ladies' tailor whose grandfather had to leave France because of religious persecution, established himself in Mulhouse and obtained the rights of the middle class. Opulence, however, never accompanies a refugee, and his family was living from hand to mouth. Lucas Lambert, the father of Jean Henri, had to work hard to feed his family and reared his son destining him for his own trade. Such was his stand, and he used his parental power accordingly without thinking or foreseeing that some day that son will be able to leave his narrow world and reach the confines of the universe.

With time, the tailor's family had much grown in size, and the tasks of the young Lambert, who was one of the eldest, became more numerous and, so to say, more conducive to degeneration. He had to serve all the needs of his brothers and sisters demanded by their age, and, to say exactly, to alternate as apprentice and servant.

Nevertheless, his education was not altogether neglected. Until the age of twelve his father had been sending him to the city free (publiques) schools where he distinguished himself by his application, leaving all his comrades far behind and from the very beginning revealed indications of a most ardent wish to study. This, however, did not lead his parents to the idea of encouraging him; on the contrary, he was unconditionally subordinated to the trade and obliged to replace the pen by the needle.

The teenager who through all the periods of his life had been steadfastly determined in his wishes and unable to abandon them, made known, respectfully, to say the truth, but firmly that it was impossible for him to follow such a way of life which was moreover repugnant to his constitution, feeble at the time. He could not and would not rebel but redoubled his [statements] and at the same time took all possible means for studying something.

Rocking a cradle with his foot, he opened some book and read it with greatest application once the weeping became weaker. And here is a story that will still better show what kind of obstacles he had to overcome and his courage to oppose them. His mother, wishing to prevent him to study by night, refused him candles. Young Lambert turned to calligraphy that later proved extremely useful to him since he wrote and drew very well. He drew small pictures and sold them to his comrades for a quarter or a half of a sou depending on the number of people showed there. The money earned he spent on candles and lit them when the house quietened.

Providence took advantage of that vigil for saving the family. Ashes still hot were once imprudently thrown out on the loft and they set fire anew to the coal contained in them. The floor above the boy's room began to burn, but he caught sight of that and had time to awaken everyone. The vehement fire that would not have delayed to guzzle the house was extinguished.

[2] It was impossible to resist such perseverance, and furthermore the boy's teachers often noted his capabilities and informed his father about them. And he conceded, and asked those same teachers to take his son by the hand and pave the first steps of the way he chooses.

It is appropriate to mention that in those times the number of men of letters in Mulhouse was restricted to half a dozen theologians since it was thought that there did not at all exist any other science except theology or otherwise that only theologians were able to develop sciences. The natural conclusion arrived at was that only those picturing for themselves that sublime knowledge ought to be encouraged and assisted. There was no choice and the father applied for a grant for his son to study theology which was however refused. Neither were many repeated petitions, or most insistent requests able to soften the managers of those blessings.

How to show the youngster's grief or rather despair manifested after seeing that the only reasonable hope to continue his studies had entirely disappeared. His parents became as severe as before and declared him, although he complained, that he ought to work, that only his own hands must become the tool of his subsistence. He moaned but submitted and once more became apprentice and servant. That double burden only became heavier and he would have probably succumbed had not one of his brothers, who still plies his trade, help him often concluding his tasks, unfinished since beyond his strength.

At the height of these domestic occupations one of his comrades brought him a book on arithmetic and geometry, and as soon as he opened it, he sensed a sphere existing for him and felt a burning desire to go ahead. And this is how it is almost always with those original geniuses who contain a hidden germ in themselves and owe their original development to some happy chance.

This is how La Fontaine with whom Lambert was often compared apparently came out of a lethargy of sorts by which he had been absorbed when hearing the emphatic reading of Malherbe's ode:

What will you, the future races, say
If an honest discourse will sometimes
Recite the adventures
Of our abominable days?

It would be curious to know which book Lambert started with; what he did was to study it so diligently that he finally understood it from cover to cover. However, what provides a still more complete proof of the power of his genius is that he revealed many blunders and mistakes that were contained there although was unable to correct them.

We have not finished with these favourable singularities. The family house threatened to fall to pieces and workers were employed to repair it. The young man observed their work, his book in hand, asked many questions about the practical application of the principles into which he was initiated and showed great intelligence the more so since they were formulated by a simple apprentice tailor.

One of the main workers therefore struck up an acquaintance with him and promised him another book of the same kind as his own but more comprehensive and supplemented with many figures. The young man winced with joy at the promise of such a treasure, went with that worker to his place and took the book at once. He devoured it and was ravished by discovering that, due to a most amazing concurrence of circumstances, it was precisely destined to correct those mistakes which he noted. And so, a light that could only brighten replaced the feeble clarity that guided him until then. Having neither mentor nor help except those two books, he mastered arithmetic and geometry and repeatedly assured himself (and his veracity was never questioned) that in spite of the dryness of those two sciences no difficulty ever repelled or arrested him.

Such a phenomenon manifested at the heart of Beotie[2] should have caused some sensation. Therefore, respected people of Mulhouse, not being theologians, nevertheless not only encouraged Lambert but particularly and freely instructed him and felt themselves abundantly rewarded by the astonishing advance of their student.

[3] And thus, always remaining in the midst of his fatherland, he laid the foundations of his philosophical knowledge and even applied them to study Eastern languages. At the same time he perfected his calligraphy since he felt that it will provide him his first means of subsistence. And actually it became useful for securing employment as a copyist in the chancellery whose chief then was Reber.

At the age of fifteen Lambert felt the wish to study French but his parents were unable to pay for an instructor. So he went to work as a clerk or book-keeper to a certain de la Lance from Montbéliard [now department Daubs in eastern France] who owned a business having to do with mines in Sepoix, Upper Alsace. After two years, believing that he had acquired sufficient knowledge of French, Lambert decided to live in a place where he could satisfy his passion for study. He was quite happy to become secretary to Iselin, counsellor of the Markgrave of [a tiny mark] Bade-Dourlach residing in Basel where he then published political newspapers[3].

Iselin experienced great affection for Lambert and did not cease to prove it. And what convinces us in the generosity of his sentiments is that Iselin, in spite of desiring to keep and be attached to him, preferred to become deprived of Lambert and procured him a position with which, as it can be stated, began all the well-being of our illustrious scholar. That was the post of tutor of the grandsons of Count de Salis in Chur.

Lambert entered his home on 17 June 1748 and remained there for eight years. I would have wished to make use of that the wealth of materials which allows me to enter into an Eulogy and enlarge on the respectable family of de Salis and render it all the justice that it deserves. I would have certainly found that Eulogy completed in Lambert's heart if only that were possible. At least an initial attempt of an eulogy is recorded in an original letter written by Lambert fifteen days after he entered the house of de Salis to the late Counsellor and treasurer of Mulhouse, Nicolas Heilmann, his relative and godfather[4].

I read that letter; it contained details most honourable for the family describing the virtues, the piety established in their home and the methodical order of education reigning there. When putting that letter alongside the letter of the Podestat de Salis[5] written last year on 11 November to the archiatre [senior physician] Hirzel expressing regret at Lambert's death and the precious memory he still had of the time they spent together, we see the beginning and the end of that intimate connection lasting almost 30 years whose nature would have allowed it to last many centuries had human life been extended so much.

If Lambert's life is to be written (sources are not lacking), these justifying pieces would have been very advantageous. However, I ought to restrain myself, and the description below will be all that the boundaries of this Eulogy permitted me to report about Lambert's stay in Chur and his voyages with his pupils.

First of all, to acquaint ourselves with the home where Lamber had been staying during that period, I say that it belonged to Pierre de Salis, Count of the Saint [Roman] Empire, previously Envoy Extraordinary at the court in London and one of the negotiators of the Peace of Utrecht [1713; rather a series of separate peace treatises]. That dignitary was then 80 years old. He possessed all the eminent qualities of a statesman, patriot and Christian philosopher; his wife, Dame Angloise, of a best disposition, was in addition lively.

[4] Lambert's pupils were the Count's grandsons, sons of his son-in-law Antoine de Salis, Podestat of Chur and President of the Ligue de la

Maison de Dieu [a very small political unit], who died in 1765. While he was an instructor, Lambert found ready to hand every means previously lacking for educating himself. Ever more feeling his power, he indiscriminately embraced physics, astronomy, mathematics, and mechanics, and did not consider himself unfit for theology, metaphysics, eloquence and poetry. He even wrote verses in every language he knew, – in German, French, Latin and Italian, – but did not dare versify in Greek [Remy 1910]. Verses were not of the highest rank, but he found more substantial reward in the devotion that hymns inspired in him.

We believe, however, that our duty is to insist on the fruits of his knowledge rather than of his eloquence, so let us return to his veritable objects. Having read at some time that Pascal had invented an arithmetical machine solely by the effort of his genius, and being unable to imagine something similar, he could not rest at all. He also constructed by his own hands a clock, or a mercury pendulum that went for 27 minutes and used it to measure exactly the time during physical experiments[6]. His arithmetic scale [(?) échelles] and the tool for facilitating the drawing of a perspective are no less remarkable.

A singular happy chance (since chance seems to have been subjected to order in his favour) led Lambert to his last-mentioned invention. He proposed an algebraic problem to one of his students who then made a mistake in his calculations. Being unable to correct it, he abandoned his attempts and handed the business over to his tutor.

For some days Lambert vainly occupied himself with that problem and finally, after thinking it over for a long while, cried out like another Archimedes: *I discovered the mistake, and it led me to a discovery*! The same day he constructed his tool [1759/6]. The notions about combinations that Lambert later formulated after writing his *Logique Algébraique* [?] and *Neues Organon* [1764/15] were also conceived at that same time.

The speed of his advance and the great knowledge that he acquired led him to the literary society established by many distinguished people in Chur, and he especially valued the ties with the gifted and in particular having a rare natural ability to mathematics Professor Martin Planta. That person, apart from other obligations, regulated the arrangements of a seminar that he founded in Haldenstein [in the canton Grison (Graubünden)].

In 1753 the situation with the city of Chur, the capital of Grison, and its bishop was sorted out and on that occasion Lambert compiled memoirs in favour of the city[7]; their thoroughness honoured him. The same year he became member of the Basel Société Helvétique to which he sent many mathematical and physical memoirs inserted in the *Acta Helvetica*.

[5] And thus eight years have passed. If only I am not mistaken, the most favourable years of his life, and they announced even more glorious times but ended too soon. On 1 September 1756 he left the house of de Salis with the third son of the Podestat and one of his nephews, at first to stay for a year at Göttingen University, and then to go further. While in Göttingen, he travelled to Harz and visited the famous mines in these mountains. There also, he was appointed correspondent of the Royal Society of Sciences.

From Göttingen the pupils and their guide went to Utrecht and passed a year in Holland where Lambert gave his treatise [1758/5] for publication to a publisher in The Hague. However, when compiling the [their further] itinerary Lambert repeated the experience of the astrologer who fell in a well. A most fateful accident brought him to the brink of death and weakened his constitution so much that I suspect that he felt it all his life.

Lambert had a habit as strange as it was invariable. He only turned to his interlocutors sideways, changed his position if that person had occurred opposite him and moved back as much as that person approached him. But that time Lambert stepped back without thinking about the staircase behind him and fell to its foot. The fall was terrible; he completely lost consciousness and only came to his senses after 24 hours, opened his eyes entirely bloodshot and did not wish to trust at all the physician who certified the duration of his previous state.

I do not know whether he had in that respect resembled the famous Bossuet who after some hours of unconsciousness told those who had gathered around him: *How can a man like me remain without thinking for so long*? Be that as it may, Lambert had to spend considerable time before recovering under the care of that same physician, the celebrated Professor Hahn from Utrecht[8] who advised him to abstain for a few years from serious reflections. However, among all the regimes there was not a single one to which Lambert could have less submitted.

In Leyden a pleasant adventure happened with Musschenbroek and it seems to me that it is easy to present that scene as a most laughable. The professor, already grey-haired in his profession, when receiving Lambert, thought that his visit was a respect rendered by a student or at best a novice. Musschenbroek therefore began to instruct Lambert, to tell him usual things about which the latter knew much more, answered his host fluently in a firm tome at his command and soon lost him. The roles of the interlocutors switched; Lambert became the teacher and Musschenbroek, the student.

The voyagers entered France. While in Paris, Lambert visited the main geometers, astronomers and physicists and made himself known to D'Alembert who understood his merit. Above all, however, he received many signs of friendship from Messier, famous for his observations and discoveries in astronomy. From Paris they went back to Grison through Marseille, County of Nice, Piedmont [Italy] and Milan. Lambert made use of these voyages for extending his knowledge of various subjects.

[6] After returning to Chur Lambert stayed some more time at the house of de Salis and finally left it in May 1759 for seeing his homeland once more. When passing through Zürich, he gave his *Perspektive* [1759/7] for publication. Upon returning to Mulhouse Lambert found his mother still living (his father died in 1747), stayed with her for three months, then separated from her forever; he lost her even the same year.

To avoid returning to his family, we will say here that four brothers and two sisters outlived him. He always felt a liking for his brother, the tailor Jean George and wished to take to Berlin one of the sons of that brother, 14 years old, whose talent was, so to say, cut out in his image.

In September 1759 Lambert was in Augsburg and stayed there for some time for completely finishing his *Photometria* [1760/9] and giving it for publication. The same time the nascent Electoral Academy of Sciences in Munich[9] admitted him as member. They even wished to attach him more strongly and made an agreement with him: he promised to send them his memoirs and to assist them in general with his advice; in turn, he received the title of honorary professor, a pension of 800 florins and retained his freedom to establish himself beyond Bavaria wherever he pleased.

That connection did not last long. They reproached him for not sufficiently taking to heart their interests whereas he complained, perhaps more justifiably, that they neglected his advices and did not remedy the disorders he indicated. They quit paying his pension, and he did not deign making any steps to recover it.

Lambert was too much occupied with abstractions to think about his material circumstances although his situation could not have been worse. He was able to live only by the production of his writings, to live as a philosopher from one contribution to another, as Scarron had formerly lived by the revenue of his *estate of Marquis Quinet*. This is how he called the royalties given him by the publisher Quinet for his burlesques.

Writings produced by Lambert would have been priceless were the rate of royalties regulated by the intrinsic quality of books or if sales favoured that rate. Trifles, however, are taken whereas serious contributions remain stored. Nevertheless, connoisseurs had been at once appreciating Lambert's writings which earned him a distinguished reputation and invariably fixed him a rank in the empire of sciences that he held from then onward.

In 1760 he combined his still uncoordinated pieces into the *Neues Organon* [1764/15]; soon he published in Augsburg his contribution on the orbits of comets [1761/10]. The torrent of ideas flowing continuously and rapidly from his mind carried him the material for his *Architektonik* [1771/37] as well. Those were his treasures and he quite conformed to the saying *Everything I have I carry with me*[10].

I do not wish nor can I provide an exact chronological list of all of Lambert's contributions, and still less do I want to analyse them. Above (la-dessus), illustrious colleagues had already formulated their judgement to which no one has objected[11]. Those writings have earned their reputation and the posterity will confirm that what our century had decided.

What I would, however, wish to turn the attention of this respected Assembly to, is a unique of its kind and almost unbelievable circumstance. And that is the history of Lambert's mind during 25 years, the advancement of his genius, the thread of its operation that he himself indicated with as much veracity as simplicity in a *Diary* of sorts [1915/70] covering the period from January 1752 to May 1777. Here are separate sheets more valuable than the sheets of the prophetess Sibylle[12] still in existence. Never were there other more deserving to be preserved and I am asking the Academy to permit their publication as a supplement to this Eulogy[13] for which they will be a soul of sorts and impart value to it.

[7] Let us go back to the year 1761 and collect Lambert's various journeys until the time when he went to Berlin. We left him in Augsburg; he came to see the University of Erlangen and visit the waters in Pfeffer[14], then returned to Chur and passed the next winter in Zürich. A concealed tendency invariably called him back to Grison. In summer of 1762 he returned once more to Chur and stayed there until the autumn of 1763. He toured Valteline [in Italy] and usefully occupied himself with regulating the boundary between the Duchy of Milan and the Republic of Grison. He was in Leipzig in December 1763 and at the beginning of 1764 printed there his *Neues Organon* [1764/15].

Berlin attracted him from long ago by its pleasant place and there also, especially, was his infinitely precious friend Sulzer who had repeatedly invited him and finally in February 1764 had the pleasure of hugging him.

Here begins a new chapter on which I will dwell less; there are writers not less informed than I am about everything that happened in Berlin. Nevertheless, it is necessary to say enough for presenting the facts. Preceded by his reputation, escorted, so to say, by his knowledge, Lambert, however, was a person with whom eyes and ears had difficulty to accustom themselves. Being clothed pitifully and unusually, showing himself awkwardly, either ignorant of almost every conventional usage or choosing not to comply with them, he seemed only to be occupied by himself. Always meditative, he started to talk to everyone he met, and the flow coming from his philosophical mouth only became exhausted when he found himself alone. I still see Lambert beginning a conversation with someone who had quitted him, continuing and completing it as though someone was hearing him out[15]. Coupled to this lack of self-respect often were the manifestations of most elevated pride so that the consequences were contrary to the premises. It is seen that if he wished to attract attention, that was not at all due to arrogance that compelled him to speak. His passion was more justified and did not tend to its goal by such coarse means. It was a pure and simple intuitive feeling of his own value, a deep belief in his knowledge and its price and especially a personal satisfaction based on the manner in which he acquired all these treasures, all by himself, by the power of his genius and diligent work.

Without troubling himself in the least about what others can think, he did not care to appear either likeable or not and presented himself unadorned and thus prevailed over the prejudice and compelled others to identify themselves with his behaviour. He always felt good enough the inconvenience caused by his manner of acting and conversing but he thought it to be compensated by the excellent qualities of mind and heart which, I assure you, gentlemen, he regarded in a final reckoning as an ingot of pure gold whose form did not change its value.

[8] In March, the King [of Prussia, Friedrich II] invited Lambert to Potsdam which provided a very critical situation for his future and at first it seemed to have led to a negative result. The resolute tone of his answers, the assurance with which he answered without hesitating the questions

– What do you know?
– Everything, Sire.
– How did you learn it?
– By myself

astonished the ears hardly accustomed to such language and can lead us to conclude that the abundance of his mind had somewhat worsened his competence. The audience therefore proved fruitless and apparently could not have been repeated.

However, the King allowed for Lambert's peculiar traits which our worthy colleague, whom His Majesty honours by receiving him daily, assured Him that He would not wish to deprive His Academy of so promising a member. Lambert was therefore admitted with a pension and delivered his opening discourse at a public Assembly in January 1765.

From that time onward the King manifested frequent signs distinguished by his respect for Lambert, included him in the Economic commission of the Academy and the Building department, granting him the title of *Counsellor superior* and considerably increased his pension. During those twelve years that really passed as a dream, Lambert, being in his element, never ceased furthering sciences or working for the general weal.

He published a large number of excellent contributions and spread an incalculable number of papers in our *Mémoires*, in the Berlin astronomical yearbooks and elsewhere. All of these writings are worthy of him and possess two great qualities, universality and originality.

Lambert was enormously inventive, a trait apparently derived from his paramount necessities. Having no instruments needed for various observations, no machines for physical experiments, and no possibility of obtaining them, he constructed them from most common objects at hand. And his skill in using these constructions compensated the imperfection of their structure. It is impossible to imagine the boundaries to which that attitude had led him. However, it cannot be passed over in silence that he had probably went too far; although [in Berlin] he had secured everything he needed, he kept to his own constructions either by force of habit or because of some stubbornness and that hindered him from attaining the precision of which his mind had been capable of.

[9] Allow me to decompose Lambert better to understand him. I never admix satire with eulogies but neither do I exaggerate. And I believe that, like in painting, that kind of writings admits some shadows that only help to mark out the luminous totality.

Lambert knew everything in geometry. In that science he achieved worthy success although possibly without attaining either deep views or even [?] the skill of calculation peculiar to three or four best geometers of our century. He excelled in all branches of mechanics, never ceased considering interesting subjects and going further than his predecessors. His knowledge of astronomy and cosmology was sublime; and due to some sort of affinity between his mind and light, Lambert traced all the paths of light and analysed all its properties in a manner that would have attracted the attention of the great Newton had he known the contributions of that worthy rival of his.

In the words of Daniel Bernoulli,

The comet observed by Lambert in his early youth seemed to have had somehow influenced his later works. It provided the first occasion for his *ingenious contribution on the orbits of comets* [1761/10] *and his various worthy memoirs on comets included in his famous Beyträge* [1765 – 1772/17] *and elsewhere. And he developed that particular talent of geometric constructions.*

In general, Lambert wished to measure all measurable. And perhaps there was not a single dimension possible to estimate that he did not, or did not attempt to measure. Apart from what is testified by his contributions, I find *Pithometry*, or the art of judging, in the list of his occupations. He diligently studied it, and on 16 May of the previous year [1777], as attested by the last line of his *Diary* [1915/70], he concluded his *Pyrometrie* [1779/66].

Logic and ontology exercised the activity of his mind; two of his greatest contributions [1764/15] and [1771/37] are respectable monuments of that genre. It seems to me, however, that they are only respected. This point concerns new paths, and I did not decide whether Lambert had opened them up; I only paid attention to those he followed and I see that they are almost deserted either since preferred are the beaten tracks or because scholars are not enough convinced in the benefits of leaving them[16].

Lambert was remote from the three kingdoms of nature[17]. He never paid attention either to the individual or to separate facts. His points of view were restricted to the starry heaven, to the straight line [imagined] before his eyes and to the interior of his brain where he was almost always stationed even when someone thought that he completely or partly attracted Lambert's attention. No digression left or right; always in the realm of the abstract whereas objects called concrete only slightly touched him.

Finally, he barely had likings or preferences. Not that he did not stroll over all the scenic countryside with its beautiful flowers; we even saw him elevating himself to composing verses [§ 4]. However, he still asked himself about everything concerning taste, *What does it prove*? I did not wish to speak to him about it. I knew that he claimed to possess a subtle wit and I encountered his memoir in a form of a dialogue [1754/1] which he desired to sprinkle with astuteness but in which the fancy-dressed academician sufficiently resembles an actor playing an alien role. When manifesting no love of humanity, great men strongly depress those inferior [?].

[10] I am only discussing the moral aspect, but it really ought to be considered, and I only wish to discuss one trait. Lambert was upright in every possible sense. Straightforwardness of views, intentions and actions. It is sufficiently understood that I do not pretend to attribute to him either blamelessness or infallibility. However, if it is possible to say about us, as Horace said about authors, *Vitiis nemo sine nascitur: optimus ille est qui minimis urgetur*[18], that optimism is incontestably peculiar to the deceased.

When concluding his eulogy on Ozanam, de Fontenelle reported that that academician literally used to say that *Mathematicians have the right to proceed to Paradise along a perpendicular*. And it is certain that Lambert, when leaving us, followed that path. He did not at all need a chariot for arriving in heaven; for him, a ray of light was the vehicle.

Variable and numerous were the occupations of his mind, as we have shown, and to the same extent unity and uniformity ruled his life. All his days began, continued and ended in the same way. He was neither an enemy of society nor insensible to some of its pleasures. There possibly even were occasions when he was obliged to follow its established laws more exactly. But neither had he violated them because of intemperance just as he had not defied the laws of modesty when speaking quite openly about his knowledge and merits. He went along his path without turning or stopping; however, strictly speaking, this never led him to excesses.

His uprightness begot his firmness often carried over to inflexibility. It was necessary to get out of his way, otherwise he pushed away or knocked down anyone without respect, distinction or consideration. He was inattentive to the customs of society rather than not knew them. Not that his [self-]education did not lead to reach an age late for adopting the habits of the society and learning its suppleness which for so many people degenerates into grimaces and contortions. He was tardy in having access to what is called high society or *beau monde*.

However, feeling greater and really nobler than most of those whom he encountered, he assigned a place for himself from which it was difficult to dislodge him. And that is the most important prerogative, the effect of *mens conscia recti* [the spirit senses the truth?].

We conclude this part of the Eulogy by repeating that Lambert was religious and even devout, more a Christian than a philosopher and that he was absolutely unaware of all the kinks of distorted philosophy. He was too great for degrading himself on that point. In January 1755 his *Diary* [1915/70] noted [his] composition entitled *Oratio de caracteribus Christiani, ejusque praestantia prae philosopho*, and his life had been its invariable commentary and unchallenged proof.

[11] And such a man died. He did not live for half a century and we will not see him anymore. I recall Flêchier's exclamation, much admired although more brilliant than judicious, during the oration at the grave of Turenne. Announcing the death of that hero, he cried out: *You, powers, enemies of France, you are alive*! I will say, although much more justifiably:

Lambert is dead, but you, ignoramuses, you are alive, enemies of knowledge; alive, Earth's useless burden born for grabbing the boons but unable to produce anything!

When I glance at the place where we had been used to see our illustrious colleague, and saw him with such pleasure and so often heard [him] *quasi ex tripode*, I say to myself, without infringing upon anyone's merits: Is that place occupied? Will that ever happen?

I postpone and try to avoid somehow the story of the catastrophe but I ought to continue, to approach that gulf where the perishable remains of that immortal man have found their refuge. From his first years, Lambert's constitution was feeble, and the accident that we described [§ 5] seriously undermined it and possibly caused some irreparable alteration. Finally, he was not sufficiently attentive to certain precautions that could have prevented the exhaustion of his organs for a longer time.

All that, however, was very remote from announcing some breakdown and still much less proclaiming an imminent end of life. We had seen him for a few years, well-nourished and blooming, a real figure of health, a sign of health more real than solidity [?]. A serious illness was needed for undermining it, and self-treatment was necessary for shattering it.

That illness was an intense cold during the winter of 1775. At first Lambert took no notice, had not applied any simple remedies that could have soon cured a still active man able to help himself. Then, exhausted by the abundance of phlegm that he had to expectorate, he resorted to a trick which I would have supposed incredible had I not heard it from him himself. And when he described it, I did not really approve it.

That trick was to precipitate the gathered phlegm by swallowing it on small crusts of dry bread. He thus choked his stomach with the most fetid filth, never ceased to add this rotten stuff to his [gastric] juice and therefore to his blood. This is how, who wishes to be invariably inventive, achieves his aim at his own expense: *Artifex periit arte sua* [The art of the skilled destroys him].

The illness proved to be protracted, but its course was manifest; he was the only one who did not understand the danger. He only consulted physicians at a very late stage and as though for conscience' sake, invariably followed his own principles and according to his alleged rules. We had seen how he melted like heated sealing wax until all that remained of him was dry yellow skin sticking to bones.

Being in that condition and showing obvious signs of general feebleness, he asked a physician as though out of curiosity whether such a state can last for a long time, about 15 years, say. I saw him on 18 August drinking coffee in the Park and spoke with him. He told me that he understood very well his illness of which he was not at all afraid, was sure how to overcome it. "I have got rid of five or six hundred catarrhs, and there are no more", were his own words[19].

He had a good reason to say so: the source of the radical humidity had been exhausted. Nevertheless, he was hardly able to keep on on his feet although that enfeeblement seemed not to have affected his mind. We saw him once more at the Assembly of 18 September, more dead than alive, and he even experienced convulsive symptoms which frightened those who noticed them.

On 22 September, Monday, feeling that he will not be able to come to the next Assembly, he wrote me a note accompanying a memoir by Segner to be presented there. The day of that Assembly, 25 September, was actually the day of his death, which he did not however foresee. He busied himself as usual and almost like Leibniz, a few moments before he died, he thought about the method by which Furtembach [a famous alchemist] had transmuted a half of a nail into gold. As usual, he had a small supper with the same appetite, but then a slight stroke of apoplexy carried him over from the society of mortals to that of immortals where no one needs titles to be admitted and everyone has the same rights.

Notes

1. An obvious mistake; read 26 August. O. S.

2. A region in ancient Greece; according to the context, remote from (the Greek) science. Boeotian: crass, dull person; nation derided by Athenians (*Concise Oxford Dict.*). O. S.

3. Iselin was also a professor of law, respected by Basel lawyers. He was a member of the previous Royal Society of Sciences in Berlin and remained member of the Berlin Academy. He died in 1779. J. H. S. F.

4. Heilmann could not have been Lambert's *relative and godfather*, otherwise the circumstances of the young man would have been quite different. It follows that Heilmann was at least 96 years old since his godson de Salis was then 80 (end of this section). O. S.

5. Podestat means chief magistrate of a city state, also local administrator. The text should have been: Podestat of such-and-such city de Salis. The next lines are strange. Apparently, *spent time together* referred to the Podestat and Hirzel (certainly not Lambert!), but why then discuss the duration of human life?

Then, at the beginning of § 4 a certain Antoine de Salis, Podestat of Chur, is mentioned. Apparently, that was another person.

6. I can only surmise that a mercury column was attached to the pendulum and time was measured by the height of the mercury. O. S.

7. Two memoirs, in 1753. O. S.

8. He then passed to Leyden University. J. H. S. F.

9. The Curfürstl. Bayer. Akademie der Wissenschaften. O. S.

10. The real meaning of that saying is *Essential is not the temporal, but the spiritual*. O. S.

11. No references are offered. Anyway, the volume of the *Nouv. Mém.* containing Formey's Eulogy does not include that judgement. O. S.

12. Overall, nine or ten sibyls are thought to have existed. O. S.

13. As I mentioned just above, the *Diary* was only published in 1915. O. S.

14. I have only found Pfeffers in Oberbayern. O. S.

15. This contradicts the statement just above. O. S.

16. This is unclear. Formey actully says that even the beaten tracks were "almost deserted". O. S.

17. Nevertheless, he was sufficiently learned in chemistry, experimented with salts and reported about that work at the Academy. J. H. S. F.

18. Instead of *Urgetur* read *urgetir*, see [ix, § 7]. No one is born flawless, and the best is he who least hurries. O. S.

19. When going to press, I hesitated whether to preserve these details. However, since they are peculiar, I decided to retain them. J. H. S. F.

IX

R. Wolf

Joh. Heinrich Lambert von Mühlhausen, 1728 – 1777

Biographien zum Kulturgeschichte der Schweiz,
3. Cyclus. Zürich, 1860, pp. 317 – 356

[1] On 26 August 1728 Johann Heinrich Lambert was born into the family of the tailor Lukas Lambert from Mühlhausen in Upper Alsace and his wife Elisabeth Schwermer. That place was more than for 200 years belonging to the Swiss Confederation as a *zugewanndten Orte*[1] and J. H. invariably considered himself a Swiss and until he earned any scientific title his contemporaries called him *Mülhusino-Helvetus* [Mühlhusino?][2]. I cannot therefore hesitate to describe that great thinker as a Swiss scientist.

Joh. Heinrich Lambert's parents were honest but had to live from hand to mouth and were only enriched by children. When being a little boy, he came back home from school and had to help his mother and look after his younger siblings. While his friends of his own age were merrily playing outside, he sat at home near the cradle and his only pleasure occurred when, rocking it, he was able to grab a book and read it.

When he was twelve years old, his father took him from school and, in spite of all his loathing, set him in the box room. His mother frowned upon his intention to satisfy his thirst for knowledge by reading books in the evening by candlelight and sent him early to bed. Lambert, however, did not allow to be deterred. When everything in the house became quiet, he got up and read either by moonlight or candlelight. Candles he bought for money earned by selling clumsily executed drawings or delivering clothes.

His pursuits certainly could not have been quite concealed for a long time the less so since he had saved the lives of the family and their house when hot ashes thrown away on the attic caused fire. Prompted by the efforts of his son and encouraged by Johann's former teachers, the father decided to let him study since in addition he had realized that Johann was not born to be a tailor.

No sooner had that permission gladden the poor boy when ill fate came up once more: the father had asked the magistrate for a grant to allow his son to study theology, but his request was turned down. And then the parents explained him anew that only by the needle will he be able to secure his wellbeing. For some time Joh. Heinrich submitted to the iron necessity without, however, being depressed. Weather permitting, he climbed by night on the roof and studied the starry heaven; if cloudy, he swallowed such scientific books that he was able to get hold of. Thus, without any other guidance he worked through two books on arithmetic and geometry borrowed from manual workers.

[2] The sensible questions and apt remarks directed to anyone who could have hopefully instructed him ever stronger drew attention to him. Joh. Jakob Zürcher, an assistant teacher, freely taught him French and Latin. It was Joh. Heinrich Reber, the city scribe, however, who proved himself the most helpful. Owing to Johann's good handwriting, he took him on in his office as a copyist thus at last freeing him forever from the tailor's workshop.

Later Reber recommended him as a book keeper to a certain Lalance from Montbéliard [now in France] who owned an ironworks in Seppois [Upper Alsace]. There, he was able to perfect his French, obtain some technical knowledge and even make some experiments. After being there for about two years, scientifically somewhat furthering himself and, among other studies, following with great interest the course of the comet of 1744, Lambert became, again on Reber's recommendation, secretary to Professor Joh. Rudolf Iselin in Basel who was then publishing a political newspaper and had to correspond extensively.

Iselin took Johann on very friendly, allowed him to attend his lectures and educate himself during the day. It seems, however, that Lambert did not attend any lectures but continued his private studies. Later he reported about them[3]:

About four years ago I had basically learned Latin and French and then the late city scribe Reber recommended me to Dr Iselin in Basel to be helpful to him with his correspondence and newspaper articles. In that capacity hardly half a day am I occupied so that I have got myself some books for learning the elements of wisdom. I have understood at once that my first efforts should be directed at perfecting my knowledge and making myself happy. However, I also understood at once that naturally depraved intentions cannot be improved without freeing the mind from prejudices and properly enlightening it.

That was therefore my first reference point, and I find those rules, which are very useful for cognizing the mind itself and its faults and for investigating the truth, in the writings of Wolff on the power of the human mind, of Mallebranche on the investigation of truth, and of Locke's thoughts on the human mind. All this is above all revealed in the mathematical sciences and especially in algebra and mechanics which provided me with clear and thorough examples enabling me to confirm the previously learned rules and to transform them, so to say, into my own flesh and blood. Until now, I have found no reason to regret my efforts since now I am able all the better to learn other sciences easier and more thoroughly and since I ought to teach others, to explain everything much better and more skilfully. That is what I have done and still do for improving my mind and laying the foundation for strengthening the will. I knew well enough that, anyway, the will desires goodness and avoids evil, but I also saw that, having assumed that, it was necessary to know exactly and beforehand what is good or bad and do not choose bogus goodness instead of the real good and do not allow Satan to trick the world and our passions because of our mistake. And I have not therefore refrained from examining ethics of the Holy Scriptures. Since then I obtained Pufendorf's small book [1673] *on the duties of men and citizen and other philosophical writings on morals. I had the opportunity to recognize clearly enough the advantage of the divine ethics over the other ones and to devote myself to it all the more earnestly. However, others do not hurry to clear the way for justifying nature, therefore so as to achieve a better knowledge of ethics I must not let it out of sight. In other respects, I am following the advice of Rollin*[4] *who offered it to those willing to apply themselves to fine arts. It is* *nevertheless true that when reading that and other writings I have really felt the deficiencies of oral instruction and was obliged to leave many of my questions about those sciences without discussing them. However, I have therefore attempted to compensate that circumstance by even more application and have now with divine assistance already come so far that I am able to recall the learnt and experience much pleasure in my capabilities.*

And I also foresee that during the next three years I will go travelling with two young men, whom I am now teaching languages, catechism according to Osterwald [1726], *arithmetic, surveying, military construction, geography and history, to Utrecht University, then throughout England and France. With God almighty's help, I am justifiably hoping to continue to be happy.*

[3] The end of Lambert's letter directs us to an important change that occurred in the spring of 1748. At that time Lambert came on Iselin's recommendation to Chur, to the house of the Count Peter de Salis, for teaching his grandson Anton and two other boys of the same family, Baptista and Joh. Ulrich. There also, he found best possibilities for learning to be in a fashionable society and had enough free time to augment his knowledge by using the rich private library and studying by himself.

He occupied himself alternately with the new and ancient languages, music and philosophy, mathematics and physics. Already here, in Chur, he laid the foundation of most of his greater scientific writings to be discussed below [1759/6 and 7; 1760/9; 1779/66; 1761/11; 1764/15]. In spite of his strong tendency to speculate theoretically, his common sense did not let him ever forget to gain more reliable foundations by observation. Instruments and tools were often lacking but he became used to attain his aims by most simple means which, owing to the unavoidable need, he had to construct all by himself. In 1750 he initiated a long series of regular meteorological observations[5], later took the opportunity of making relevant investigations during some excursions to the mountains and surveyed the surroundings of his place. Having been admitted to a literary society established by the most respected men, he became acquainted with Martin Planta. His versatile talent was ever more appreciated and that process was much facilitated by his two memoranda of 1753 favouring the city of Chur which at that time had been experiencing difficulties with its bishop in Zwift.

[4] And so elapsed eight years of Lambert's life in the de Salis' house, pleasantly and usefully, and the time had come when both his elder pupils, Anton and Baptista, should have begun their travel under his guidance. Late in 1756 they went to Göttingen where they attended lectures in law and privately read the *Pandects*[6]. In the summer of next year the university grinded to a halt because of the French occupation of the city. That caused Lambert, who was meanwhile appointed Corresponding member of the Göttingen Academy, to move with his pupils to Utrecht[7]. There they continued those studies for a whole year although not without short trips to Amsterdam and The Hague where Lambert gave for publication his first independent work [1758/5][8], then to Leyden. There, Lambert met

Musschenbrok who treated him at first somewhat haughtily, but Lambert's superior knowledge soon properly impressed the grey-haired scientist.

In summer 1758 they visited Paris where Lambert became friendly with Messier whereas D'Alembert did not yet correctly appraise the young man's worth, cf. below. The trip home carried them through Marseille, Nice, Turin and Milan, and about the last days of 1758 Lambert became happy to return his wards home safe and sound.

On 18 August 1758 Lambert wrote from Paris to Albrecht von Haller:

When accompanying my charges on their journey, I had enjoyed the agreeable effect of your recommendations that you had given me to Göttingen and Hanover and I always recall that all the more pleasantly since you gave them to me directly. Add to this, Sir, that I am indebted to you for a favourable review in the <u>Nouvelles littéraires de Goettingue</u> of my dissertation [1755/2] (see Note 8 – R. W.). *I would never desire anything either more advantageous or more expressive, and how I wish that my dissertation would have merited it!*

However, basing yourself, as you have done it, Sir, on your own merits, it was natural for you to shed liberally lustre on pieces much inferior to your own. And I feel how much your review encouraged me to continue following the path along which I had aleady started going. How delighted I will be, Sir, if my small appended treatise [1758/5] *could have confirmed my gratitude. It is at least for that reason that I am presenting it to you and I have also presented it to the illustrious Royal* [Scientific] *Society of Göttingen that honoured me by admitting to corresponding membership, and to Professor Kästner, who, in addition to being friendly to me, had published its extract in the <u>Nouvelles littéraires</u>* [<u>de Goettingue</u>].

Although the subject that I have treated can interest astronomers and geometers, and althought of all my discoveries the appended table of the fall of the barometer [with the height of the place] *pleases me most of all the more so since it was the most unexpected, I nevertheless admit that it was the topic of my Introduction which mainly compelled me to publish that piece. I consider it important to announce in advance my photometry and to make possible to perceive the scope of what I will treat. The subject of my published work on heat is the same as my pyrometry but it is only a small sample. All the material is prepared and I only have to arrange it and connect one of my systems with another* [photometry and pyrometry].

My service in the de Salis family will end before October and I ought to regret [the imminent loss of] *the free time they had been willingly leaving me for working on such subjects. I do not know when will I be able to resume working on them. You certainly know, Sir, that free time is necessary and you will easily imagine how much can it influence your writings which are praised by the entire republic of literature and especially by those who came to sacrifice hypotheses to experiments.*

I sincerely admit, Sir, that I expect to find it again in Göttingen and nothing would have pleased me more than an invitation to the chair of philosophy. I understand well enough that when competing for that position it is easy to become an instructor waiting for a vacant chair, and I know not less that the prime minister von Münchhausen[9] *much prefers literature to provide means for those, who, armed with a recommendation, ask to teach.*

I know very well, however, what does it mean to give lessons for a living and how much time necessary for working at the furthering of sciences is lost thereby. You know it, Sir, and your example vividly proves that the glory of a university much less depends on those who are only teaching than on those who in addition acquired reputation by their writings. I do not deny that it is that glory to which I aspire, and I do not wish anything so much as achieving successful development [in science]. *You are sufficiently enlightened, Sir, for discerning it. How satisfied will I be if your recommendations will assure me such a possibility or if the actual circumstances at Göttingen University will permit an invitation which can benefit me.*

And I am venturing to turn to you, Sir, understanding the influence that the superiority of your merits provides you with the illustrious and generous curator of that university [Münchhausen?]. *Please regard favourably my frankness with which I venture to propose my plan and reject it if you find obstacles which can destroy it or have eluded me. If nevertheless a list of original works neither compiled nor translated which I propose to bring to some degree of perfection can contribute something, it will not at all be difficult to inform you briefly about those that I will gradually offer to the public. At least your role in advancing sciences, Sir, assures me beforehand that my efforts in that direction will not displease you. They are the fruit of hours of free time I had from the age of 24 to 30, that is, from the time that I began to apply my previous attempts.*

Apart from my photometry and pyrometry I have written a paper [1760/8] *at the invitation of the Société Helvétique determining the effect of the Moon on the barometer*[10] *for the third volume of their Actes and I already find that it will occupy up to four or five lines and I will see whether other causes follow some determinable law*[11]. *I am experimenting with natural evaporation and was compelled until now to study its laws and measure.*

And I am beginning to make similar inquiries about the variation of the magnetic needle. I am applying my own discoveries and those of others to establish the paths that it takes, and expect to purge successfully logic from the remainders of scholasticism and substitute them by practical rules for meditating and inventing. I will compile a second part of ontology that differs from the first one like practical geometry [geodesy] *differs from the simple theory because in general I attempt to make abstract sciences soemwhat useful even in ordinary life. And I am dealing the same way with German eloquence.*

Those, Sir, are the fruits of my free time, but they need much more until becoming sufficiently ripe for publication. If you believe that, as I expect, I can find free time in Göttingen, or that some invitation can provide it, I will invariably acknowledge your efforts by everything depending on me. I beg you to answer me in a word to what extent you will be able to open the way for me.

Although Haller seemed to have actively interceded on Lambert's behalf, Göttingen missed the opportunity to secure for themselves that young and so much promising scientist. Anyway, on 28 January 1759 Lambert wrote Haller from Chur:

If the two reasons that made both your efforts and my expectation fruitless can restrict me to wait for reconcilliation (paix) *or the next vacancy, I will not have a case to regret the facilities to be found for studies. I do not dare however redouble my importunities and will not allow you to burden yourself with all possible obligations concerning my requests. I ardently wish to find occasions for letting you see that by the results and I am asking you to turn to me each time you believe me capable of being useful to you.*

[5] After Lambert had rested in the house of de Salis that became his second home, he experienced a desire to see his mother once more (his father died back in 1747). He travelled through Zürich where Joh. Geßner, Heidegger and others heartily received him, and stayed there many weeks, gave his *Perspective* [1759/6 and 7][12] for publication, and, in particular, on 3 May 1759 visited the new observatory of the Physical Society (Wolf 1858, p. 303). Because of his strange clothes that usually consisted in a scarlet jacket, a light blue waistcoat and black breeches, boys, seeing him in the city, at first ran after him until noticing with surprise how much respect did the stern mayor and other high-ranking persons show for that odd visitor.

Lambert stayed with his mother in Mulhouse for about three months, then went to Augsburg where he made friends with the superb mechanician Joh. Georg Brander, participated in his work and lived in his place for a long time[13]. In that city, his main goal was to conclude definitely some of the works he mentioned in his letter to Haller and give them for publication. He began with his *Photometria* [1760/9] and wrote to Geßner already in October 1759:

I have concluded an agreement with Frau Wittib Kletin to begin giving her gradually [pieces of] *my <u>Photometria</u> and will be presumably occupied with it all winter so as to clear up my material and, God willing, will speedily conclude that task by Lent* (Fastenzeit)[14].

And he also wrote to Haller [no date given]: “I will pass the winter here to have my photometry published. It is more complete than I promised”. And the next month:

I reckon that I have fourfold fulfilled my public promises about my photometry although I had not either committed myself or proposed to make it complete. Light, reflected from the surface of glass; reflected and absorbed by white bodies, – for example, gypsum, paper, – or even by coloured bodies; comparison of the clarity of illuminated objects with that of the illuminating light; clarity [transparency] *of the atmosphere; of the lunar phases and of Venus, etc, these are the phenomena equally curious and interesting for physics since experiments enter here as much as theory. A white wall or gypsum absorbs 2/3 of the light and only reflects 1/3; a glass mirror absorbs almost a half and reflects the other half. This is the result of my experiments and there is also a number of similar experiments.*

This statement should be sufficient for providing a notion about the richness of that contribution which Brandes [H. W. Brandes?] even much later called “a book that treated photometry very thoroughly with perfect mathematical exactness and elegance”, and Wilde [1838 – 1843] the details of whose description I am compelled to omit, devoted to that writing 46 pages.

On the other hand, I ought to add that the credit of being the first to deal scientifically with photometry must still be attributed to Bouguer[15] who had somewhat anticipated Lambert. Nevertheless, Lambert’s merit in studying that important branch of optics is at least not less; even if he knew Bouguer’s previous work, he revised it all by himself and made further investigations in many parts of that discipline. Moreover, Lambert had proposed and made use of the photometer usually named after Rumford.

After the *Photometria* came a smaller work on the properties of the paths of comets [1761/10] whose essence is ill-suited for being described. It suffices to indicate that, in particular, we find there the following celebrated theorem named after Lambert and very advantageously applied later, especially by Olbers:

Given a parabolic orbit, the time during which a certain arc is described only depends on its chord and the sum of its both radii vectors.

Partly because of that contribution and partly owing to later writings [1765 – 1772/17, Bd. 2/2, pp. 200 – 322; 1773/44] Lambert forever connected his name with that section of astronomy.

[6] Almost more interesting are Lambert’s *Cosmological Letters* [1761/11] which can be considered as an attempt to provide a fit notion about the whole space similar to what Fontenelle [1686] strove for with regard to the Solar system, – as an attempt to show the impressive view of the superb and systematic structure of the visible part of the universe.

Without knowing anything about Wright [1750], Lambert came to his first ideas about that subject at the beginning of his stay in Chur while observing the starry heaven exactly at the same time as Kant, whom he much resembled in general and with whom he lively latter corresponded, having been prompted by that English astronomer, wrote his history of the heaven (1755). Moreover, Lambert somewhat dwelt on that subject in his *Photometria* [1760/9].

Like Kant, Lambert perceived each fixed star as a sun about which moves a certain number of planets and comets and together with which it comprises *a system of the first order*. According to Lambert, our Sun belongs to a spherical star cluster having a diameter of about 150 distances of Sirius[16], consisting of about 1.5*mln* stars scattered in all directions, visible to us and comprising *a system of the second order*.

All these stars belonging to it circulate about a dark central body or common centre of gravity and for us their actual movement is combined with the apparent movement caused by the movement of our Sun and make up the so called proper motion of the fixed stars revealed by observations. Later it became possible to separate these two components and to indicate the direction of the Sun’s motion.

There is a large number of such systems of the second order and they all constitute a *system of the third order*, the Milky Way that appears in the form of a comparatively thin disc with a diameter of about $150\cdot10^3$ distances of Sirius, and also presumably has a central body about which its separate stars are circulating. And there again should be a large number of such milky ways which, taken together, constitute *a system of the fourth order*, and thus we may possibly go on still further if only the power of our comprehension is sufficient.

It certainly could not have occurred to Lambert to wish to justify formally the rightness of his systems about which we have provided the main points. He was only able to attempt to show them plausibly and for many statements he had only put forward theleological reasons [reasons of expediency]. For him, they carried much weight; he was a very confident Christian and repeatedly argued that *it would be a* **miserable principle** *only to believe in what you can understand although this is what we ought to do so often in our everyday life*[17].

In the newer times the knowledge of the starry heaven had enormously widened and many separate statements made by Lambert were opposed, but in general his ideas were confirmed. Thus, [W.] Herschel, Prevost, Argelander et al had accomplished the separation of the components of the stars' motion foreseen by him and proved that the Sun was really moving.

The influence of Lambert's contribution "full of genius and knowledge" (Lalande 1803/1985, p. 475) on thinking contemporaries can be ascertained by two letters of Bonnet to Haller of 22 November and 24 December 1771:

Did you read the <u>System of the World</u> *of the famous Lambert that appeared last year? I read it the second time and believe that I read a revelation of sorts that exhausts all the faculties of my soul and fills me with deepest veneration of that adorable intellect which rules the immense machine of the universe by such simple and fruitful laws. Lambert is Newton's interpreter and rival. How the immortal inventor of the law of universal gravitation would have applauded that admirable application of his principles! Nevertheless,* <u>general ideas</u> *are largely lacking in that fine work; a commentary is really necessary for those readers who are not initiated in the mysteries of higher astronomy.*

I know well enough that it is impossible to reveal these mysteries to all the readers but I also see that here and there the Editor wished to fil lin advanageously the gaps left between ideas very remote from each other. A Fontenelle is needed for investing that divine astronomy with a human form.

What I have written about the <u>System of the World</u> *of the profound Lambert will not seem to you exaggerated at all had you only read that admirable work. I dare assure you that you have never yet read anything about the universal harmony that can be compared with it. He wished to present us the heaven as a revelation of sorts of the existence* [of matter], *of the perfection and unity of the original cause. You have answered me somewhat coldly: "I would not have looked for beauty in astronomy which you have discovered in it". But read and reread that book and you will change your language.*

And later the admiration for that work quite properly continued. Thus, Usteri (1821, pp. 371 – 372) quoted Merian:

Lambert, one of the most astonishing geniuses of the 18th century, shows us in his *__Letters__* *grand, magnificent, and new ideas about the extent of the perceptible universe, the continuity and harmony of the worlds, the number and destination of the fixed stars and comets. It may be said that he broadened all the proportions and spread before our eyes the amazing immensity of space.*

He added, nevertheless:

Lambert, however, does not know how to write, and his work is a sort of a chaos that needs to be disentangled. Merian cleansed it from all the scientific details, of all the alien obscuring objects and let scientific Europe see the universe, dazzling and ravishing, in all its simplicity, order and magnificence.

It seems that Usteri judges Lambert too severely, whereas Merian credits him too much, and I cannot resist to contrast his verdict with the opinion of the famous Struve (1847, p. 12) who called his *Letters* "remarkable for the clarity of exposition and penetrative views"[18].

[7] Nevertheless, I ought to take this occasion to say that in general many scientists complained that Lambert, in his German works, had sometimes used unclear expressions and was long-winded[19]. Thus, Jeanneret, in a letter to Jetzler, argued that it was often difficult for him to understand Lambert, and the latter answered on 17 March 1782:

It is a pity, as you noted, that it is so difficult to understand Lambert. Apparently, he, like Newton, attempted to be admired as much as to inform. I am sure that many times he intentionally concealed his route that led him to interesting truths. He differs here from Euler who always shows the whole analysis that he had applied, and clearly at that; Lambert himself admits that that clarity is typical of Euler, the great geometer.

And then Jeanneret wrote on 3 April:

Actually, scientists are greatly wrong when attempting to be obscure for being admired. I recall what Bernoulli told me: he had found Lambert's *__Photometria__* [1960/9] *so obscure that he would have written it just as well as read it. Here, then, is a very little useful book. If it is difficult to such a mind as D. B., what about others: this is what hinders me up to now from buying it, but I am plucking up courage yet again. And scientists of the first rank read little, each is occupied with his own thoughts; therefore, they do not read even books as scientific or as obscure as possible, and, anyway, it is not them that need be instructed, such a thought is absolute folly. If they resemble D'Alembert,* [reading] *frequently only excites their jealousy, they read each other solely in order to criticize.*

And on 11 November 1773 Fontana[20] wrote Kästner from Pavia [Italy]

I have received the German works of Lambert. He is a great genius, it is impossible to deny it. However, he drags out the subject he treats, he extends it a bit too much, his prolixity is sometimes repulsive. It seems that he ignores the art even rarer than talent, the most difficult art of deleting. It is not amiss to call him the Dryden of geometry about whom Pope reasonably remarked:

The copious Dryden wanted, or forgot
The last and greatest art, the art of blot.

Be that as it may, he is so virtuous that I almost forget his deficiencies, and where is the man lacking them? Optimus ille est qui minimiis urgitur [The best is he who least hurries].

On 28 March 1759, shortly before Lambert came to Augsburg, an Academy of Sciences was established in Munich[21], and it did not miss the opportunity of connecting a rising luminary in their surroundings to itself. Formey [viii, § 6] reported that the

Electoral Academy of Sciences in Munich admitted him as member. They even wished to attach him more strongly and made an agreement with him: he promised to send them his memoirs and to assist them in general with his advice; in turn, he received the title of honorary professor, a pension of 800 florins and retained his freedom to establish himself beyond Bavaria wherever he pleased.

That connection did not last long. They reproached him for not sufficiently taking to heart their interests whereas he complained, perhaps more justifiably, that they neglected his advices and did not remedy the disorders he indicated. They quit paying his pension, and he did not deign making any steps to recover it[22].

[8] The breakdown of relations between Lambert and that Academy seemed to occur at the end of 1763 or beginning of 1764 since the first volume of the Academy's *Abhandlungen* still included two of Lambert's contributions [1763/13 and 14] whereas on 21 July 1764 Auguste Reizenstein[23] wrote from Munich to Geßner: "What will now become of the good Lambert? Here, he is forever done with".

Lambert possibly never was, or at least never was for a long time in Munich since in the summer of 1761 we find him again [?] in Pfeffers, in autumn he was in his beloved Chur and in winter in Zürich where he was admitted Honourable member of the Physical Society

As a person whose penetrating mind reveals the truths in the most difficult sciences, discovers new truths and exposes secrets.

The *Monatliche Nachrichten* of 1778 reported that

He lived and took meals at a citizen and, because of the small amount of money given for his board, his life had to be meagre and simple. Once in winter, together with many members of the local Physical Society, and especially with our Geßner, he had been all evening helping with observations at the Observatory of that Society. Then, to be able to continue their work, they had a joint supper. Lambert, however, was afraid that his restraint will be tempted, went home for supper and returned back in half an hour.

From summer 1762 until autumn 1763 Lambert had been again living in Chur, made short trips from there to Veltlin [Lombardy, Italy] and Cleven [Chiavenna] and was engaged in demarcating the border between Bünden[24] and Milan [the Milan Suchy]. Then, via Augsburg, he went to Leipzig and gave there his *Organon* [1764/15] for publication[25]. He had been mostly occupied for the latest years with that work; Ernst Reinhold (1828 – 1830) mentioned it and stated:

In that work he attempted to solve the problems of logic more thoroughly than previously and separated them in four parts.
1) Does the human mind lack power for reliably proceeding to the truth?
2) Does the truth in itself admit of being sufficiently known so as to be distinguished from falsity?
3) Does the language in which the truth has to be presented and described place obstacles to its cognition?
4) To what extent does the mind allow itself to be hoodwinked by a fake appearance of truth and is unable always to penetrate truth?
Lambert therefore derived four scientific disciplines which the mind ought to apply for consciously recognizing and describing truth as such and distinguishing it from falsity and fakes. He considered and dealt with all of them as with an organon of human knowledge consisting of
1. Dianiology, or the doctrine of reason, common sense, that, concerning its scope and aims, largely coincided with Wolffian logic.
2. Alethiology, or the doctrine of the criteria of truth. Here, he was concerned, as Locke before him, with establishing simple notions and applying them for discussing the basis of all scientific knowledge.
3. Semiotics, or the doctrine of naming ideas and things.
4. Phenomenology, or the doctrine of the fake.

I do not think that I ought to look for Lambert's significance in his philosophical writings[26], so let that short description be sufficient. I will only add that in that field he was judged in extremely differing ways. In general, his contemporaries little valued him as a philosopher; histories of philosophy mostly mentioned him quite briefly with not much praise among Kant's predecessors. Erhardt (1829), the author of an eulogy on him, called the *Organon* a work "worthy of amazement" and stated that each reader sufficiently patient for thinking about all of it will be rewarded by "many splendid remarks, unexpected new ideas and meaningful examples". However, he nonetheless rather attributed to Lambert an ephemeral significance.

On the contrary, on 12 July 1764 a certain Moses Mendelsohn wrote to his friend Abbt:

Had I read Lambert's Organon *a few years ago, I would have certainly left my competitive paper (on the evidence in metaphysicial sciences)*[27] *in my desk or perhaps had experienced a volcanic rage. Only a Lambert knows how to find out the hidden routes of the mind, the most secret approaches to the temple of truth. His work is the most excellent of that kind. His Dianiology contains the main propositions of the art of inventing; his Phenomenology – fruitful notions about the logic of the probable; and his doctrine of indicating the truth is equally worthy. Only his Alethiology pleased me somewhat less. Read this book for heaven's sake as soon as possible so that we will be able to discuss it in detail. Such writings are appearing! And foreigners still belittle the state of science in Germany.*

And Erman (1828) still placed Lambert even alongside Leibniz during whose jubilee of birth he made a report at the Berlin Academy. In particular he stated:

Leibniz is the German Plato, and Lambert, the German Aristotle. Had Leibniz moved with an ancient naturalism in the realm of ideas instead of writing clumsily in a foreign language, he would have been the entire Plato. And had Aristotle, in his sketch of the logical functions of thinking, applied the mathematical talent and mathematical knowledge of the author of Architektonik [1771/37], *he would have been the entire Lambert*.

Just as differing are the opinions about the style of that contribution [the *Organon*][28]. Thus, on 20 March 1776 Jeannert wrote to Jetzler:

As to Lambert, I believe that his science is accompanied by a bit of charlatanism, I am hearing that being said in a manner that makes me trust it. In our world, it is necessary to be valued, and that is of what often consists the entire merit of some persons.

And here is a trait. A Genevan[29] *translated into French his* Organon [1764/15], *and since it contained unintelligible places, asked the author for explanations so as to adduce them to his translation. Lambert, however, had not deigned to consent and said that it was useless: for those, who will not understand his book just as it is written, the study of metaphysics was more harmful than useful.*

And J. [Jacob II] *Bernoulli told me that Béguelin, who had wished to isolate a part of that book and present it for a wider circle of readers, found it so obscure that refused to read it. I think, however, that Béguelin is not accused of having a mind not good enough for studying metaphysics.*

On the contrary, Erhardt (1829), although noting that Lambert was long-winded and readily repeated himself, said that

His language is free from provincialism and peculiar idioms; his expressions are unaffected and unsweetened, always appropriate for the subject and notions, short, energetic, clear to a high degree and prove that who thinks clearly must be able to speak clearly and it is a sign not of a great talent but really of a poor mind to wrap up thoughts in an incomprehensible terminology. Lambert's is the precise language of a mathematician rather than a puffed out language of a Romantic of our day. Elegance was not in his nature.

[9] At the beginning of 1764 Lambert journeyed through Halle, where he became acquainted with Segner, to Berlin, supposedly to stay there only for a while and then to look for his fortune in Russia. He was also probably hoping that his appearance in person at the Berlin Academy, that had already in 1761 elected him Foreign member, will secure him a pension and then to remain in Berlin.

That he had wished to do so even earlier is evident from a letter of Auguste Reizenstein to Joh. Geßner written in Chur on 25 October 1762:

Lambert had said that Prof. Sulzer and Euler are attempting to achieve a pension for him and he is greatly touched by those good efforts. I, for my part, intensely wish that those worries will soon bear fruit since it appears to me that that industrious scientist does not at all have a substantial livelihood which is painful. Love of mankind, my dearest friend, in itself encourages Prof. Sulzer to say all the advantageous about Lambert, otherwise I would have to exert every effort to say trivial words to them in his favour.

At that time Euler and Sulzer did not achieve their aim. However, when Lambert came in person and visited Sulzer once more, the latter became so favourably disposed towards him that decided to dare an attempt once more. That was soon possible since a few days after Lambert's arrival he was anyway summoned to Potsdam. Sulzer [1809] described that episode in his autobiography:

I had so much admired that splendid man that, while on my way to Potsdam and there also, I could not think about anything else except his great talent. In Potsdam, I spoke about him to a few people who saw the King daily with such passion that they became unable to abstain from informing the King about my admiration for that extraordinary mind. That resulted in a letter from von Catt, the King's reader, waiting for me when I returned back to Berlin.

My friend informed my that the King wished to speak to the arrived philosopher and that I ought to ensure that he comes to Potsdam the next day to be presented to the King the very same evening.

And so, although the business started well, it should have been feared that, owing to Lambert's odd behaviour during the personal audience [surely to be manifested], comparable to that of a man fallen down from the Moon, he will be disliked. However, the King wished to see him, and nothing could be done. And at least for the moment the audience really came to nothing. Formey [viii, § 8] reports:

The resolute tone of his answers, the assurance with which he answered without hesitating the questions

– *What do you know?*
– *Everything, Sire.*
– *How did you learn it?*
– *By myself*

astonished the ears hardly accustomed to such language and can lead us to conclude that the abundance of his mind had somewhat worsened his competence. The audience therefore proved fruitless and apparently could not have been repeated[30].

[10] Sulzer [1809] continued:

His Majesty did not discover the great philosopher in that good man whom he expected according to the reports. Von Catt informed me about it at once, and rather pitifully at that. Lambert, however, too little experienced for noting that he was not liked, came back all pleasure. I was not a little embarrassed: the good man was sent back to Berlin with a promise that I will inform him about the consequences of the audience. I told him in strictest confidence that a man like him will not be let to leave once more if only everything will start properly, that the King has good intentions about him but some time can pass until that happens, and he was satisfied.

Meanwhile, I diligently wrote to von Catt about Lambert, expressed my great regret that the King saw him from the wrong side etc.

More than half a year had passed. The Russian envoy, Prince Dolgorukiy made the acquaintance of Lambert and the Petersburg Academy expressed desire to invite him. That encouraged me anew to urge von Catt to tell the King that he will forever regret if, as it appears, that man leaves Berlin.

That attempt had the desired effect and the King offered him a pension of 500 thalers and a seat at the Academy.

Merian [Sulzer (1809)] added to that story that

Von Catt had asked me to inform Lambert about that which I have willingly done at once, and since I was sure how strongly he wished to remain in Berlin, I thought that that news will just as well gladden him. And I was all the more astonished to see him take it in most indifferently and hear him say that he still wished to consider that offer.

I answered him straightly that there was nothing to consider: either he accepts the invitation at once, or denies it in which case he surely will be approached never again. After that I went to Sulzer to inform him about that. And since Lambert came to him the same day, Sulzer told him in his sometimes domineering tone: "Sit down and write what I will dictate". That was a thank-you letter to the King. Lambert obeyed and thus happily ended that business.

The Cabinet order by which the King appointed Lambert effective member of the physical class was dated 9 January 1765, and the end was lucky since otherwise D'Alembert's letter of 1 March 1765 to Friedrich II could have again easily cancelled the appointment[31]. D'Alembert wrote somewhat boisterously

I know only one work of Lambert. It is good but it did not seem to me comparable to those of Euler. And if the latter is kneeling down before him, as Your Majesty had honoured me by writing it [in a letter], *it should be said about Euler what La Fontaine had said, that it would be very stupid to believe that Aesop and Phaedrus were more intelligent than he is.*

Meanwhile, on 24 January, Lambert very successively read his maiden speech [1767/18] on the influence of experimental physics on other sciences and soon made himself heard again. On 30 April 1765 Sulzer informed Jetzler that

We have finally got Lambert, but he is not quite satisfied by his pension of only 500 thalers, certainly too little for such a man, but all that was possible given the present situation. This summer we will continue the experiment with cannonballs and hope to determine rather precisely their motion simply by observation. Lambert had begun to read out his pertinent memoir[32] *at the Academy; In spite of so many previous writings on that subject, it contains very much of what is new and special.*

[11] Nevertheless, during his first years in Berlin, Lambert was not always been on intimate terms with his new colleagues. On 11 October 1766 Johann III Bernoulli[33] wrote to Mallet that

Lambert casts a shadow on his great merits by unimaginable conceit. Partly he caused us to lose Euler[34] *and among his colleagues he is only getting along with me. I do not quarrel with him although we had been taking meals together all the time he lived in my place. His conversation on all the sciences is instructive. If you do not ask him about anything except his own ideas, and do not interrupt or contradict him, he will speak for three hours as though reading from a book.*

How high he was, however, already then valued, is shown by Sulzer's letter of 22 November 1766 to Jetzler:

The Russians threaten to take away Lambert as well. However, he strings rather highly. I hope that we will retain him and that that [threat] *will be an occasion to arrange something more advantageous for him. Although he withdraws from all contacts and his behaviour becomes ever stranger and more childish, I confess that I would rather give him away my own pension than see him leaving the Academy.*

Later Lambert's personal relations with his colleagues had been also becoming ever better partly because he himself began changing to his own advantage, and partly because it was gradually understood that most of what was at first thought to be bad temper, arrogance, etc, should be reckoned as lack of external form and excessive naivety and that his essence was entirely superb. Even Friedrich II ever better discerned that a somewhat peculiar shell hides a man of great talent and many times defended him against mockers by noting that “in that man we ought to see the vastness of his insight rather than trifles”. He repeatedly increased Lambert’s pension and in 1770 appointed him head of the building department (Oberbaurat).

Graf [1829] reported that Thiébault [1813], in his recollections, wrote:

I wanted to congratulate Lambert the same day on which I saw that appointment in a newspaper but he remarked:

“It is very strange that the King announces such news without discussing the matter with me. It concerns me and above all I should have been asked whether I wished to agree or not. I have not yet decided since I do not need that appointment”.

His friends exerted much effort to make him agree. And after accepting the new position he went to the appropriate officials (Ministern) and told them:

“Your Excellencies must not think that I will go through the usual construction bills and amend them. That is a task which can be done by your clerks if only you will not wish to do it yourselves. I will not concern myself with things that anyone else can attend to and therefore would only mean a waste of my time. However, if you encounter difficulties and will be unable to overcome them, you may only turn to me”.

In that new position Lambert had also been in the King’s good graces and retained them to the end of his life as is seen from Jetzler’s letter of 15 July 1776 from Berlin to the mayor of Meyenburg [Brandenburg]:

The contacts with Lambert are especially important for me since he unquestionably belongs to the greatest philosophers and mathematicians. Three weeks ago the King increased his pension by 400 thalers although neither Lambert nor anyone else knew about it beforehand. That was a certain sign of his merits since it is sufficiently known that the King does not willingly give too much.

The pensions of some other academicians, mostly Swiss, had also been increased and it is honourable for Switzerland that they stand so well with the King of Prussia.

[12] Lambert’s later scientific works concern almost all branches of pure and applied mathematics. However, their number is much too great for discussing all of them separately and it will suffice only to add the following to what was already casually noted. Lambert had permanently inserted his name to

Divisibility of numbers [1765 – 1772/17, Bd. 1, pp. 1 – 33; 1946 – 1948/71, Bd. 1, pp. 91 – 116];
The theory of equations [1765 – 1772/17, Bd. 3, pp. 184 – 249; 1770/31, 32];
Series [1758/4], later a series was called after him[36], then considered by Euler (1783) and generalized by Lagrange;
Interpolation [1765 – 1772, Bd. 3, pp. 66 – 104; 1774 and 1777/49; 1946 – 1948/71, Bd. 1, pp. 333 – 358 and Bd. 2, pp. 291 – 293];
Theory of probability [1760/9; 1765 – 1772/17, Bd. 1, pp. 1 – 313 and 424 – 488; 1772/40; 1799/69];
Integrability, determining conditions of [1769/26], etc.
Geometry: for example, elements of the so-called *géométrie de la règle* [?], perspective [1759/ 6 and 7; 1768/24], perfection and enrichment of trigonometry [1765 – 1772/17, Bd. 1, pp. 369 – 424; 1770/34], first sketch of tetragonometry [1765 – 1772/17, Bd. 2/1, pp. 175 – 183], a very thorough study of the art of sighting [1765 – 1772/Bd. 1, pp. 314 – 368 and Bd. 3, pp. 12 – 84][37];
Chorography[38]: many works (later developed by Lagrange), for example [1765 – 1772/17, Bd. 3, pp. 105 – 199, reprinted: *Ostwald Klassiker* No. 54, 1894], [also see Wallis & Edney (1994, pp. 1108 – 1110)].
Mechanics: theoretical investigations of its principles [1765 – 1772/17, Bd. 2/2, pp. 363 – 628]; study of the so-called three-body problem [1769/28]; various works on friction [1774 – 1779/46]; fluidity of sand [1774/47]; water mills and windmills [1777/59 – 62]; human power [1779/64], etc.
Physics: he just as well enriched almost all its branches. A long series of works in hygrometry [1771 and 1774/36], meteorology [1763/14; 1773/42; 1779/65], acoustics [1776/57; 1777/58][39], optics [1772/38; 1773/43], magnetism [1768/21 and 22], etc.

Suffice it however to indicate [additionally] that Lambert was one of the first to apply widely graphical presentation for studying series of observations [Gray & Tilling (1978)], and mention one more of his main works, the *Pyrometrie* [1779/66].

We have seen [§ 4, letter to Haller] that Lambert very early busied himself with the theory of heat and already then thought about writing a more comprehensive work on that subject. Other tasks had, however, intervened and only in the twilight of his life did he find necessary time to pick up that early project. His last contribution thus connected with the first one [1755/2]. Lambert concluded his *Pyrometrie* on 16 May 1777, and a few days before his death gave it for publication, and his friend, Wenceslaus Johann Gustav Karsten added the lacking Introduction. Daniel Huber (1829b) described that book in the following way:

In that valuable work he very thoroughly and extensively dealt with all the measureable according to the contemporary knowledge of heat. He made use of almost all the relevant experience of his predecessors and of very many of his own observations and inserted many meaningful comparisons and calculations.

The heating and cooling of bodies in an indefinitely spread [infinite] *media, the exchage of heat between various bodies are there investigated and a penetrating theory compared with experience* [experiments]. *The law of the emission and reflection of heat is considered and even the perception of heat is subjected to calculation with appropriate formulas being provided. The power of heat is very wittily compared with the cohesive force of bodies.*

Prompted by the laws of mutual heating of bodies, Lambert made the same experiment as that, accomplished by Fahrenheit according to Boerhaave's desire, which led to the notion of specific heat. Lambert varied his experiments and in some formulas he even introduced a coefficient completely corresponding to that notion. It is seen therefore how very near was our physicist to the theory established only a few years later by Crawford and Wilke. Only the insufficient amount of material at hand can explain why did not the broad viewpoint of his talent pursue these ideas further and why did he overlook its most important consequences.

The last part of the Pyrometrie considered the solar warmth and contained extremely thorough and complete comparisons of its change over various seasons, times of day and latitudes. Lambert derived a theory based on principles more correct than previously assumed and checked them by many careful observations[40].

And it should not be forgotten that Lambert's Pyrometrie was situated on the boundaries of the old theory of heat. Only after its appearance was the new doctrine of specific, latent, emanated etc. heat constructed and provided that part of natural sciences with a completely new form.

Lambert's contribution remains a not less rich but yet insufficiently applied treasure of collected experiments, important viewpoints and interesting mathematical investigations really suitable for furthering science even in its present more perfect state.

Finally, **astronomy**, namely in its more practical parts, is very grateful for Lambert's later works. Above all, we ought to mention his numerous tables with which he attempted to facilitate considerably astronomical calculations: his eccliptical tables for calculating future eclipses [1765/16] with a French translation appearing at the same time; supplements to logarithmic and trigonometric reference books [1770/29][41]; and, quite especially, his collection of astronomical tables [1776/56] compiled and enriched by him.

Referring to that collection, Lalande [1803/1985], who usually only provided the titles of the writings he included in his *Bibliographie* without saying much about them, stated:

This collection of tables is the most extensive and most complete among those published until now. It contains everything necessary for astronomical calculations and observation, – tables of the Sun, the Moon, planets and satellites, semidiurnal arcs [?], *amplitudes* [?], *refractions etc. and many new tables for promoting astronomical calculations due to Lambert, Bode, Schulze* [1778] *and Lagrange*[42].

The reference to Bode recalls that Lambert, realizing that, because of Johann III Bernoulli's poor hearing and sickliness (see Note 33), practical astronomy retreated ever further into the background[43], prompted, in 1772, the invitation of the young and very promising Elert Bode from Hamburg to the Berlin observatory and then persuaded Bode to publish an astronomical yearbook which occurred from 1774 onward.

The Notes that Lambert added to the supplement to his own ephemerides[44] became the basis for it to be ever more elevated to a very valuable and influential astronomical source facilitating for many decades the scientific contacts between astronomers. Only Zach, Lindenau, Bohnenberger and Schumacher replaced it by their own publications intended for that purpose.

Besides those discussed above, Lambert wrote several longer astronomical papers for the publications of the Berlin Academy some of which [1775/53; 1778/63; 1781/67; 1775/54; 1775 – 1776/55] ought to be mentioned in the first place. The first three discussed the inequalities of Saturn and Jupiter and became the groundwork for Laplace's important pertinent investigations; the others were an attempt to calculate the elements [of the orbit] and tables [of motion] of the satellite of Venus observed, as it seemed, at different times by the Cassini, Short, Montaigne et al. Even now his attempt is interesting although the efforts of either to find that satellite once more or to explain convincingly how those experienced observers were so seriously mistaken.

[13] No one can wonder that because of all those writings, numerous reviews for the *Allgemeine Deutsche Bibliothek* and extensive correspondence[45] Lambert's strength became rapidly exhausted. In addition, when in winter of 1775 a serious and persistent cold had overtook him, in spite of being persuaded by his friends he had not turned for help to physicians and believed that he can cure himself. Graf [1829] reported:

Once the amount of phlegm from the windpipe had increased, he swallowed it on small crusts of bread and thus poisoned his [gastric] *juice*.

As a consequence of applying such unsuitable means he visibly lost weight and his illness, in spite of his previous stronger health supported by rare moderation, transformed into tuberculosis. And still he did not at all lose hope, estimated that "he expectorated eight thousand small abscesses from his lungs and therefore feels himself better", and will live 15 or 20 years more. He worked on and on, went outside almost daily although barely being able to keep on on his legs.

Even on 18 September 1777, although more dead than alive, he attended the meeting of the Academy and continued working the following days until on 25 September, shortly after pleasurably having supper, a stroke prematurely ended his life and, as Formey [viii, last lines] put it,

carried him over from the society of mortals to that of immortals where no one needs titles to be admitted and everyone has the same rights.

[14] With deep regret Lambert's friends and admirers, among whom the great Friedrich was not at all the last one, got to know about the demise of that excellent man whose small singularities were forgotten long ago due to the considerable advantage ensured for him by his character and mind. He was ever more recognized as one of those rare men "whom nature needs centuries to create". And, what means even more, the memory of the deceased had not weakened in those who had been close to him, – it had been inherited by the new generations until the approach of the jubilee of his birth [1828] gave the occasion to prove it.

The outcome of the various discussions was that on the square in Mühlhausen, where the house where Lambert was born, is situated, will be known henceforth as Lambertsplatz; and a memorial pillar with Lambert's portrait and a suitable inscription will be erected and inaugurated during the jubilee. Owing to some local difficulties the celebration was postponed for a day, i. e., until 27 August 1828, and then took place in fine weather and many participants from far and near.

The compiled pieces about Lambert's life and work were later collected and published. Decorated by his expressive portrait and a picture of the monument, it became a nice festive present for friends and admirers of the great scientist. Regrettably, on the other hand, no action was taken concerning Joh. Kaspar Horner's stated opinion. Already on 14 December 1827, in a letter to Daniel Huber, he noted that

Instead of a wooden, stone or steel monument Lambert's countrymen should have erected an incomparably more durable paper monument, – a complete collection of all of his works which remain unknown, buried in the publications of the Berlin Academy and elsewhere. They would have found, as I think, so many donations and subscribers that a proper deposit for another monument could have been still possible.

And again, on 27 February 1828, after Huber had been asked to describe Lambert's scientific works and informed him about it and decribed the difficulties, that, according to the opinion of minister Graf, the founder and manager of the Lambert Society, will hinder Horner's plan:

I am very glad that you are asked to dignify Lambert's scientific merits. That request could not have been addressed to any more capable hands. Indeed, not only knowledge but thoroughness, zeal and fervour of the older generation are also needed. A younger man could have accomplished that business easier and less carefully and it would have been really sorrowful for the present epoch to celebrate that man worthy of glory without perfectly illuminating his merits.

Our new transcendental mathematicians, although possibly surpassing Lambert in mechanical, analytical skill, do not at all possess similar philosophical minds.

I would still like to insist on my idea that a complete collection of all of Lambert's writings would have been his best, most general and lasting memorial. I think that your friend, the minister Graf, imagines *too serious difficulties. My idea does not demand anything except an unhurried reprint of the separate pieces in a chronological order so that the publisher will be able to give them to his typesetter as though on the side. It would be possible to publish in 12 – 18 volumes the separate works* [1758/5; 1759/7; 1760/9; 1779/66; 1761/10 and 11; 1764/15; 1765 – 1772/17; 1771/37], *the correspondence with* [Daniel I] *Bernoulli and the numerous memoirs now lying in the scientific cemetery of the Berlin Academy.*

The expenses will be partly covered by Lambert's countrymen and partly by subscriptions in Germany, France, England and Italy. No annoucements or comments are needed at all, and corrections will present the only difficulties. If this plan will be found too extensive, and much of what was written by Lambert not anymore up to date, I would recommend a collection of his mathematical works. After all, all his pertinent publications are rarities. A few years ago I had to send Plana in Turin what I could get hold of from Lambert's works. He is a great admirer of Lambert and argues that Laplace, in his theory of Jupiter and Saturn, had borrowed much from Lambert without naming him.

Like all my ideas, my proposal is not at all authoritative and should not be opposed to any prepared decision.

As noted above, Horner's proposal did not regrettably come true and nowadays it has a slim chance to be realized. Future will decide whether this can happen at the time of Lambert's second jubilee but we must not doubt that when that festive day happens, the generations to come will also honour Lambert, as Eberhard concluded in his speech at the jubilee: "His monument is in your midst because he insistently strove to the great goal, to promote truth and science"[46].

Notes

1. Connected with the Swiss Confederacy by a system of treatises. O. S.

2. In addition to Huber (1829a) and Formey [viii] I am partly issuing from other biographies, Lambert's publications and the correspondence of Haller (1846), Jetzler, Horner, Huber et al. One of Lambert's countrymen scientists is mentioned on p. 238 of this collection, another one was Peter Witz born in 1766, who was an assistant teacher in Mühlhausen, then […] a minister […]. He became known because of his widely applied treatise on calculations (1808). R. W.

3. In a letter of 6 December 1750 from Chur to minister Rißler in Mühlhausen, see Johann III Bernoulli (1782 – 1784). On this occasion it is possible to mention that the Basel Library is keeping that copy of the correspondence which Johann II Bernoulli received from his son, the Editor, Johann III […]. R. W.

4. Lambert wrongly named that treatise, see Bibliography. O. S.

5. Among other pursuits, Lambert constructed there a 15-foot gnomon and determined the height of the pole [the latitude] of Chur, – 46°50'. In his biography of Lambert appended to No. 3 of his *Nouvelles littéraires de Goettingue*, Johann III Bernoulli reports:

It would be perhaps said that probably he had no instruments in Switzerland, but he had a unique possibility in him himself. For his experiments he always applied the simplest means, the most meagre in appearance instruments which as a rule he produced himself. His feeling was so sure and his mind so sensible that almost always he benefited as much as others did with large and very expensive instruments. However, he was unable to get rid of that habit which degenerated into inedequacy: he had been keeping to it even when being an academician and living in such a city as Berlin. In many instances he could have easily attain perfection impossible in spite of all his talent when only applying simple means.

Later Lambert published his meteorological observations in the *Acta Helvetica*, the periodical of the Swiss scientific society that admitted him in 1753. Then, in 1755, he sent his forst work [1755/2] for publication [in the same *Acta*]. R. W.

The latitude of Chur, as shown on the map of Switzerland, is about 46°45'. O. S.

6. *Pandects* is a complete body of Roman law. O. S.

7. In Utrecht Lambert almost perished. Because of bad breath he always stood sideways to his interlocutor. Graf (1829) describes that episode:

After going out from a room, he went back a few steps from his friend accompanying him without noticing the staircase behind him and tumbled down to its foot. He seriously injured his head and his eyes became bloodshot. He completely lost conscience and only regained it after 24 hours. When opening his eyes, he did not wish to believe the physician who testified the duration of his previous state. It was Friday, but he stated that it was still Thursday. Considerable time had to pass before he completely recovered. His physician, the famous Professor Hahn, wished to forbid him any mental work for a few years.

8. Tempelhof edited a German edition (Berlin 1773) of that contribution, where the terrestrial refraction was somewhat neglected. R. W.

9. In 1753 **–** 1770 G. A. Baron Münchhausen was prime minister in the Electorate Brunswick – Hanover which included Göttingen. O. S.

10. Heinrich (1807) indicated that Lambert was the first who dared make such investigations. R. W.

In this letter Lambert mentions his future *Photometria* [1760/9] and as though already written *Pyrometrie* [1779/66] that he actually only concluded at the very end of his life. A line (ligne) is 1/12 of an inch. O. S.

11. The influence of the Moon on various meteorological elements had been studied from the beginning of the 18th century and acknowledged until mid-19th century (Sheynin 1984, pp. 56 – 62). Much later and elsewhere Lambert published another paper on the same subject [1773/42].

Lambert and Daniel Bernoulli discussed the phenomenon mentioned above and (Radelet-De Grave et al 1979) on 13 June 1759 the latter expressed his opinion (p. 62): if the influence of the Moon on the "air" is similar to its influence on the seas, it should be observable because the Moon's distance varies. The "elasticity" of air and its weak inertia should be, however, allowed for. And further:

Your considerations […] *are quite justified; publish them without hesitating* […] *whatever are the results* […] *only try to establish them properly.* O. S.

12. The German edition of 1774 of that contribution is also interesting because of historical comments, see Kästner (1796 – 1800, Bd. 2, pp. 3 – 7). Lambert also published rules for drawing perspectives [1768/24]. R. W.

13. This prompted Lambert to compile his books [1761/12; 1769/27] and the paper [1768/25] as well as to enter into an extensive correspondence with Brander that comprises volume 3 of Johann III Bernoulli (1782 – 1784). R. W.

14. Note, however, that Protestants, except those belonging to the Chuch of England, do not have any obligatory fasts. In any case, perhaps *until Easter*. O. S.

15. Bouguer (1729) had early made known his first ideas and his more comprehensive contribution appeared much later and posthumously (1760). R. W.

In 1726 and 1757 he published two other pertinent writings. O. S.

16. The distance to Sirius (certainly unknown to Lambert) is 8.8 light years; multiplied by 150, it becomes 1320 light years. O. S.

17. In the somewhat Frenchified or Volterian Berlin of that time his diligent attendance at services and communions stood out the more so since such an attitude was not widespread, especially not among philosophers. R. W.

18. However, Struve (pp. 17 – 18) also criticized Lambert. O. S.

19. Cf. the discussion of Lambert's philosophical works below. R. W.

20. This indication is not sufficient: there were two Italian scientists of that name, see *Information* at the end of this biography. O. S.

21. The exact name of that Academy is seen in the bibliographic information about Lambert's contributions, e. g., [1763/13 and 14]. O. S.

22. Bierman (1988, p. 94) quoted Euler's letter (although without a complete reference) who had thought that Lambert's relations with that Academy worsened because of religious discord between the Swiss Protestant and Bavarian Jesuits. O. S.

23. That learned lady certainly lived in Chur for a long time. I was regrettably unable to find out anything about her […]. R. W.

Wolf's reasoning above is unconvincing since Lambert later published one more memoir in that source [1768/25]. O. S.

24. More correctly Drei Bünde, an independent state situated in the contemporary canton Graubünden (Grisons). O. S.

25. Its Latin translation made by Pfleiderer for an Italian scholar became owned by Lord Stanhope, a great admirer of Lambert. It apparently was not published. R. W.

26. Lambert had also published the *Architektonik* [1771/37] that prompted both Trembley to compile his book (1780) and Johann III Bernoulli with the assistance of Professor Chr. H. Müller to publish a collection (1782) of Lambert's posthumous manuscripts. R. W.

28. Cf. § 6 (opinion of Usteri) and § 7. R. W.

29. Apparently Trembley, see Note 26 and Wolf (1859, p. 264). R. W.

30. Graf (1829) described the audience in the following way.

The King: Good evening, dear sir. Give me pleasure, please tell me which sciences are you especially learned in.
Lambert: All.
The King: And you are also a skilled mathematician?
Lambert: Yes.
The King: And who had taught you mathematics?
Lambert: I myself.
The King: So you are the second Pascal?
Lambert: Yes, Your Majesty.

At that, the King turned around, since he was hardly able to refrain from laughing and went away to his study. At dinner the monarch remarked that the greatest fool that he ever saw was proposed to him for his Academy.

In a serious article about Lambert by Servois (*Biographie* 1811 – 1828, t. 23, pp. 265 – 274) most of the similar conversations concerning Lambert and a great part of the anecdotes about him are nevertheless fabricated. R. W.

31. Concerning D'Alembert cf. p. 324 of this collection. Later he came to respect Lambert duly and mentioned him to Friedrich very approvingly. R. W.

32. Lambert published three relevant papers [1766/19; 1767/20; 1775/52]. R. W.

33. Johann III Bernoulli, the son of Johann II, whose name ought to be mentioned very often in Lambert's biographies, was born 4 November 1744 in Basel. Partly under his father's guidance he made such rapid progress that already in 1757 became a laureate [a bachelor] on which occasion he delivered an *Oratiuncula* on the inoculation of smallpox which is included in one of Haller's letters from vol. 4 of his correspondence. The speech was occasioned by the favourable inoculation of him himself and his two younger brothers. Then he spent a year in Neuenburg [Neuchâtel] (see p. 162 of this collection), in 1758 received the degree of master, took to the law according to his father's wish and earned a doctor's degree in the same year (in 1763).

In addition, he studied mathematical sciences under the guidance of his father and uncle [Daniel Bernoulli] and was so successful that Friedrich II invited him to Berlin and on 7 January 1764 appointed him member of the mathematical class of the Berlin Academy. After Huber's departure (Wolf 1858, pp. 442 – 445) he should have activated the orphaned observatory; however, in 1767 he moved too soon in a newly built [and cold] room, became sickly and spoiled his hearing. In spite of many holidays spent in southern regions attempting to recover there, he did not sufficiently recuperate.

The more his practical [astronomical] activity became hindered, the eagerly he worked as a writer, The British Commissioner of the Longitude rewarded his tables (1779). Some of his numerous publications in the *Mémoires* of the Berlin Academy and the Berlin astronomical yearbooks are also of eminent merit as are also his writings (1771; 1771 – 1779; 1777 – 1779) and his various travel notes (Reisewerke) and Sammelwerke [?] are also very useful and provided me many valuable notes for my contributions (1859 – 1860). Later Bernoulli became Director of the mathematical class of the Academy and died while holding that position on 13 July 1807 in Köpenik near Berlin.

For this Note that can be supplemented by Notes 3, 26 and 45 I have partly issued from Merian (1860). That writing contains much which can be interesting for extending and correcting my biographies of the Bernoullis and for the history of mathematical sciences. Thus, we see that the mathematical correspondence of Nikolaus I is being kept at the Basel Library, and find out that the just claims of the Bernoulli family during the appointment of the chair of mathematics in Basel in 1748 were considered to a certain degree more than I had thought [v, § 10]. It contains additions to Leibniz' letters published by Gerhardt (1849 – 1859) etc. R. W.

I have omitted much when translating [v, § 10]. There, in particular, on p. 162 mentioned in this Note by Wolf, he reports his discovery, in the papers of the Bern Economic Society, that Johann III was one of the six foreign scientists whom they had sent a barometer, a thermometer and a rain-gauge. O. S.

34. Michajlov (1957) and Bierman (1988) described the relations between Euler and Lambert. At first they respectfully referred to each other; sharp discord between them arose in connection with the work of the commission investigating the economic activities of the Academy and especially because Euler had been unsatisfied with his position at the Academy. He was the effective president but had too little rights, whereas the commision infringed upon him. Lambert, as can be understood, acted in that commision objectively and in any case did not at all wish to drive Euler out, but (Bierman, p. 101) he "finally ended the Berlin period of Euler's life". Note also that in one of his memoirs appearing during the time of his death, Euler (1783) in the very first lines called Lambert "most acute and talented". Bierman only referred to Wolf (and attempted to refute him) whereas Johann III Bernoulli (whom he did not mention) was apparently better acquainted with the consequences of the commision's psychological pressure on Euler. I did not regretfully see Youshkevitch (1979) who had undoubtedly dwelt on that episode. O. S.

35. Cf. end of Note 30. R. W.

Speiser (2008, pp. 237 – 238) described other similar episodes also as reported by Thiébault. O. S.
36. The series named after Lambert is (Fichtenholz 1951/1974, p. 323)

$$\sum_{n=1}^{\infty} a^n \frac{x^n}{1-x^n}. \text{ O. S.}$$

37. Tetragonometry, as Lambert called it, was the solution of quadrilaterals, a term similar to trigonometry one of whose aims is the solution of triangles. Sighting (for determining the amount of wine in barrels) should have meant measurement by means of a *Visierstab*, a measuring rode. O. S.
38. As though supplementing geography, chorography deals with smaller territories, for example provinces. O. S.
39. Cf. that important memoir [1768/22] with Horner's memoir in Gehler (1787 – 1795/1831 – 1837, Bd. 6, pp. 746 – 756 and 820 – 822). R. W.
40. Cf. Wolf (1859, pp. 188 – 192). R. W.
41. Schulze (1778) borrowed many tables from the new edition of that source. R. W.
42. See Lalande, p. 551. Wolf had also quoted Lalande in his § 6, but that source contains much more. Here are his statements about other publications of Lambert.
About [1758/5]: "excellent work" (p. 465).
About [1765 – 1772/17, Bd. 1]: "contains interesting remarks" (p. 491).
About [1770/29]: "It merits to be translated [into French] as are the other German works of that skilled astronomer" (p. 508). He mentions [1765 – 1772/17, Bde 1 and 2].
About [1765 – 1772/17, Bd. 2]: "excellent use of mathematics" (p. 515).
About many "interesting" memoirs of Bernoulli and Lambert in the Berlin astronomical yearbook of 1774 for 1776: "from this time, [French] astronomers ought to study German" (p. 539). O. S.
43. The author apparently also had in mind the so-called method of eye and ear applied before the invention of the chronograph. The observer noted the time as shown by his table-chronometer, continued to count out silently the seconds as measured by its ticking (which Johann III was apparently unable to hear) and noted the moment that the chosen star passed the crosshairs of the eyepiece of his instrument. O. S.
44. There are about 50 of them covering all the conceivable branches of astronomy. I certainly cannot say much about them. The Ephemerides for 1775 were connected with Lambert's chart of the Moon [1774/50] and mostly based on his observations of our satellite. R. W.
45. Concerning Lambert's German correspondence with Holland, Brauder, Kant et al see Note 3 and about the regrettably mostly lost French correspondence, p. 195 of this volume. However, I hope to return to that later in my next volume [that appeared in 1862] in a contribution about Lesage. Here, I only add that Joh. III Bernoulli stated, in the Introduction to vol. 1 of Lambert's German correspondence (1782):

In a published notice of Lambert's posthumous papers that had become sufficiently known, I have already indicated that the lot directed me to find them after the Academy of Sciences in this city had bought them from the heirs of the deceased.

Nevertheless, during my stay in Berlin in 1847, in spite of the kindness of the librarian Friedländer and Privy Councillor Ulrici, I was unable to find that "sufficiently known" notice not to mention the manuscripts themselves the existence of which I might have suspected because of Dan. Bernoulli's earlier letters.
The Akte of the Academy, as Ulrici asserted, do not contain a single word about such a purchase. Some of Lambert's manuscripts that I had an opportunity of seeing out of the goodness of Director Encke from the Berlin Observatory certainly are of a lesser value and do not provide even a tiniest hint about the fate of the other manuscripts. R. W.
46. Until now, Lambert's "paper monument" consists of contributions [70 – 72]. Also see (the now dated) statement about the same subject by Jaquel (1969). O. S.

Information about Some Scientists and Others Mentioned by Formey and Wolf

Argelander F. W. A., 1794 – 1875, astronomer
Bode J. F., 1747 – 1826, astronomer
Boerhaave H., 1668 – 1738, physician
Bohnenberger J.-G. F., 1765 – 1831, astronomer
Bonnet Ch., 1720 – 1793, natural scientist
Bossuet J.-B., 1627 – 1704, French bishop, theologian
Brandes H. W., 1777 – 1834, astronomer and natural scientist
Buegelin N. de, 1714 – 1789, mathematician
Cassini G. D., 1625 – 1712, astronomer
Cassini J., 1677 – 1756, astronomer
Chodowiecki W., 1765 – 1805, illustrator
Dryden J., 1631 – 1700, poet
Encke J. F., 1791 – 1865, astronomer
Erhardt S., philosopher
Erman P., 1764 – 1851, physicist
Fléchier E., 1632 – 1710, historian, writer
Fontana F., 1730 – 1805, physicist
Fontana G., 1735 – 1803, mathematician
Formey J.-A.-S., see [v]
Geßner J., 1709 – 1790, botanist and astronomer
Haller A. von, 1708 – 1777, physician, botanist and poet
Heidegger K., 1710 – 1778, mayor of Zürich
Hirzel H. K., see [v]
Horner J. K., 1774 – 1834, natural scientist
Huber D., 1768 – 1829, astronomer
Jeanneret, see [v]
Jetzler C., see [v]
Karsten W. J. G., mathematician
Kästner A. G., see [v]
Lafontaine J., 1621 – 1695, poet and writer of fables
Lavater J.K., 1741 – 1801, writer. Connected the spiritual cast of mind with the outline of face and skull
Lesage G.-L., 1724 – 1803, physicist
Lindenau B. A., 1780 – 1854, astronomer
Malherbe F., 1555 – 1628, poet
Mallet, see [v]
Mendelsohn M., 1729 – 1786, philosopher. Wolf wrongly mentioned him disdainfully
Merian J. B., 1723 – 1807, philosopher
Messier Ch., 1730 – 1817, astronomer
Montaigne M., 1533 – 1592, philosopher and writer
Müller Ch. H., 1740 – 1807, historian and philosopher
Musschenbrock P. van, 1692 – 1761, physicist
Osterwald J.-F., 1773 – 1850, theologian
Ozanam J., 1640 – 1717, mathematician
Pfleiderer Ch. F., 1736 – 1821, mathematician and physicist
Phaedrus, ca. 15 BC – ca. 70, writer of fables
Plana G., astronomer
Planta M., 1727 – 1772, physicist
Pope A., 1688 – 1744, poet
Prevost P., 1751 – 1839, physicist and philosopher
Rumford (Sir Benjamin Thompson, Count Rumford), 1753 – 1814, physicist
Scarron P., 1610 – 1660, writer, founder of parodying poetry
Schulze J. K., 1749 – 1790, astronomer
Schumacher H. C., 1780 – 1850, astronomer
Segner J. A., 1704 – 1777, physicist and mathematician
Short J., 1710 – 1768, mechanician and astronomer
Stanhope C., 1753 – 1816, politician and scientist
Sulzer J. G., 1720 – 1779, philosopher

Trembley J., see [v]
Turenne H., 1611 – 1675, viscount and military leader
Wilde H. E., 1793 – 1859, philosopher, mathematician and physicist
Zach F. X., 1754 – 1822, astronomer

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X

J. H. Lambert

List of Publications

Checked against Steck M. (1943), *Bibliographia Lambertiana*. Hildesheim, 1970

Abbreviation
AH = *Acta Helvetica*
Berlin Mém. = *Mém. Acad. Roy. Sci. et Belles-Lettres Berlin*

1. Dialogus Socraticus. In *Zürcherische Sammlung gelehrten Schriften*, 1754.

2. Tentamen de vi caloris, qua corpora dilatat ejusque dimensione. AH, vol. 2, 1755, pp. 172 – 242.

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70. *Monatsbuch*. München, 1915. Editor K. Bopp.
71. *Opera mathematica*, Bde. 1 – 2. Zürich, 1946 – 1948.
72. *Philosophische Schriften*, Bde 1 – 9. Hildesheim, 1965 – 2007.

Speiser (2008, p. 251) mentioned the following contributions although without providing the sources

Mémoires sur la partie photométrique de tant d'art de peindre, 1768.
Sur la perspective aérienne, 1774.
Anmerkungen über Strahlenbrechung, 1779.
Von Glanze der Venus, 1780.

XI

Oscar Sheynin

Supplement: Daniel Bernoulli's Instructions for Meteorological Stations

Tikhomirov (1932) discovered instructions written by Daniel Bernoulli in 1733. Here is the English abstract of his Russian paper:

A rough copy of an instruction for geophysical observations (meteorology, terrestrial magnetism, gravimetry, hydrology, etc.) compiled in 1733 by […] *Daniel Bernoulli has been discovered by the author in the archives of the Central Geophysical Observatory. This instruction was intended for the members of the Great North Expedition* [the Second Kamchatka Expedition of Vitus Bering] *leaving for Siberia.*

The original is revised [?] *in Latin, but a translation of it into contemporary Russian has also been found by the author in the archives of the Conference of the Academy of Sciences* [that is, in the *Protokoly* (1897 – 1911)]. *This archaic text is inserted in the present paper in extenso. Beside that observer handbooks compiled by the members of the expedition J. G. Gmelin and L. de la Croyère intended for meteorological stations organized in Tomsk, Yenisseysk, Irkoutsk and Touroukhansk have also been found in the archives of the Conference* […].

The paper gives an analysis of all these instructions. The present research as well as a series of previous ones referring to the history of Russian meteorology in the 18th century led the author to the conclusion that a network of regularly functioning meteorological stations existed in Siberia in the 1730s and 1740s whose life history was intimately interrelated with the fortune of the Great North Expedition of 1734 – 1743. A part of observations effected at these stations was secretly removed to France by the Academician I. N. [*J. N.*] *Delisle and later printed in P. Cotte's* Traité de meteorology*. Paris, 1774.*

The instructions themselves rather than their copy were kept in the Archive of the USSR Academy of Sciences (Gnucheva 1940, p. 46)

Fond 3, Inventory 1, No. 2331, lists [sheets] 76 – 78, 81 – 84, 159 – 160.

Tikhomirov published that Russian translation but at least for me its text is too difficult to understand. He also found general instructions compiled from instructions written by Delisle, Gmelin and Miller (Müller). It consists of 45 sections the last 20 of which are "mostly a brief version" of the instructions written by Bernoulli. Finally, he also discovered four instructions written for ordinary observers rather than for qualified personnel as those he mentioned previously, including that written by Bernoulli.

The first volume of the *Protokoly* (1897 – 1911) contains a related passage (p. 61, 10 October 1732, also quoted by Tikhomirov):

Daniel Bernoulli proposuit instructionem pro itinere Kamtschatkensi, et novom methodum observandi differentias altitudinum in barometro.

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